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# OPTIMAL DESIGN OF TRUSS STRUCTURES WITH STRESS AND DISPLACEMENT CONSTRAINTS USING A HYBRID ALGORITHM

*Carlos Millan[1], Jair Arrieta[2], Jose R. Hernandez Avila[3]*

[1,3]Faculty of Engineering, Universidad de Sucre, Sincelejo, Colombia

[2]Civil Engineering Department, Universidad de Cartagena, Cartagena de Indias, Colombia

*Corresponding author: [1]carlos.millan@unisucre.edu.co

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## ABSTRACT

In this article, the hybrid algorithm called SAwPSO (Simulated Annealing with Particle Swarm Optimization) is proposed to optimize truss structures with stress and displacement constraints. SAwPSO works as follows: SA selects the PSO parameters and then PSO optimizes the problem. This allows the designer to not need to tune the PSO parameters to solve the problem. The reliability of the SAwPSO is demonstrated through three optimization problems of truss structures with continue design variables. Numerical results indicate that SAwPSO can minimize the overall weight of truss structures subjected to stress and displacement constraints.

## INTRODUCTION

The stresses and displacements of a truss structure are essential characteristics to determine the behaviour of the structure. In design practice, this allows establishing the kind of work that can be carried out, as well as the quantity and kind of material used for its elaboration. In such situations, the optimal design of structures is of great practical importance because it provides an effective way to control and manipulate the characteristics of a structure, and thus improve its performance.

On the other hand, researchers have given considerable attention to develop efficient optimization algorithms to solve structural optimization problems. However, most of the optimization metaheuristics ((Arora, 1989; Dorigo, Maniezzo, & Colorni, 1996; Erol & Eksin, 2006; Geem, Kim, & Loganathan, 2001; Holland, 1975; Kaveh & Farhoudi, 2013; Kaveh & Khayatazad, 2012; Kaveh & Talatahari, 2010; Kennedy & Eberhart, 1995; Kirkpatrick, Gelatt, & Vecchi, 1983; Yang, 2010), among others) present parameters that must be adjusted to solve a certain type of problem. This makes users spend time looking for the optimal parameters to solve the problem and most of the time they find results that are not globally optimal.

To solve this inconvenience in this work the hybrid algorithm SAwPSO is proposed and applied to size optimization problems of truss structures with stress and displacement constraints. The validity of SAwPSO is confirmed by testing for three size optimization problems of truss structures. The remainder of this article is structured as follows. Section 2 describes the mathematical formulation of truss optimization. The SAwPSO is briefly presented in Sect. 3. Section 4 presents three benchmark numerical examples to illustrate the efficiency of the SAwPSO. Finally, in Sect. 5, our conclusions are presented

## PROBLEM DEFINITION

The objective is to minimize the weight of the structure while satisfying some stress and displacement limitations. The mathematical formulation of these problems can be expressed as follows:

$$\begin{aligned} &\text{Minimize:} && W(A) = \sum_{j=1}^{n} A_j L_j \rho_j \\ &\text{Subject to:} && \begin{cases} \delta_{\min} \leq \delta_j \leq \delta_{\max} \quad , \quad j = 1,2,\dots m \\ \sigma_{\min} \leq \sigma_j \leq \sigma_{\max} \quad , \quad j = 1,2,\dots n \end{cases} \end{aligned} \tag{1}$$

where $A_j$ is the cross-sectional area of element j; $L_j$ is the length of element j; $\rho_j$ is the material density of element j; W(A) is the total weight of the truss; n is the total number of elements. The vector A represents the cross-section vector of the element that can be selected from a set of discrete or continuous variables. $\delta_j$ is the displacement of node j, m is the number of nodes; $\sigma_j$ is the tension (tension/compression) that occurs in element j. The “min” and “max” subscripts are the minimum and maximum values that the restrictions can reach.

### *Sawpso Algorithm*

The basis of the hybrid algorithm is very simple. Each algorithm is working separately, each one evaluating a different function. In the SAwPSO, the SA selects the initial parameters of the PSO ($c_1$, $c_2$, N, α), and the PSO is evaluating the objective function (weight optimization of truss structures). In this way, both algorithms perform the optimization work until it meets the established stopping criterion. Figure 1 shows the SAwPSO flowchart.

