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FLOW SHOP SCHEDULING PROBLEMS USING BRANCH AND BOUNDTECHNIQUE UNDERLR-FUZZY NUMBER

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ABSTRACT:

This paper is an analysis of flow shop scheduling problem in LR-fuzzy numbers where processing time of the jobs are associated with probabilities and solved using Branch and Bound technique to obtain the optimal sequence. Here processing time of the jobs are represented by LR fuzzy numbers. Using under various reference function in the Yager'sranking formula the total duration of the jobs were calculated. A numerical example is illustrated to obtain the completion time of the jobs by using the Branch and Bound technique in LR -fuzzy numbers with various reference functions.

INTRODUCTION

Generally fuzzy numbers plays a vital role to handle uncertainties. Fuzzy sets provide a membership value for each individual in the universe.LR – fuzzy number is an extension of fuzzy numbers with left and right shapes. Dubois and Prade[3]define the idea of LR-fuzzy numbers with some arithmetic operations.

Scheduling deals with the idea of planning and arranging jobs in an orderly sequence. Flow shop scheduling is the process in which all jobs pass through all machines in the same order. It is majorly used in the area of production management. To increase the profit and to minimize the total completion time of the jobs,scheduling helps to order the jobs in a particular way. Johnson proposed the rule to calculate the makespan of the jobs.

Application of Branch and Bound technique in flow shop scheduling was developed by Ignall and Scharge [5].Brown and Lominick[2]analysed branch and bound technique in machinescheduling.Maggu and Das[6] introduced the idea of Job-block criteria in flow shop scheduling problems.Uthra,Thangavelu and Shunmugapriya [10] compared the optimal solution of FSS problems in intuitionistand fuzzy environment.

Thorani and RaviShankar [7]discussed the fuzzy transportation problems under LR- fuzzy numbers with various types of reference functions to get the optimal solution.Thorani and RaviShankar[8] developed the idea of solving assignment problem by considering LR fuzzy numbers as Generalised Trapezoidal fuzzy numbers and by applying centroid technique, under Linear programming model the optimal solution of the multi objective assignment problem were discussed.

In this paper, the optimal solution of flow shop scheduling problem were discussed by usingvarious reference functions derived from the Yager's Ranking formula for LR- fuzzy numbers. By applying Job-Block criteria and by using Branch and Bound method the optimal sequence was obtained and the minimum total elapsed time was calculated for various reference functions in Yager's formula.The results were compared to decide the minimum completion time for the entire processunder various reference function.

Preliminaries

Fuzzy Set

Let X be a nonempty set, and a fuzzy set$\widetilde{A}$ is defined by $\widetilde{A}$ = {(x, $\mu_{\widetilde{A}}$(x)): x∈A}. In the pair (x, $\mu_{\widetilde{A}}$(x)), the first element belongs to the classical set A, the second element $\mu_{\widetilde{A}}$(x), belong to the interval [0, 1] is called the membership function.

Fuzzy number$\widetilde{A}$is a fuzzy set on the real line$\mathfrak{R}$, must satisfy the following conditions.

- $\mu_{\tilde{A}}(x_0)$ is piecewise continuous
- There exist at least one $x_0 \in \mathfrak{R}$ with $\mu_{\tilde{A}}(x_0)$ =1
- $\tilde{A}$must be normal & convex

LR-Fuzzy Number

A fuzzy number $\overline{A} = (m, n, \alpha, \beta)_{LR}$ is said to be an LR fuzzy number if

$$\mu_{\tilde{A}}(x) = \begin{cases} L\left(\frac{m-x}{\alpha}\right), x \le m, \alpha > 0 \\ R\left(\frac{x-n}{\beta}\right), x \ge n, \beta > 0 \\ 1\ otherwise \end{cases}$$

If m = n then $\overline{A} = (m, n, \alpha, \beta)_{LR}$will be converted into $\overline{A} = (m, \alpha, \beta)_{LR}$and is said to betriangular LR-fuzzy number. L and R are called reference functions, which are continuous,non-increasing functions that defines the left and right shapes of $\mu_{\widetilde{A}}$(x) respectively and L(0)=R(0) = 1.Two special cases are triangular and trapezoidal fuzzy numbers for which

L(x) = R(x) = max $\{0,1-|x|\}$ are linear functions. Non reference function which are commonly used with parameters p, denoted by $RF_p(x)$.Linear and Non- Linear functions with their inverses are given in the below table.

