PalArch's Journal of Archaeology of Egypt / Egyptology

SANSKRUTI INDEX OF SOME GRAPH OPERATIONS

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S. Sowmya.Sanskruti Index of Some Graph Operations-- Palarch's Journal of Archaeology of Egypt/Egyptology 17(7), 4809-4815. ISSN 1567-214x

Keywords: Sanskruti Index, Topological Index, Cartesian Product, Tensor Product, Wreath Product.

ABSTRACT:
A topological index has a vital role in molecular chemistry.Inmathematical chemistry, graph operations are very significant since certaingraphs of chemical interest can be evaluated by various graph operations ofdifferent simple graphs. In this paper, we have obtained analytical expressionsfor Sanskruti index of various graph operations such as cartesian product, tensor product and wreath product of graphs.

INTRODUCTION

In recent years graph theory is substantially used in the branch of mathematicalchemistry due to the fact this idea is associated with the realistic purposes ofgraph theory for solving the molecular problems. Over the years topologicalindices like Wiener index, Balaban index, Hosoya index, Randic index and so onhave been studied significantly improved and currently the research and attentionin this area has been accelerated exponentially. Throughout this paper we willfocus on finite, simple and connected graphs. Let $G = (V(G), E(G))$ be a graphwith $V(G)$ is a set of all vertices and $E(G)$ is a set of all edges. The degree of v,denoted by $deg(v)$ or $d(v)$, is the number of edges incident with v in G.

Definition 1.1. Hosamani [2] in 2016 introduced Sanskruti index of graph $G$and is defined as:

$$S(G) = \sum_{uv\in E(G)} \left[\frac{S_u S_v}{S_u + S_v - 2}\right]^3 \qquad (1)$$

where$S_u$isthe sum of the degree of the neighbourhood vertices. That is,$S_u = \sum_{uv} d_v$. This index gives good correlation with entropy ofan octane isomers.

Product graphs are applicable in a number of areas, including automata theory,communication networks, information theory, computer architecture, algebraicstructures and chemistry. They help to construct many network topologies forinterconnection networks[3].

Definition 1.2.[1] Cartesian product of two graphs $G_1 = (V_1, E_1)$ and $G_2 = (V_2, E_2)$is denoted by$G_1 \times G_2$, containing vertex set $V_1 \times V_2$where $(u_1, u_2)$is adjacent with(v_1, v_2) iff $[u_1 = u_2$and$v_1v_2 \in E_2]$or $[v_1 = v_2$ and $u_1u_2 \in E_1]$.

Definition 1.3.[1] The wreath product of two graphs$G_1 = (V_1, E_1)$ and $G_2 = (V_2, E_2)$having vertex sets V_1 and V_2 with no common vertex and E_1 and E_2 asedge sets is the graph $G_1[G_2]$containing vertex set $V_1 \times V_2$ and (u_1, u_2) is adjacentto (v_1, v_2)if and only if $(u_1u_2 \in E_1)$ or $(u_1 = u_2$and $v_1v_2 \in E_2)$. Wreath productof two graphs is also known as composition of two graphs.
Definition 1.4. [1] Tensor product of two graphs $G_1 = (V_1, E_1)$ and $G_2 = (V_2, E_2)$is denoted by $G_1 \otimes G_2$has the vertex set $V_1 \times V_2$ and (u_1, v_1) is adjacent with(u_2, v_2) iff$u_1u_2 \in E_1$ and $v_1v_2 \in E_2$.

In this paper, we investigate the exact formulae of Sanskruti index for thedifferent versions of graph operations.

MAIN RESULTS

Sanskruti Index of Wreath Product for Two Graphs

Theorem 2.1. The Sanskruti index of wreath product for the two graphs P_n and $P_2$is

$$S(P_n[P_2]) = 2\left[\frac{169}{24}\right]^3 + 8\left[\frac{273}{32}\right]^3 + 2\left[\frac{441}{40}\right]^3 + 8\left[\frac{525}{44}\right]^3 + (5n-24)\left[\frac{625}{48}\right]^3.$$