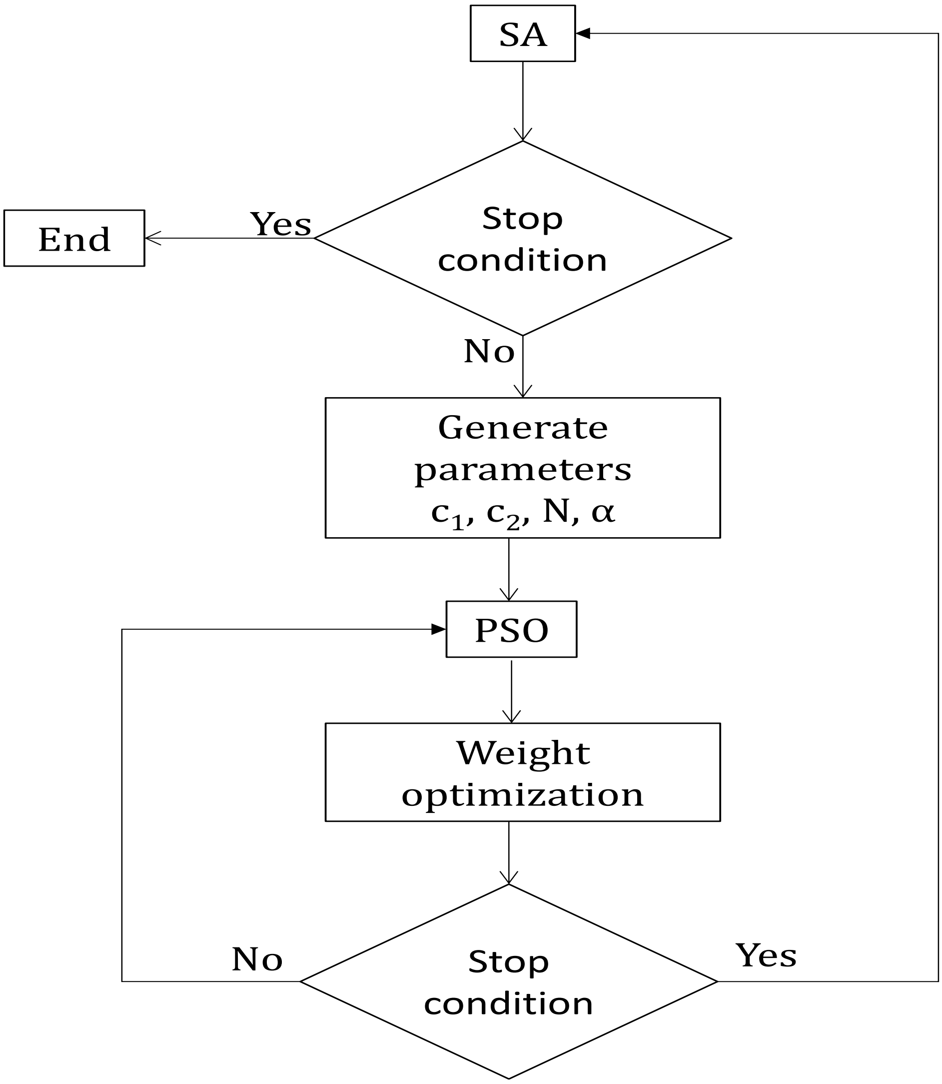

**Figure 1.** The SAwPSO flowchart.

## TRUSS PROBLEMS AND DISCUSSIONS

To evaluate the feasibility and validity of the SAwPSO, the following three classical truss sizing problems are optimized, and the results are compared with the previous results obtained through various existing metaheuristics: (1) 10-bar planar truss; (2) 25-bar spatial truss, and (3) 72-bar spatial truss. 50 runs were executed for each problem. The results in the tables are in terms of minimum weight, mean, standard deviation (SD) and number of iterations (NI).

### *10-bar planar truss*

The 10-bar planar truss (Figure 2) is a common problem in the field of structural optimization, being widely used to verify the efficiency of a new proposed optimization algorithm (KAVEH et al. 2015). There are 10 design variables in this example, and they can be selected from 0.1 to 35.0 in$^2$. The density of the material is 0.1 lb/in$^3$ and the modulus of elasticity is 10000 ksi. Offsets of free nodes must not exceed ±2 in in both vertical and horizontal directions. In addition, the allowable stresses, both tensile and compressive, must not exceed 25 ksi.