Reference Functions and their Inverses

+/Function Name	Reference Function $RF_p(x)$	Inverse of Reference function $\alpha\epsilon[0,1]$
Linear	$RF_p(x)$= max $\{0,1-x$	$RF_p^{-1}(x) = (1-\alpha)$
Exponential	$RF_p(x) = e^{-px}$, $p \geq 1$	$RF_p^{-1}(x) = -\frac{(In\alpha)}{p}$
Power	$RF_p(x)$= max $\{0,1-x$	$RF_p^{-1}(x) = -\sqrt[p]{(1-\alpha)}$
Exponential Power	$RF_p(x) = \boldsymbol{e^{-x^p}}$, $\boldsymbol{p} \geq 1$	$RF_p^{-1}(x) = \sqrt[p]{-In\alpha}$
Rational	$RF_p(x) = \frac{1}{1+x^p}$,$p \geq 1$	$RF_p^{-1}(x) = \sqrt[p]{\frac{(1-\alpha)}{\alpha}}$

λ −cut for LR-Fuzzy Number

Let $\overline{A} = (m,n,\alpha,\beta)_{LR}$ be an LR fuzzy number andλ bea real number in the interval [0,1].Then $A_k = \{x\epsilon X: \mu_{\widetilde{A}}(x) \geq \lambda\} = (m-\alpha L^{-1}(\lambda), n+\beta R^{-1}(\lambda))$ is said to be the $\lambda -$ cut of $\overline{A}$.

Yager's Ranking Formula for LR-Fuzzy Numbers

If $\overline{A} = (m,n,\alpha,\beta)_{LR}$ is a LR -fuzzy number then

$R(\bar{A}) = \frac{1}{2}[\int_0^1((m-\alpha L^{-1}(\boldsymbol{\lambda}))\boldsymbol{d\,\lambda} + \int_0^1(n+\beta R^{-1}(\boldsymbol{\lambda})\boldsymbol{d\,\lambda}]$

Let $\overline{A}$and $\overline{B}$be two fuzzy numbers then

1.$\bar{A} > \bar{B}$if $R(\bar{A}) > R(\bar{B})$2. $\bar{A} = \bar{B}$if $R(\bar{A}) = R(\bar{B})$3. .$\bar{A} < \bar{B}$if $R(\bar{A}) < R(\bar{B})$

Case: 1 IfL(x)= R(x) =max $\{0,1-|x|\}$then

$R(\bar{A}) = \frac{1}{2}[m-\frac{\alpha}{2}+n+\frac{\beta}{2}]$

Case: 2 IfL(x)= R(x) = e^{-x}then

$R(\bar{A}) = \frac{1}{2}[m-\alpha+n+\beta]$

Case: 3 IfL(x)= max $\{0,1-|x|\}$R(x) = e^{-x}then

$R(\bar{A}) = \frac{1}{2}[m-\frac{\alpha}{2}+n+\beta]$

Case: 4 IfL(x)= e^{-x}R(x) =max $\{0,1-|x|\}$then

$R(\bar{A}) = \frac{1}{2}[m-\alpha+n+\frac{\beta}{2}]$

Arithmetic Operation

Let $\overline{A} = (m_1,n_1,\alpha_1,\beta_1)_{LR}$ and$\overline{B} = (m_2,n_2,\alpha_2,\beta_2)_{LR}$ be two LR-fuzzy numbers.Then