Proof. Let P_n and P_2be two paths of length $n-1$ and 1 respectively.The wreathproduct of the path graphs P_n and P_2 yield the fence graph as shown in Figure 1(a).Let$G = P_n[P_2]$. We note that $|V(G)| = 2n$ and $|E(G)| = 5n - 4$. It is easy tosee that from Figure 1(a), only 4 vertices are of degree 3 and remaining all the verticesof degree 5. We divide the vertex set of the graph in two partitions as:

$$V_3 = \{v \in V(G): d_v = 3\} \text{ and} V_5 = \{v \in V(G): d_v = 5\}.$$

Now we can divide the edge set of $P_n[P_2]$ in three partitions:

$$E_{3,3} = \{u, v \in V(G): d_u = d_v = 3\},$$
$$E_{3,5} = \{u, v \in V(G): d_u = 3 \text{ and} d_v = 5\},$$
$$E_{5,5} = \{u, v \in V(G): d_u = d_v = 5\}.$$

It can be seen from Figure 1(a) that $|E_{3,3}| = 2; |E_{3,5}| = 8$ and $|E_{5,5}| = (5n - 14)$. Also,summation of degrees of edge endpoints of this graph have five types (13,13),(13,21),(21,21),(21,25) and (25,25) are shown in Figure 1(a). The number of edgesof these edge types are shown in following table.

Table 1. The Edge Partition of $P_n[P_2]$based on Degree Sum ofNeighbourhood Vertices of End Vertices of Each Edge

$(S_u, S_v), uv \in E(G)$	(13,13)	(13,21)	(21,21)	(21,25)	(25,25)
Number of edges	2	8	2	8	$(5n - 24)$

Using Table 1 and by the definition of$S(G)$ we can deduce the following formulafor the fence graph $P_n[P_2]$as

$$S(P_n[P_2]) = 2\left[\frac{169}{24}\right]^3 + 8\left[\frac{273}{32}\right]^3 + 2\left[\frac{441}{40}\right]^3 + 8\left[\frac{525}{44}\right]^3 + (5n-24)\left[\frac{625}{48}\right]^3.$$

The result is true for all $n \geq 5$.

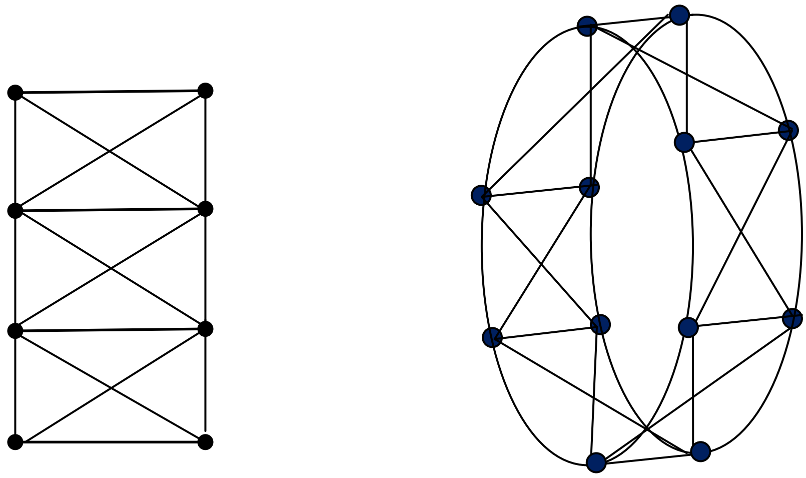

Figure 1.(a)$P_4[P_2]$and(b). $C_6[P_2]$

Theorem 2.2. The Sanskruti index of wreath product for two graphs C_n and $P_2$is $S(C_n[P_2]) = 5n\left[\frac{625}{48}\right]^3$.

Proof. Let $C_n$be the cycle of length n and P_2 be the path of length 1. Let $G = C_n[P_2]$. We notethat $|V(G)| = 2n$and $|E(G)| = 5n$ is shown in Figure 1(b). In this graph, degree ofeach vertex is 5. Thus, we can partition the edge set into the following:

Table 2. The Edge Partition of $C_n[P_2]$based on Degree Sum ofNeighbourhood Vertices of End Vertices of Each Edge.

Number of edges	$(S_u, S_v), uv \in E(G)$
$5n$	(25,25)

Using Table 2 and by the definition of $S(G)$ we get,$S(C_n[P_2]) = 5n\left[\frac{625}{48}\right]^3$.

This result is true for all $n \geq 3$.