The results show (Table 1) that the optimal design weight obtained with SAwPSO (5060.88 lb) is less than other methods (5062.39 lb for EHS, 5061.42 for SAHS, 5086.90 for MCSS, 5064.60 for IMCSS, 5063.58 for NFR and 5065.99 lb for NCO). Also, SAwPSO requires less NI than HS, EHS, TLBO, MCSS, IMCSS, NFR and NCO (15000 for SAwPSO, 20000 iterations for HS, 16872 for TLBO and 62950 for NFR) to converge to the optimal solution. In terms of solution stability, SAwPSO is more stable than EHS, SAHS and TLBO with the lowest DP (0.18 lb for SAwPSO, 1.98 lb for EHS, 0.71 lb for SAHS and 0.79 lb for TLBO). Figure 3 presents the box plot for this problem.

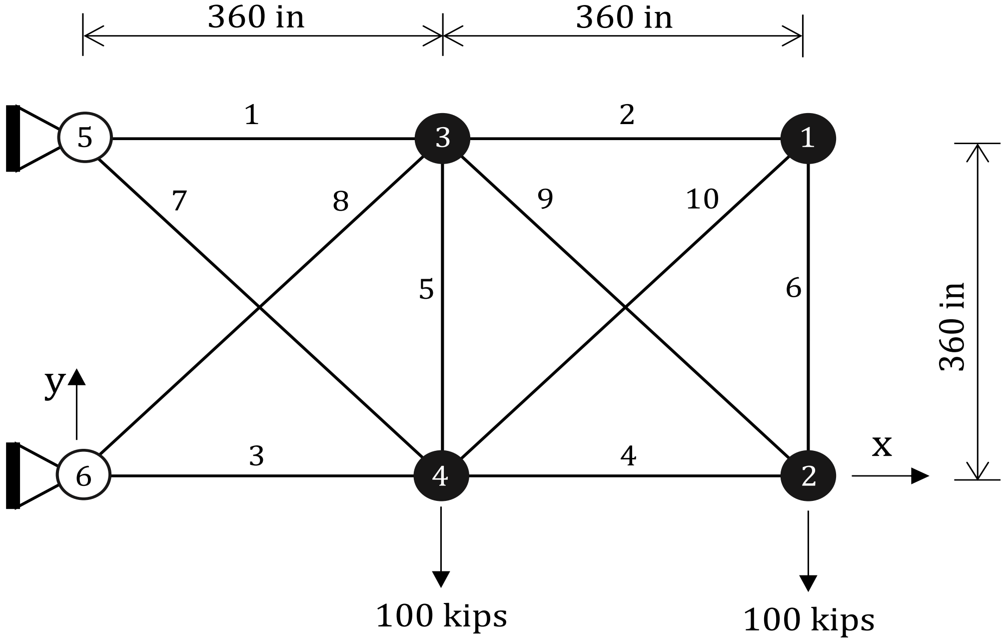

**Figure 2.** Schematic of the 10-bar planar truss.

**Table 1.** Optimal design parameters for the 10-bar planar truss by different algorithms

| **Variables ($in^2$)** | **Lee & Geem (2004)** | **Degertekin (2012)** | | **Degertekin & Hayalioglu (2013)** | **Kaveh et al. (2015)** | | **Moez et al. (2016)** | **Vezvari et al. (2018)** | **SAwPSO** |
|---|---|---|---|---|---|---|---|---|---|
| | **HS** | **EHS** | **SAHS** | **TLBO** | **MCSS** | **IMCSS** | **NFR** | **NCO** | |
| $A_1$ | 30.150 | 30.208 | 30.394 | 30.429 | 29.577 | 30.026 | 30.6206 | 31.1567 | 30.443 |
| $A_2$ | 0.102 | 0.100 | 0.100 | 0.100 | 0.114 | 0.100 | 0.1058 | 0.1004 | 0.100 |
| $A_3$ | 22.710 | 22.698 | 23.098 | 23.244 | 23.806 | 23.628 | 23.1368 | 22.3469 | 23.149 |
| $A_4$ | 15.270 | 15.275 | 15.491 | 15.368 | 15.888 | 15.973 | 15.3435 | 14.9622 | 15.227 |
