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MATHEMATICAL THINKING SKILLS LEVEL AMONG INTERMEDIATE SCHOOL STUDENTS

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ABSTRACT

The current study aims to reveal the level of the second intermediate grade students' possession of mathematical thinking skills. The descriptive approach was used to achieve the objectives of the study. A test that measures mathematical thinking skills was prepared and applied to 500 female students in public schools in the city of Hail, Saudi Arabia. The results of the study showed that the participants had a low level of mathematical thinking skills, which the order of which was as follows: the skill of visual-spatial perception, followed by the skill of symbolic expression, and finally the skill of logical thinking. The study recommended the necessity of spreading awareness of the importance of mathematical thinking skills in teaching mathematics, adopting educational programs that contribute to the development of mathematical thinking skills among female students, and highlighting the importance of including mathematical thinking skills in the curriculum for those in charge of preparing mathematics curricula.

INTRODUCTION:

Mathematics is one of the basic subjects that are studied in the general education stage. It provides the learners with the ability to think, promoting accuracy in analysis, and clarity in reasoning (Alasmar, 2016). Mathematics is also a means of training in sound thinking methods, as we note that one of the goals of teaching this subject in different countries of the world is to develop thinking skills (Alamer, 2010).

Thinking is a process carried out by the mind that is reflected in the individual's performance, language, and view of things (Osman, 2015). Alhams (2019) indicated that mathematical thinking is one of the important types of thinking, where it increases the learner's ability to understand mathematics and as well as other subjects. Mathematical thinking helps connect the learner's ideas with shapes, symbols, tables, drawings, and words. Mathematical thinking helps in guessing, intuition, and problem solving (Herlina, 2015). Alghamdi (2020) defined mathematical thinking skills as the mental activities carried out by students in order to process mathematical data through many skills to reach the correct solutions.

Mathematical thinking skills are among the most important mathematical skills that affect the learning of mathematics. The importance of mathematical thinking skills lies in the fact that they are tools that help improve innovative thinking and problem-solving skills in a productive and creative way. Also, they help improve levels of self-confidence among students while they are learning mathematics (Aslan, 2012).

Although many studies (e.g., Barham & Al-khateeb, 2012; Hirzalla, 2016) have indicated the importance of mathematical thinking skills in students' academic achievement, the results of Saudi Arabia in international tests, such as the TIMSS, indicate a weakness in students' achievement. Almusharraf (2009) confirmed the willingness of many students to solve direct arithmetic problems, and their reluctance to solve problems that require mathematical thinking skills. The problem of poor mathematical thinking skills is exacerbated by the fact that many mathematics teachers continue to use traditional teaching methods on the one hand, and focus on the lower levels of thinking on the other hand.

Study problem:

This study seeks to identify the level of mathematical thinking skills of second intermediate grade female students. Therefore, the problem of the study can be identified in the following question: What is the level of possession of mathematical thinking skills of the second intermediate grade female students in the city of Hail?

THEORETICAL FRAMEWORK

Definition of mathematical thinking skills:

Mathematical thinking skills are one of the outcomes of the learning process. Students need to develop different types of mathematical thinking skills, such as induction, deduction, symbolic expression, generalization, and others (Al-Kaisi, 2014).

Mathematical thinking skills are defined as an organized, flexible mental activity that aims for the student to use all or some forms of thinking when dealing with mathematical content, using thinking skills to reach the correct solution (Almaliki, 2016). Odeh (2016) believes that mathematical thinking skills are a series of mental activities that an individual's brain performs to

research a specific topic, judge the reality of something, or solve a specific problem in mathematics. This behavior has specific characteristics, the most important of which is the existence of the linking property, which links mathematical information to reality, and the ability to use foresight, choice, and reorganization. Jaafar (2013) indicated that it is a set of mental processes centered on specific mathematical problems, which aim to produce ideas that are used as a means or strategy to solve these problems. Kazmel (2013) considers mathematical thinking skills to be a mental activity that aims to use all or some forms of thinking when facing mathematical problems and dealing with various exercises. Mathematical thinking is determined by several skills related to mental operations, such as induction, inference, symbolic expression, visual-spatial perception, and mathematical proof. This type of thinking occurs when an individual faces a problem that is difficult to solve in simple ways.

In light of the foregoing, mathematical thinking skills procedurally are defined in this study as the ability of the second intermediate grade female student to perform, master, and understand a set of mathematical thinking skills (symbolic expression, visual-spatial perception, and logical thinking). These skills are measured by the score that each student obtained in the mathematical thinking skills test prepared for this purpose.