Sanskruti Index of Tensor Product for Two Graphs

Theorem 2.3. The Sanskruti index of tensor product for two graphs P_n and $P_n$is $S(P_n \otimes P_n) = 4\left[\frac{36}{11}\right]^3 + 4\left[\frac{36}{10}\right]^3 + 8\left[\frac{72}{16}\right]^3 + 8\left[\frac{72}{15}\right]^3 + 4(2n-10)\left[\frac{96}{18}\right]^3 + 4\left[\frac{144}{22}\right]^3 + 4\left[\frac{144}{23}\right]^3 + (2n-10)(n-5)\left[\frac{256}{30}\right]^3 + 4(2n-10)\left[\frac{192}{26}\right].^3$

Proof. Let P_n be the path graph taken two copies of length $n-1$ respectively and let $G = P_n \otimes P_n$.Wenote that $|V(G)| = n^2$ and $|E(G)| = 2(n-1)^2$ is shown in Figure 2(a).Since all the vertices in this graphare of degree 1,2 and 4, we can divide the vertex set in three partitions:

$$V_1 = \{v \in V(G): d_v = 1\},$$
$$V_2 = \{v \in V(G): d_v = 2\},$$
$$V_4 = \{v \in V(G): d_v = 4\}.$$

Also, we can divide the edge set of G in four partitions.

$$E_{1,4} = \{u, v \in V(G): d_u = 1 \text{ and} d_v = 4\} \Rightarrow |E_{1,4}| = 4.$$

$$E_{2,2} = \{u, v \in V(G): d_u = d_v = 2\} \Rightarrow |E_{2,2}| = 4.$$
$$E_{2,4} = \{u, v \in V(G): d_u = 2 \text{ and} d_v = 4\} \Longrightarrow |E_{2,4}| = 8(n-3).$$
$$E_{4,4} = \{u, v \in V(G): d_u = d_v = 4\} \Longrightarrow |E_{4,4}| = 2(n-3)^2.$$

Now from the Figure 2(a), we can see that the summation of degrees of edge endpointsof this graph have nine types (4,9), (6,6),(6,12),(8,9),(8,12),(12,12),(9,16),(16,16)and (12,16). The number of edges of these edge types are shown in followingtable.

Table 3 The Edge Partition of $P_n \otimes P_n$ based on Degree Sum ofNeighbourhood Vertices of end Vertices of Each Edge.

Number of edges	$(S_u, S_v), uv \in E(G)$
4	(4,9)
4	(6,6)
8	(6,12)
8	(8,9)
$4(2n-10)$	(8,12)
4	(12,12)
4	(9,16)
$(2n-10)(n-5)$	(16,16)
$4(2n-10)$	(12,16)

Using Table 3 and by the definition of $S(G)$ we get,

$$S(P_n \otimes P_n) = 4\left[\frac{36}{11}\right]^3 + 4\left[\frac{36}{10}\right]^3 + 8\left[\frac{72}{16}\right]^3 + 8\left[\frac{72}{15}\right]^3 + 4(2n-10)\left[\frac{96}{18}\right]^3 + 4\left[\frac{144}{22}\right]^3 + 4\left[\frac{144}{23}\right]^3 + (2n-10)(n-5)\left[\frac{256}{30}\right]^3 + 4(2n-10)\left[\frac{192}{26}\right]^3.$$

This result is true for all $n \geq 6$.

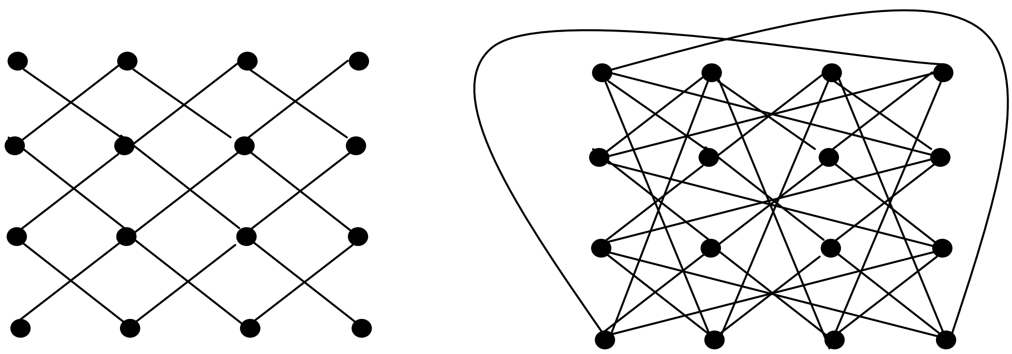

Figure 2(a)$P_4 \otimes P_4$(b). $C_4 \otimes C_4$

Theorem 2.4. The Sanskruti index of tensor product for two graphs C_n and C_n is $S(C_n \otimes C_n) = 2n^2 \left[\frac{256}{30}\right]^3$.