- $\bar{A}+\bar{B} = ((m_1+m_2, n_1+n_2, \alpha_1+\alpha_2, \beta_1+\beta_2)_{LR}$

- $\bar{A} - \bar{B}$=$(m_1 - n_2, n_1 - m_2, \alpha_1 + \beta_2, \beta_1 + \alpha_2)_{LR}$
- Scalar Multiplication
- If $x > 0$, $x\epsilon R then x \times \bar{A} = (xm_1, xn_1, x\alpha_1, x\beta_1)_{LR}$
- If $x < 0$, $x\epsilon R then x \times \bar{A} = (xn_1, xm_1, -x\beta_1, -x\alpha_1)_{LR}$

Mathematical Formulation of LR -fuzzy Number in Flow Shop Scheduling Problem

Assume that some jobs are to be processed in three machines in a sequential order.

The problem is formulated under the parameters of processing time and their corresponding probabilities for each machinesType equation here.

jobs	Machine A		Machine B		Machine C	
	p_i	l_i	q_i	m_i	r_i	n_i
1	$(m_1, n_1, \alpha_1, \beta_1)_{LR}$	l_1	$(a_1, b_1, \gamma_1, \varepsilon_1)_{LR}$	m_1	$(f_1, g_1, \theta_1, \mu_1)_{LR}$	n_1
2	$(m_2, n_2, \alpha_2, \beta_2)_{LR}$	l_2	$(a_2, b_2, \gamma_2, \varepsilon_2)_{LR}$	m_2	$(f_2, g_2, \theta_2, \mu_2)_{LR}$	n_2
3	$(m_3, n_3, \alpha_3, \beta_3)_{LR}$	l_3	$(a_3, b_3, \gamma_3, \varepsilon_3)_{LR}$	m_3	$(f_3, g_3, \theta_3, \mu_1)_{LR}$	n_3
….	……..	….	……..	…..	……..	….
….	……….	….	……..	……	………	….
N	$(m_n, n_n, \alpha_n, \beta_n)_{LR}$	l_n	$(a_n, b_n, \gamma_n, \varepsilon_n)_{LR}$	m_n	$(f_n, g_n, \theta_n, \mu_n)_{LR}$	n_n

Assumption

- The tasks to be processed are independent of each other
- Pre-emption of employment are not allowed
- An appointment is not available to the next machine until and unless the current
- processing device is completed.
- Machines never breakdown and are available throughout the scheduling process.
- (v)Each job must be completed once it is started.

Notations

p_i − Processing time of i^{th}job on Machine A

q_i − Processing time of i^{th} job onMachine B

r_i − Processing time of i^{th} job on Machine C

(All the processing times are being taken in the form of

$$(m, n, \alpha, \beta)_{LR})$$

l_i −Probability associated with processing time p_i

m_i −Probability associated with processing time q_i

n_i −Probability associated with processing time r_i

A_i – Expected processing time for Machine A

B_i – Expected processing time for Machine B

C_i – Expected processing time for Machine

S_k – Sequence formed from jobs(k =1,2,…5)

Proposed Algorithm for LR – Fuzzy Flow Shop Scheduling Problems

Step 1: Assumethe processing time as an LR-fuzzy number .

Step 2: Calculate the expected processing time as

$$A_i = p_i * l_i, B_i = q_i * m_i, C_i = r_i * n_i$$

Step 3: Defuzzify the LR – fuzzy numbers by using various reference function in theYager's Ranking formula.

Step 4:Calculate the expected processing time of the job block $\delta = (y,z)$ on the fictitiousmachines A,B and Csuchthat

$$A_\delta = A_y + A_z - \min(A_z, B_y)$$

$$C_\delta = C_y + C_z - \min(B_z, C_y)$$

$$B'_\delta = B_y + B_z - \min(A_z, B_y), B''_\delta = B_y + B_z - \min(B_z, C_y)$$

$$B_\delta = \frac{B'_\delta +, B''_\delta}{2}$$

Step 5: Define the new reduced problem by using the job-block criteria.