| $A_5$ | 0.102 | 0.100 | 0.100 | 0.100 | 0.114 | 0.100 | 0.1017 | 0.1011 | 0.100 |
| $A_6$ | 0.544 | 0.529 | 0.529 | 0.575 | 0.100 | 0.517 | 0.5517 | 0.4386 | 0.546 |
| $A_7$ | 7.541 | 7.558 | 7.488 | 7.440 | 8.605 | 7.457 | 7.5205 | 7.6323 | 7.467 |
| $A_8$ | 21.560 | 21.559 | 21.189 | 20.967 | 21.682 | 21.437 | 21.0745 | 21.6152 | 21.109 |
| $A_9$ | 21.450 | 21.491 | 21.342 | 21.533 | 20.303 | 20.744 | 21.3645 | 21.2733 | 21.539 |
| $A_{10}$ | 0.100 | 0.100 | 0.100 | 0.100 | 0.112 | 0.100 | 0.1 | 0.1 | 0.100 |

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| Weight (lb) | 5057.88 | 5062.39 | 5061.42 | 5060.96 | 5086.90 | 5064.60 | 5063.58 | 5065.99 | 5060.88 |
| Mean (lb) | – | 5063.73 | 5061.95 | 5062.08 | – | – | – | – | 5061.08 |
| SD (lb) | – | 1.98 | 0.71 | 0.79 | – | – | – | – | 0.18 |
| NI | 20000 | 9791 | 7081 | 16872 | 8875 | 8475 | 62950 | 8400 | 15000 |

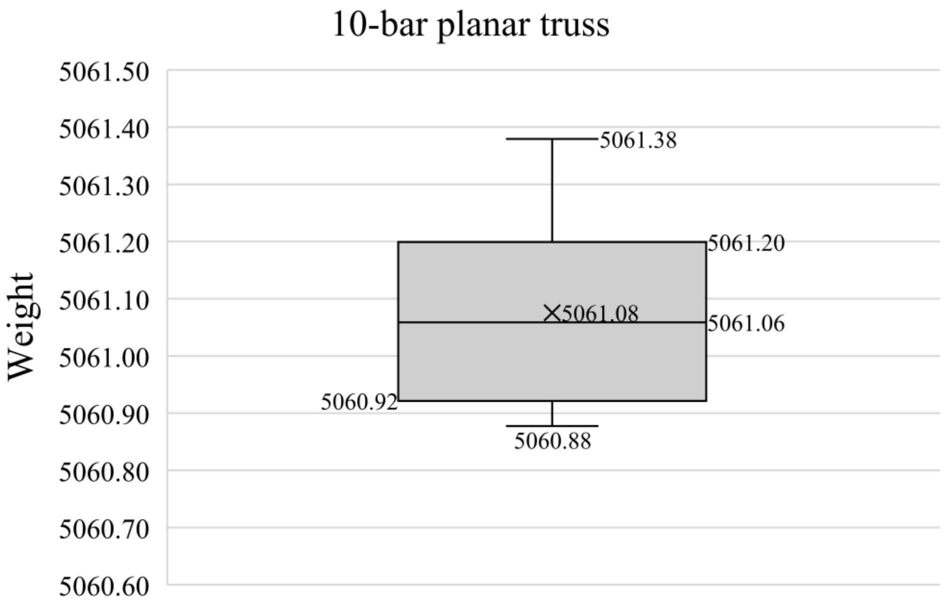

**Figure 5.** Box plot for the 10-bar planar truss.

### *4.2 25-bar spatial truss*

Figure 4 shows the geometry of the structure to be analyzed. The range of cross-sectional areas is 0.01 to 3.4 in$^2$, The material density is 0.1 lb/in$^3$ and the elastic modulus is 10000 ksi. The structure is subject to the two independent load conditions listed in Table 2. Free node offsets must not exceed ±0.35 in in all directions. Because of structural symmetry, the bars are divided into eight groups and the allowable stress values for all groups are listed in Table 3.