It should be noted that educational researchers have differed among themselves about determining mathematical thinking skills, due to the different characteristics of students at each stage, the nature of mathematics at each stage, and the multiplicity of names for the same concept. To determine the mathematical thinking skills for the second intermediate grade, the author limited these skills and presented them to a group of arbitrators to express their opinion on the importance of each skill, with the possibility of deleting, adding, or modifying these skills. The author concluded with a list of mathematical thinking skills needed for the second intermediate grade, which are as follows: symbolic expression, visual-spatial perception, and logical thinking.

Symbolic Expression:

Symbolic expression is thinking through symbols and abstractions, not through tangible data. Humans use symbols to think in different ways, including mental images, meanings, words, numbers, relationships, and mathematical formulas (Waleed, 2019). The symbol is used instead of the name. The name is given to the concept that identifies a group of things that share some characteristics (Alhobby, 2018). Al-Zadjalia (2013) defines symbolic expression as the learner's ability to use symbols to express mathematical ideas, verbal problems, a concept, or a mathematical process. The symbol can be a letter, a shape, or a relationship, in order to facilitate mathematical operations and facilitate mathematical thinking, or thinking about tangible things and data through symbols and abstractions. One of the most important characteristics of symbols is that they are universally negotiable because they eliminate the difficulty of understanding between different languages, and facilitate understanding of mathematics. Symbolic expression in this study is defined as a skill that the student possesses that enables her to convert verbal data, mathematical ideas, and shapes into symbols and vice versa.

Symbolic expression is inculcated by training students to convert words into symbols, and turn sentences into symbolic expression (Alsayari, 2015). Almaqati (2009) indicated that symbolic expression skills can be identified in the following:

1. Understand a given verbal statement, generalization, or issue.
2. Determine the relationships contained in the statement, generalization, or issue.
3. Define the terms in this phrase.
4. Define mathematical symbols for words, terms, and relationships.
5. Provide a verbal translation of the given phrase, generalization, or issue.
6. Demonstrate the ability to translate from verbal forms into symbolic forms.

Visual-Spatial Perception:

Alsayari (2015) defines visual-spatial perception as thinking that depends on the shapes, drawings, and images presented in the situation, and the real relationships involved in them. Almokly (2013) indicated that it is the ability to imagine the new image of geometric shapes and solids resulting from performing mental engineering transformations, and to know how to apply them mentally. Hanlon (2010) believe that it is the mental processing of a visual image, and the tasks associated with it. In this study, visual-spatial perception is defined as the student's ability to mentally process visual stimuli (geometric shapes, solids, images), perceive and interpret the relationships between them, and visualize them from different perspectives.

Almokly (2013) believes that the importance of visual-spatial perception lies in the fact that it:

1. It helps the learner to understand, reorganize, process, and interpret visual relationships.
2. Contributes to making education more accessible and improving understanding of scientific facts.
3. It trains memory, strengthening it through practice.
4. It contributes to making the learner more in control of dealing with abstract things.

Logical Thinking:

The logical thinking skill is defined as the process in which concepts are formed based on students' previous concepts (Alghamdi, 2020). Alhams (2019) indicated that it is the ability of the student to analyze the information presented to him and link it with previous knowledge in order to reach a specific result. Logical reasoning is the process of making judgments and other forms of dynamic thinking to reach the right conclusion (Aminah et al., 2018). Abou Ghaly (2010) defined it as the ability to perform reasoning in order to reach evidence that supports or rejects a particular point of view through the ability to collect, organize, and produce information, as well as to save it, analyze it, and evaluate it. On the other hand, logical thinking in this study is defined as a

mental process carried out by the student to draw a specific result, based on understanding, analyzing, interpreting, and linking verbal data.

Some educational studies (e.g., Abou Ghaly, 2010; Gharabi, 2017) agree that logical thinking skills depend on collecting, memorizing, organizing, analyzing, producing, and finally evaluating information.

LITERATURE REVIEW

Many educational studies have discussed the issue of mathematical thinking and its skills due to its importance in the educational process. Aljaberi (2014) conducted a study aimed at examining the levels of mathematical thinking among 80 pre-service female student teachers and their attitudes towards mathematics in Jordan. The results indicated that there is a positive correlation between the levels of mathematical thinking of student teachers and their attitudes towards mathematics. The overall level of the study group in mathematical thinking was average. Their best performance was in modeling and induction, and the level was weak in mathematical proof and generalization. Onal et al. (2017) conducted a study in Turkey aimed at revealing the mathematical thinking skills of 229 high school students. The researchers used the descriptive analytical method, and the results of the study showed that there is an average level of mathematical thinking among the study sample.