Step 6: For the 3 machine with processing time A_i , B_i , $C_i$calculate the lower bound by using the formulae

- $I_1 = t(J_r, 1) + \sum_{i \in J_r} A_i + \min(B_i + C_i)$
- $I_2 = t(J_r, 2) + \sum_{i \in J_r} B_i + \min(C_i)$
- $I_3 = t(J_r, 3) + \sum_{i \in J_r} C_i$

Step 7: Calculate L = max .$\{I_1, I_2, I_3\}$ and evaluate L first for the n classes of permutations i.e. for these starting with 1,2,3….respectively and labelling the appropriate verticesof the branch from the calculatedvalues.

Step8: Exploring the vertex with the lowest value and calculating the remaining subclasses starting with the vertex and again concentrate on the lowest label vertex. The process is continued until the end of the tree represented by two single permutationis reached. The total work duration is calculated.

Step 9: Minimum Total Elapsed Timeis calculated by preparing in-out table for thesequence obtained.

Step 10: Step 3 to 9 is followed for all the reference functions.

Step 11: Comparative analysis is made**.**

Numerical Example

A famous jewellery shop in T .Nagar had some Goldsmith for cutting,cleaning and polishing the ornaments. During weekends, they give equivalent work for some jobs.Calculate the minimum time taken by them to complete their work in busy schedule. They usually get bangles(J_1), ear rings(J_2), neck set(J_3), bracelets(J_4) and rings(J_5), to do all the works.

Here the processing time for doing each job is given in LR-fuzzy numbers. Jobs(2,3) are given an equivalent work .

Works Jobs	Cutting	l_i	Cleaning	m_i	Polishing	n_i
J_1	$(7,8,2,4)_{LR}$	0.1	$(4,5,2,3)_{LR}$	0.2	$(6,7,3,5)_{LR}$	0.1
J_2	$(6,7,4,5)_{LR}$	0.2	$(3,5,1,2)_{LR}$	0.1	$(2,4,1,3)_{LR}$	0.1
J_3	$(2,4,1,3)_{LR}$	0.3	$(6,7,2,4)_{LR}$	0.2	$(3,5,2,4)_{LR}$	0.2
J_4	$(3,5,2,4)_{LR}$	0.2	$(8,10,4,6)_{LR}$	0.1	$(8,9,4,7)_{LR}$	0.3
J_5	$(8,9,4,6)_{LR}$	0.2	$(5,7,3,4)_{LR}$	0.4	$(7,9,1,6)_{LR}$	0.3

Solution

When the processing time of the machines multiplied with their probabilities we get

Machines Jobs	Cutting	Cleaning	Polishing
J_1	$(0.7,0.8,0.2,0.4)_{LR}$	$(0.8,1,0.4,0.6)_{LR}$	$(0.6,0.7,0.3,0.5)_{LR}$
J_2	$(1.2,1.4,0.8,1)_{LR}$	$(0.3,0.5,0.1,0.2)_{LR}$	$(0.2,0.4,0.1,0.3)_{LR}$
J_3	$(0.6,1.2,0.3,0.9)_{LR}$	$(1.2,1.4,0.4,0.8)_{LR}$	$(0.6,1,0.4,0.8)_{LR}$
J_4	$(0.6,1,0.4,0.8)_{LR}$	$(0.8,1,0.4,0.6)_{LR}$	$(2.4,2.7,1.2,2.1)_{LR}$
J_5	$(1.6,1.8,0.8,1.2)_{LR}$	$(2,2.8,1.2,1.6)_{LR}$	$(2.1,2.7,0.3,1.8)_{LR}$

Defuzzifying the LR- fuzzy numbers by using the Yager's ranking formula with the reference function