The results obtained by SAwPSO and other optimization metaheuristics are compared in Table 4. SAwPSO found an optimal design with a weight of 545.17 lb similar to those provided in the literature. However, the NI required by SAwPSO to obtain the optimal solution is the lowest among the metaheuristic algorithms compared in this study (19750 for WEO and 15000 for SAwPSO). The SD obtained with SAwPSO shows that it is more stable compared to the other metaheuristic algorithms. WEO achieved a lower SD than SAwPSO (0.08 for WEO and 0.55 SAwPSO), but at a considerably higher computational cost (19750 and NI versus 15000 NI). Figure 5 presents the box plot for this problem.

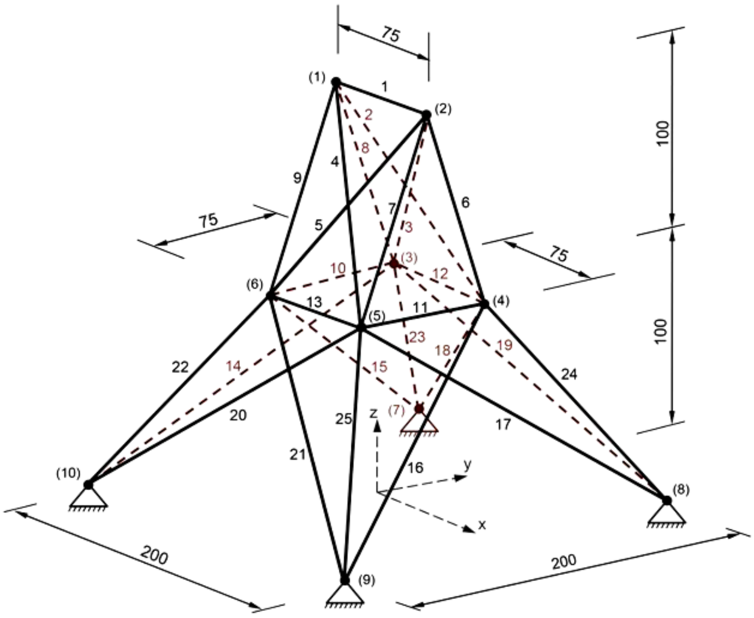

**Figure 4.** Schematic of the 25-bar spatial truss (all the dimensions are in).

**Table 2.** Two load cases for 25-bar spatial truss

| **Node** | **Case 1 (kips)** | | | **Case 2 (kips)** | | |
|---|---|---|---|---|---|---|
| | **Fx** | **Fy** | **Fz** | **Fx** | **Fy** | **Fz** |
| 1 | 0.0 | 20.0 | -5.0 | 1.0 | 10.0 | -5.0 |
| 2 | 0.0 | -20.0 | -5.0 | 0.0 | 10.0 | -5.0 |
| 3 | 0.0 | 0.0 | 0.0 | 0.5 | 0.0 | 0.0 |
| 6 | 0.0 | 0.0 | 0.0 | 0.5 | 0.0 | 0.0 |

**Table 3.** Bar grouping and stress levers

| **Member group ($in^2$)** | **Compressive stress limit (ksi)** | **Tensile stress limit (ksi)** |
|---|---|---|
| $A_1$ | 35.092 | 40.0 |
| $A_2$-$A_5$ | 11.590 | 40.0 |
| $A_6$-$A_9$ | 17.305 | 40.0 |
| $A_{10}$-$A_{11}$ | 35.092 | 40.0 |
| $A_{12}$-$A_{13}$ | 35.092 | 40.0 |
| $A_{14}$-$A_{17}$ | 6.759 | 40.0 |
| $A_{18}$-$A_{21}$ | 6.959 | 40.0 |
| $A_{22}$-$A_{25}$ | 11.082 | 40.0 |