The study of Alsalamah (2018) aimed at identifying the level of mathematical thinking and its relationship to creative thinking skills among gifted secondary school students in the city of Riyadh. The descriptive correlational approach was used, where the study sample consisted of 105 male and 105 female students. The results of the study indicated that the level of mathematical thinking among the respondents was moderate. The results of the study also indicated that there is a positive correlation between mathematical thinking and creative thinking skills.

Samawi et al. (2019) studied the level of mathematical thinking and its relationship to critical thinking and mathematics achievement among middle school students in Jordan. The descriptive analytical method was used on a sample of 139 students. The results of the study indicated a high level of mathematical thinking, and the mediation of critical thinking.

Alghamdi's study (2020) aimed to identify the level of achievement in the mathematics course and the level of performance of mathematical thinking skills among fifth grade students in the city of Jeddah. The descriptive analytical method was used, and the sample consisted of 150 male and female students. The results showed that the overall level of performance of mathematical thinking skills among the respondents was average. The results also showed that there are positive correlation coefficients between mathematical thinking skills and achievement.

METHODOLOGY

The descriptive survey method was used to reveal the level of the second intermediate grade students' possession of mathematical thinking skills. Darwish (2018) indicated that the descriptive approach is a general study of a

phenomenon that exists in a group of people, and it is a method of analysis and interpretation in an organized scientific manner.

Research population:

The study population consists of all the female students of the second intermediate grade who were enrolled in public schools in the city of Hail, in the academic year 2019/2020.

Research sample:

The number of the sample was 500 female students from the second intermediate grade in the public schools affiliated with the education offices in the city of Hail, which are (East Hail, North Hail, and South Hail). Khader (2013) indicated that in descriptive studies it is recommended to use 10% for a large population (a few thousand), where the sample of this study was estimated to be 15% of the original study population.

Instrumentation:

The study instrument was a mathematical thinking skills test. The test was prepared according to the following steps:

1. The objective of the test was to measure the level of possession of the second intermediate grade female students in the city of Hail for mathematical thinking skills.
2. A list of mathematical thinking skills, which is suitable for second year intermediate students, has been identified. The opinions of specialists were taken and the theoretical literature was referred to determine these skills, so they were symbolic expression, visual-spatial perception, and logical thinking.
3. The test items were prepared in their initial form according to the content of the mathematics textbook for the second intermediate grade. The test in its initial form consisted of 16 multiple-choice items. The test items were distributed among each of the three skills, to be as follows (symbol expression = 5, visual-spatial perception = 5, logical thinking = 6).

Test validity:

The validity of the test was determined according to the apparent validity method, where the test was presented to a group of specialized arbitrators to ensure the following:

1. The clarity and accuracy of the test items.
2. The suitability of the test paragraphs for the level of the second intermediate grade students.
3. The appropriateness of the test items to measure the mathematical thinking skills of the second intermediate grade.
4. The ability to add, modify, or delete the test paragraphs and paragraph alternatives.

Based on the suggestions of some arbitrators, the test paragraphs were reformulated. An item was removed from the logical thinking axis because it was not suitable for the level of the students. The final test form consisted of 15 items, where one point was assigned to each of the test items.

Pilot study:

The test was applied to a survey sample of 40 female students from the study population (not the study sample). The objective of the pilot study was to ensure the clarity of the test instructions and the integrity of the language, and to determine the time required to answer the test. The time taken by the students to answer the test items was monitored by calculating the average time required for the test. It was found that the average time for administering the test is (45) minutes.

After analyzing the test results, the following were verified:
1. Internal consistency.
2. Discrimination and difficulty coefficient for each question of the test.
3. Test reliability.

Internal consistency:

The internal consistency of the test was verified by finding the Pearson correlation coefficient to calculate the correlation coefficient between each paragraph and the axis to which it belongs from the test axes. Also, the correlation coefficient of the test axes with the total score of the test was calculated.

Table 1: Pearson correlation coefficient between each item and the domain to which it belongs

Domain	Item	Correlation coefficient	Item	Correlation coefficient
Symbolic expression skill	1	0.586**	4	0.450**
	2	0.652**	5	0.532**
	3	0.450**		
Logical thinking skill	6	0.535**	9	0.529**
	7	0.311	10	0.618**
	8	0.456**		
Visual-spatial perception skill	11	0.428**	14	0.682**
	12	0.660**	15	0.440**
	13	0.458**		

Note. ** p-value is significant at 0.05

Table 1 shows that the values of the correlation coefficient of each of the items with its domain are positive and statistically significant at 0.05 level. This finding indicates the validity and appropriateness of the test items. However, there is one item, which is item 7 of the logical thinking skill domain, whose correlation coefficient was 0.311, which is a non-significant value. Eighty

percent of the arbitrators agreed on the importance of keeping this item due to the importance of the item within the logical thinking skill domain.