Case: 1 IfL(x)= R(x) =max $\{0,1 - |x|\}$

Works Jobs	Cutting	Cleaning	Polishing
J_1	0.8	0.9	0.7
J_2	1.3	0.4	0.3
J_3	1.0	1.4	0.9
J_4	0.9	0.9	2.7
J_5	1.8	2.5	2.7

Giving jobs 2 and 3 an equivalent work and by using job block criteriawe get

Works Jobs	Cutting	Cleaning	Polishing
1	0.8	0.9	0.7
δ	1.9	1.4	0.9
4	0.9	0.9	2.7
5	1.8	2.5	2.7

Table representing the value of lower bound

J_r	LB(J_r)
LB(1)	8.7
LB(δ)	10.3
LB(4)	8.8
LB(5)	11.3
LB(1δ)	10.4
LB(14)	8.9
LB(15)	11.4
LB(14δ)	10.6
LB(145)	9.6

The optimal sequence formed is1-4-5-δ

In- out Table Showing the Minimum Total Elapsed Time

Works	Cutting	Cleaning	Polishing

Jobs	In	Out	In	Out	In	Out
1	--	0.8	0.8	1.7	1.7	2.4
4	0.8	0.17	1.7	2.6	2.6	5.3
5	0.17	3.5	3.5	6.0	6.0	8.7
δ	3.5	5.4	6.0	7.4	8.7	**9.6**

Total time taken for completingthe entire process = 9.6hrs
Idle time in cutting = 9.6 -5.4 = 4.2hrs
Idle time in cleaning= 0.8 + 0.9 +2.2 =3.9hrs
Idle time in polishing = 1.7 + 0.2 +0.7 = 2.6hrs

Case: 2 IfL(x)= R(x) = e^{-x}

Works /Jobs	Cutting	Cleaning	Polishing
J_1	0.85	1	0.75
J_2	1.4	0.45	0.4
J_3	1.2	1.5	1
J_4	1	1	3
J_5	1.9	2.6	3.1

Giving jobs 2 and 3 an equivalent work and by using job block criteria we get

Works/ Jobs	Cutting	Cleaning	Polishing
1	0.85	1	0.75
δ	2.1	1.5	1
4	1	1	3
5	1.9	2.6	3.1

Table representing the value of lower bound

J_r	LB(J_r)
LB(1)	9.6
LB(δ)	11.4
LB(4)	9.8
LB(5)	12.3
LB(1δ)	11.5
LB(14)	9.9
LB(15)	12.45
LB(14δ)	11.1
LB(145)	10.4

The optimal sequence formed is1-4-5-δ

In- out Table Showing the Minimum Total Elapsed Time

Works	Cutting		Cleaning		Polishing	
Jobs	In	Out	In	Out	In	Out
1	--	0.85	0.85	1.85	1.85	2.6
4	0.85	1.85	1.85	2.85	2.85	5.85
5	1.85	3.7	3.7	6.3	6.3	9.4
δ	3.7	5.8	6.3	7.8	9.4	**10.4**

Total time taken for completingthe entire process =10.4 hrs
Idle time in cutting = 10.4 -5.8= 4.6 hrs
Idle time in cleaning= 0.85 + 0.85 +2.6 = 4.3hrs
Idle time in polishing = 1.85 + 0.2 5+0.45 = 2.5 hrs

Case: 3IfL(x)=max $\{0, 1 - |x|\}$, R(x) = e^{-x}

Works Jobs	Cutting	Cleaning	Polishing
J_1	0.9	1.1	0.8
J_2	1.6	0.47	0.4
J_3	1.2	1.6	1.1
J_4	1.1	1.1	3.3
J_5	2.1	2.9	3.2

Giving jobs 2 and 3 an equivalent work and by using job block criteria we get

Works/ Jobs	Cutting	Cleaning	Polishing
1	0.9	1.1	0.8
δ	2.4	1.6	1.1
4	1.1	1.1	3.3
5	2.1	2.9	3.2