**Table 4.** Optimal design parameters for the 25-bar space truss by different algorithms

| **Variables ($in^2$)** | **Kaveh & Bakhshpoori (2016)** | **Degertekin et al. (2017)** | **Jalili & Hosseinzadeh (2018)** | **SAwPSO** |
|---|---|---|---|---|
| | **WEO** | **HTS** | **BBO-DE** | |

| | | | | |
|---|---|---|---|---|
| $A_1$ | 0.0100 | 0.0100 | 0.0101 | 0.0100 |
| $A_2$-$A_5$ | 1.9184 | 2.0702 | 2.0256 | 1.9396 |
| $A_6$-$A_9$ | 3.0023 | 2.97003 | 3.0560 | 2.9966 |
| $A_{10}$-$A_{11}$ | 0.0100 | 0.0100 | 0.0100 | 0.0100 |
| $A_{12}$-$A_{13}$ | 0.0100 | 0.0100 | 0.0100 | 0.0100 |
| $A_{14}$-$A_{17}$ | 0.6827 | 0.6707 | 0.6839 | 0.6883 |
| $A_{18}$-$A_{21}$ | 1.6778 | 1.6171 | 1.6126 | 1.6778 |
| $A_{22}$-$A_{25}$ | 2.6612 | 2.6981 | 2.6602 | 2.6583 |
| Weight (lb) | 545.16 | 545.13 | 545.09 | 545.17 |
| Mean (lb) | 545.22 | 545.17 | 545.34 | 545.66 |
| SD (lb) | 0.08 | 0.48 | 0.36 | 0.55 |
| NI | 19750 | 7653 | 13600 | 15000 |

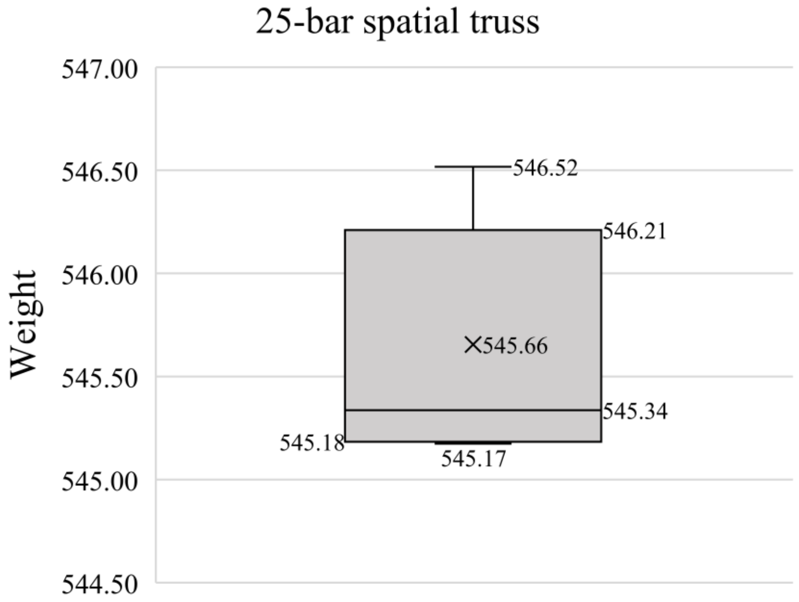

**Figure 5.** Box plot for the 25-bar spatial truss.

### *4.3 72-bar space truss*

In this problem (Figure 6) the modulus of elasticity is 10000 ksi and the material density of the truss members is 0.1 lb/in$^3$. The truss members are classified into 16 element groups because of structural symmetry: (1) A1-A4, (2) A5-A12, (3) A13-A16, (4) A17-A18, (5) A19- A22, (6) A23-A30, (7) A31-A34, (8) A35-A36, (9) A37-A40, (10) A41-A48, (11) A49-A52, (12) A53-A54, (13) A55-A58, (14) A59-A66, (15) A67-A70, (16) A71-A72. The structure is subject to the two independent load conditions listed in Table 5. Members are subject to the stress limitation of ± 25 ksi. In addition, the nodal displacements of all free nodes are limited to ±0.25 in. The maximum and minimum values allowed for the cross-sectional areas are 0.1 and 3.4 in$^2$, respectively.

The results of comparing SAwPSO with existing metaheuristic methods are presented in Table 6. It can be seen that the best design obtained by SAwPSO (379.70 lb) is similar to the metaheuristic algorithms considered in this study.