Table 2: Correlation coefficients of test domains with the total score

Domain	Correlation coefficient
Symbolic expression skill	0.639**
Logical thinking skill	0.611**
Visual-spatial perception skill	0.643**

Note. ** p-value is significant at 0.05

Table 2 shows that the correlation coefficients for all skills with the total score of the test ranged between 0.611 to 0.643, and they were all significant at the 0.05 level. Therefore, this finding indicates the validity and appropriateness of the test items.

Difficulty and discrimination coefficients:

Table 3 presents the difficulty and discrimination coefficients for the test items.

Table 3: Difficulty and discrimination coefficients for test items

Item	Difficulty coefficient	Discrimination coefficient	Item	Difficulty coefficient	Discrimination coefficient
1	60	45.5	9	77.5	28.2
2	67.5	36.4	10	75	72.7
3	62.5	36.4	11	37.5	45.5
4	62.5	36.4	12	55	63.6
5	86	45.5	13	62.5	48.2
6	65	27.3	14	50	54.5
7	80	30	15	77.5	48.2
8	70	28.2			

Table 3 shows the suitability of the difficulty and discrimination coefficients for all test items, except for item 5 of the symbolic expression skill domain. According to Al-Kilani et al. (2008), the difficulty coefficients are appropriate when their values range from 20% to 85%, and the discrimination coefficients are appropriate when they range from 20% to 80%. Therefore, 80% of the arbitrators agreed on the need to keep item 5 with the amendment of its alternatives, given the importance of the item in the symbolic expression skill domain.

Test reliability:

To calculate the test reliability, the Split-half method and Spearman-Brown prophecy formula were used.

Table 4: Instrument reliability coefficients

Split-half coefficient	Spearman-Brown coefficient
0.648	0.627

Table 4 shows that the correlation coefficient using the split-half coefficient was 0.648, and the Spearman-Brown coefficient for the test was 0.627, which means these values are suitable. Thus, the reliability of the mathematical thinking skills test was confirmed. Therefore, the test in its final form consisted of 15 items, each domain containing 5 items.

Statistical analysis:

To achieve the objectives of the study and data analysis, the SPSS program was used to calculate the following: (1) percentages, averages, and standard deviations, (2) Pearson correlation, (3) difficulty and discrimination coefficients, (4) split-half method, and (5) Spearman-Brown prophecy formula.

RESULTS AND DISCUSSION:

To answer the research question, which stated, *"What is the level of possession of mathematical thinking skills among the second intermediate grade female students in the city of Hail?"* Average and standard deviations were calculated for the sample's level of mathematical thinking skills. The level was determined as follows: high when obtaining (75% or more) of the total score, medium when obtaining (50% - less than 75%) of the total score, low when obtaining (25% - less than 50%) of the total score, and very low when obtaining (less than 25%) of the total score.

Table 5: Sample responses to the symbolic expression skill

Item	Average	Weighted Average	SD	%	Level
1	0.53	1.85	1.099	37%	Low
2	0.33				
3	0.36				
4	0.37				
5	0.26				

Note. average score out of (5).

Table 5 shows that the level of participants' possession of the skill of symbolic expression is low (% of weighted average = 34%), which is less than the hypothetical average of 50%. This result demonstrates that the level of the respondents' possession of the skill of symbolic expression is weak. Also, most of the participants (n = 166) obtained (1 out of 5), representing 33.2% of the female students participating in this study, while only one of the participants got (5 out of 5), which represents 0.2% of the participants.

Table 6: Sample responses to the logical thinking skill

Item	Average	Weighted Average	SD	%	Level
6	0.51	1.43	1.090	28.6%	Low
7	0.32				
8	0.10				
9	0.21				
10	0.29				

Note. average score out of (5).

Table 6 shows that the level of participants' possession of the skill of logical thinking is low (% of weighted average = 28.6%), which is less than the hypothetical average of 50%. This result demonstrates that the level of the respondents' possession of the skill of logical thinking is weak. Also, most of the participants (n = 177) obtained (1 out of 5), representing 35.4% of the female students participating in this study, while 25 of the participants got (5 out of 5), which represents 5% of the participants.