Table representing the value of lower bound

J_r	LB(J_r)
LB(1)	10.4
LB(δ)	12.4
LB(4)	10.6
LB(5)	13.4
LB(1δ)	12.5
LB(14)	10.7
LB(15)	13.5
LB(14δ)	12.1
LB(145)	11.3

The optimal sequence formed is1-4-5-δ

In- out Table Showing the Minimum Total Elapsed Time

Works	Cutting		Cleaning		Polishing	
Jobs	In	Out	In	Out	In	Out
1	--	0.9	0.9	2.0	2.0	2.8
4	0.9	2.0	2.0	3.1	3.1	6.4
5	2.0	4.1	4.1	7.0	7.0	10.2
δ	4.1	6.5	7.0	8.6	10.2	**11.3**

Total time taken for completingthe entire process =11.3 hrs
Idle time in cutting = 11.3-6.5= 4.8 hrs
Idle time in cleaning= 0.9 + 1.0 +2.7 = 4.6hrs
Idle time in polishing = 2.0 + 0.3+0.6 = 2.9hrs

Case: 4IfL(x)=e^{-x}, R(x) =max $\{0, 1 - |x|\}$

Works Jobs	Cutting	Cleaning	Polishing
J_1	0.7	0.8	0.6
J_2	1.1	0.4	0.3
J_3	0.9	1.3	0.8
J_4	0.8	0.8	2.4
J_5	1.6	2.2	2.7

Giving jobs 2 and 3 an equivalent work and by using job block criteria we get

Works/ Jobs	Cutting	Cleaning	Polishing
1	0.7	0.8	0.6
δ	1.6	1.3	0.8
4	0.8	0.8	2.4
5	1.6	2.2	2.7

Table representing the value of lower bound

J_r	LB(J_r)
LB(1)	8.7
LB(δ)	9.4
LB(4)	8.1
LB(5)	10.3
LB(1δ)	9.5
LB(14)	8.2
LB(15)	10.4
LB(14δ)	9.6
LB(145)	8.8

The optimal sequence formed is1-4-5-δ

In- out Table Showing the Minimum Total Elapsed Time

Works	Cutting		Cleaning		Polishing	
Jobs	In	Out	In	Out	In	Out
1	--	0.7	0.7	1.5	1.5	2.1
4	0.7	1.5	1.5	2.3	2.3	4.7
5	1.5	3.1	3.1	5.3	5.3	8.0
δ	3.1	4.7	5.3	6.6	8.0	**8.8**

Total time taken for completing the entire process = 8.8
Idle time in cutting = 8.8 -4.7 = 4.1hrs
Idle time in cleaning= 0.7 + 0.8 +2.2 =3.7hrs
Idle time in polishing = 1.5 + 0.2 +0.6 = 2.3hrs
The proposed algorithm is calculated for the other reference function derived from the yager's formula .The calculated values are shown in the below table

Reference function	**Optimal Sequence**	**Total CompletionTime**
L(x)= R(x) = max $\{0,1-|x|\}$	1-4-5-δ	9.6
L(x)= R(x) = e^{-x}	1-4-5-δ	10..4
L(x)= max $\{0,1-|x|\}$ R(x) = e^{-x}	1-4-5-δ	11.3
L(x)= e^{-x} R(x) =max $\{0,1-|x|\}$	1-4-5-δ	8.8

CONCLUSION

This paper provides an idea to solvescheduling problem in the LR- fuzzy environment. Branch and bound techniquehave been usedto derive the optimal sequence of the flow shop scheduling problems in which the processing time are being taken as LR-Fuzzy numbers.It is illustrated with the example that either linear or exponential membership function on LR-fuzzy number yields best results. Our future work is to solve the scheduling pattern in different fuzzy environment with different algorithm.

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