However, SAwPSO requires less NI to converge to the optimum (15000 NI for SawPSO, 15044 NI for EHS, 19778 NI for TLBO and 15600 NI for CBO). When examining Table 6 in terms of statistical results, it is observed that the values of mean weight and standard deviation (SD) obtained by SAwPSO are smaller than HBB-BC, EHS, SAHS, TLBO and HTS. Although the SAwPSO has a slightly higher standard deviation than the CBO method is more efficient than the latter in terms of computational effort. Figure 7 presents the box plot for this problem.

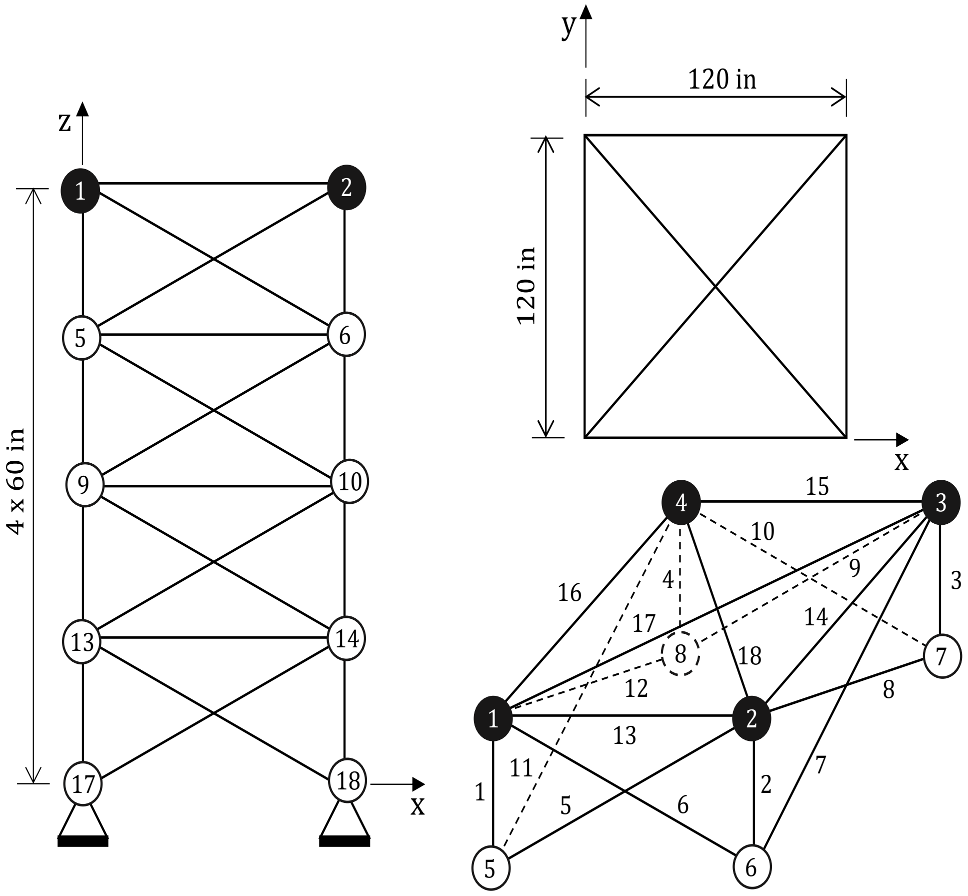

**Figure 6.** Schematic of the 72-bar spatial truss (all the dimensions are in).

**Table 5.** Two load cases for 72-bar spatial truss

| Node | Case 1 (kips) | | | Case 2 (kips) | | |
|---|---|---|---|---|---|---|
| | Fx | Fy | Fz | Fx | Fy | Fz |
| 1 | 5,0 | 5,0 | -5,0 | 0,0 | 0,0 | -5,0 |
| 2 | 0,0 | 0,0 | 0,0 | 0,0 | 0,0 | -5,0 |
| 3 | 0,0 | 0,0 | 0,0 | 0,0 | 0,0 | -5,0 |
| 4 | 0,0 | 0,0 | 0,0 | 0,0 | 0,0 | -5,0 |