Table 7: Sample responses to the visual-spatial perception skill

Item	Average	Weighted Average	SD	%	Level
11	0.53	1.85	1.099	37%	Low
12	0.33				
13	0.36				
14	0.37				
15	0.26				

Note. average score out of (5).

Table 7 shows that the level of participants' possession of the skill of visual-spatial perception is low (% of weighted average = 37%), which is less than the hypothetical average of 50%. This result demonstrates that the level of the respondents' possession of the skill of visual-spatial perception is weak. Also, most of the participants (n = 157) obtained (2 out of 5), representing 31.4% of the female students participating in this study, while 6 of the participants got (5 out of 5), which represents 1.2% of the participants.

Table 8: Average scores of participants for mathematical thinking skills

Skill	Average	%	SD	Level	order
Symbolic expression skill	1.70	34%	1.146	Low	Second
Logical thinking skill	1.43	28.6%	1.090	Low	Third
Visual-spatial perception skill	1.85	37%	1.099	Low	First
Total	4.98	33.2 %	3.404	Low	

Note. Average score out of (15).

Table 8 shows that the level of mathematical thinking skills of the participants was weak, with an average score of 33.2%, which is less than the hypothetical mean (50%). Table 8 also shows that the order of the participants' mathematical thinking skills was as follows: the skill of visual-spatial perception, the skill of symbolic expression, followed by the skill of logical thinking. The reason for this result may be attributed to the students' lack of seriousness in answering the test, or a result of the accumulated weakness of the participants based on their previous training. In addition, perhaps the reason for this weakness is due to the failure of mathematics teachers to develop mathematical thinking among their students due to the intensity of the course content, and the teachers' focus on covering the topics of the study plan in the specified time.

The result of this study agrees with the finding of the Qassi (2014) study regarding the low percentage of students' acquisition of mathematical thinking skills. This weakness was explained as a result of several variables, including the difficulty of mathematics due to its abstractness and its need for greater effort, which leads to the students' reluctance to study it. Secondly, the ineffectiveness of the teaching methods used in teaching mathematics can be cited, where traditional methods based on indoctrination are often used rather than the linking of the content of mathematics to the reality of life. Thirdly, there has been a neglecting of the development of mental arithmetic as a practice that allows students to employ their own strategies based on acquired understanding and arithmetic skills. This practice, if done, will contribute to the development of students' thinking in order to reach a solution instead of relying on a calculator, or paper and pen.

Also, the result of the current study is consistent with the finding of the Alsayari (2015) study, which revealed that the participants' mathematical thinking skills were weak. This weakness was explained by the fact that the methods used by mathematics teachers still focus on rote memorization. The teacher was the only source of learning in the classroom, and the teacher's role was centered on correcting students' answers only. In addition, mathematics teachers were suffering from poor qualifications in the field of developing and supporting mathematical thinking, which prevented teachers from using skills supporting mathematical thinking in their teaching. Moreover, the time allotted for studying mathematics was short, the content of the subject was intense, and the daily study schedule was crowded. All these factors made mathematics teachers focus on completing the teaching of the content, and not paying attention to the development of mathematical thinking among their students.

In addition to the above, this result is consistent with Hirzalla (2016), which confirmed that the low level of mathematical thinking among the participants was due to the education system that focuses primarily on achievement tests for the content of the textbook while providing less focus on the diversity of teaching methods, and the development of higher-order thinking skills. Mathematics teachers, due to the intensity of the course content, focus their attention on covering the content topics in the specified time without allocating time to train students in mathematical thinking skills. In addition, due to the accumulated weakness of students across successive classes, most of the

teachers' attention is focused on raising the level of achievement among students without caring about developing their mathematical thinking skills.

CONCLUSION:

The current study sought to reveal the level of second-grade intermediate level female students' possession of mathematical thinking skills in the city of Hail in the Kingdom of Saudi Arabia. The results of the study indicated that the level of the participants' possession of mathematical thinking skills was weak (Average = 33.2%). The results also show that the distribution of the participants' mathematical thinking skills was as follows: the skill of visual-spatial perception, followed by the skill of symbolic expression, and the skill of logical thinking came in the last place. This study recommends the importance of spreading awareness among female students of the importance of mathematical thinking skills in teaching mathematics, preparing educational programs that contribute to the development of those skills, and providing classroom and extracurricular activities that help develop students' mathematical thinking skills. The study suggests conducting future research that studies the level of possessing mathematical thinking skills among female students at other academic levels, the effectiveness of a training program to develop mathematical thinking skills among female students, and the impact of an interactive learning environment on developing female students' mathematical thinking skills.

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