**Table 6.** Optimal design parameters for the 72-bar space truss by different algorithms

| **Variables (in$^2$)** | **Kaveh & Talatahari (2009c)** | **Degertekin (2012)** | | **Degertekin & Hayalioglu (2013)** | **Kaveh & Mahdavi (2014b)** | **Degertekin et al. (2017)** | **Jalili & Hosseinzadeh (2018)** | **SAwPSO** |
|---|---|---|---|---|---|---|---|---|
| | **HBB-BC** | **EHS** | **SAHS** | **TLBO** | **CBO** | **HTS** | **BBO-DE** | |
| $A_1$-$A_4$ | 1.9042 | 1.967 | 1.860 | 1.9064 | 1.9028 | 1.9001 | 1.9018 | 1.8941 |
| $A_5$-$A_{12}$ | 0.5162 | 0.510 | 0.521 | 0.5061 | 0.5180 | 0.5131 | 0.5114 | 0.5175 |
| $A_{13}$-$A_{16}$ | 0.100 | 0.100 | 0.100 | 0.100 | 0.1001 | 0.1000 | 0.1000 | 0.1003 |
| $A_{17}$-$A_{18}$ | 0.100 | 0.100 | 0.100 | 0.100 | 0.1003 | 0.1000 | 0.1001 | 0.1000 |
| $A_{19}$-$A_{22}$ | 1.2582 | 1.293 | 1.293 | 1.2617 | 1.2787 | 1.2456 | 1.2766 | 1.3050 |
| $A_{23}$-$A_{30}$ | 0.5035 | 0.511 | 0.511 | 0.5111 | 0.5074 | 0.5080 | 0.5129 | 0.5041 |
| $A_{31}$-$A_{34}$ | 0.100 | 0.100 | 0.100 | 0.100 | 0.1003 | 0.1000 | 0.1000 | 0.1000 |
| $A_{35}$-$A_{36}$ | 0.100 | 0.100 | 0.100 | 0.100 | 0.1003 | 0.1000 | 0.1001 | 0.1000 |
| $A_{37}$-$A_{40}$ | 0.5178 | 0.499 | 0.499 | 0.5317 | 0.5240 | 0.5550 | 0.5178 | 0.5243 |
| $A_{41}$-$A_{48}$ | 0.5214 | 0.501 | 0.501 | 0.5159 | 0.5150 | 0.5227 | 0.5174 | 0.5192 |
| $A_{49}$-$A_{52}$ | 0.100 | 0.100 | 0.100 | 0.100 | 0.1002 | 0.1000 | 0.1000 | 0.1000 |
| $A_{53}$-$A_{54}$ | 0.1007 | 0.100 | 0.100 | 0.100 | 0.1015 | 0.1000 | 0.1000 | 0.1007 |
| $A_{55}$-$A_{58}$ | 0.1566 | 0.160 | 0.168 | 0.1562 | 0.1564 | 0.1566 | 0.1567 | 0.1567 |
| $A_{59}$-$A_{66}$ | 0.5421 | 0.522 | 0.584 | 0.5493 | 0.5494 | 0.5407 | 0.5428 | 0.5420 |
| $A_{67}$-$A_{70}$ | 0.4132 | 0.478 | 0.433 | 0.4097 | 0.4029 | 0.4084 | 0.4055 | 0.4021 |
| $A_{71}$-$A_{72}$ | 0.5756 | 0.591 | 0.520 | 0.5698 | 0.5504 | 0.5669 | 0.5711 | 0.5625 |
| Weight (lb) | 379.66 | 381.00 | 380.62 | 379.63 | 379.69 | 379.73 | 379.63 | 379.70 |
| Mean (lb) | 381.85 | 383.50 | 382.42 | 380.20 | 379.90 | 382.26 | 379.89 | 380.12 |
| SD (lb) | 1.20 | 1.92 | 1.38 | 0.41 | 0.08 | 1.94 | 0.18 | 0.22 |
| NI | 13200 | 15044 | 13742 | 19778 | 15600 | 13166 | 11600 | 15000 |

**Figure 7.** Box plot for the 72-bar spatial truss.

## CONCLUSION

In this work, the SawPSO algorithm was proposed for the optimization of truss structures. Three truss structures were approached to evaluate the performance of SAwPSO: 10-bar planar truss, 25-bar spatial truss and 72-bar spatial truss. The results showed that the SAwPSO algorithm finds competitive results when compared to those obtained with other optimization metaheuristics. In addition, the standard deviation values check the reliability of the algorithm proposed here.

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