Proof. Consider two copies of cycle graph C_n of length n respectively. Let $G = C_n \otimes C_n$. We notethat $|V(G)| = n^2$ and $|E(G)| = 2n^2$ is shown in Figure 2(b).Using Table 4 and by the definition of $S(G)$we get, $S(C_n \otimes C_n) = 2n^2 \left[\frac{256}{30}\right]^3$.This result is true for all $n \geq 3$.

Table 4 The Edge Partition of $C_n \otimes C_n$based on Degree Sum ofNeighbourhood Vertices of End Vertices of Each Edge

Number of edges	$(S_u, S_v), uv \in E(G)$
$2n^2$	(16,16)

Sanskruti Index of Cartesian Product for Two Graphs

Theorem 2.5. The Sanskruti index of tensor product for two graphs $K_{n,n}$ andP_n is

$$S(K_{n,n} \times P_n) = 2n^2 \left[\frac{(n^2+2n+2)^2}{2n^2+4n+2}\right]^3 + 4n \left[\frac{n^4+6n^3+13n^2+14n+6}{2n^2+6n+3}\right]^3 + 2n^2 \left[\frac{(n^2+4n+3)^2}{2n^2+8n+4}\right]^3 + 4n \left[\frac{n^4+68+23n^2+28n+12}{2n^2+8n+5}\right]^3 + (n^3-2n^2-10n)\left[\frac{(n^2+4n+4)^2}{2n^2+8n+6}\right]^3.$$

Proof. Let $K_{n,n}$be the complete bipartite graph and P_n be the path graph. Let$G = K_{n,n} \times P_n$. We note that $|V(G)| = 2n^2$ and$|E(G)| = n(n^2+2n-2)$ is shown in Figure 3(a). Wedivide the vertex set of G in two partitions:

$V_{n+1} = \{v \in V(G): d_v = n+1\}$,
$V_{n+2} = \{v \in V(G): d_v = n+2\}$.

Similarly, we can divide edge set of G in three partitions.

$E_{n+1,n+1} = \{u, v \in V(G): d_u = d_v = n+1\} \Rightarrow |E_{n+1,n+1}| = 2n^2$.
$E_{n+1,n+2} = \{u, v \in V(G): d_u = n+1$ and$d_v = n+2\} \Longrightarrow |E_{n+1,n+2}| = 4n$.
$E_{n+2,n+2} = \{u, v \in V(G): d_u = d_v = n+2\} \Longrightarrow |E_{n+2,n+2}| = n(n^2-2)$.

Using Table 5 and by the definition of $S(G)$ we get,

$$S\big(K_{n,n} \times P_n\big) = 2n^2\left[\frac{(n^2+2n+2)^2}{2n^2+4n+2}\right]^3 + 4n\left[\frac{n^4+6n^3+13n^2+14n+6}{2n^2+6n+3}\right]^3 + 2n^2\left[\frac{(n^2+4n+3)^2}{2n^2+8n+4}\right]^3 + 4n\left[\frac{n^4+68+23n^2+28n+12}{2n^2+8n+5}\right]^3 + (n^3-2n^2-10n)\left[\frac{(n^2+4n+4)^2}{2n^2+8n+6}\right]^3.$$

This result is true for all $n \geq 5$.

Table 5 The Edge Partition of $K_{n,n} \times P_n$ based on Degree Sum ofNeighbourhood Vertices of End Vertices of Each Edge.

Number of edges	$(S_u, S_v), uv \in E(G)$
$2n^2$	$(n(n+1)+(n+2), n(n+1)+(n+2))$
$4n$	$(n(n+1)+(n+2), n(n+2)+(n+1)+(n+2))$
$2n^2$	$(n(n+2)+(n+1)+(n+2), n(n+2)+(n+1)+(n+2))$
$4n$	$(n(n+2)+(n+2)+(n+2), n(n+2)+(n+1)+(n+2))$
n^3-2n^2-10n	$(n(n+2)+(n+2)+(n+2), n(n+2)+(n+2)+(n+2))$

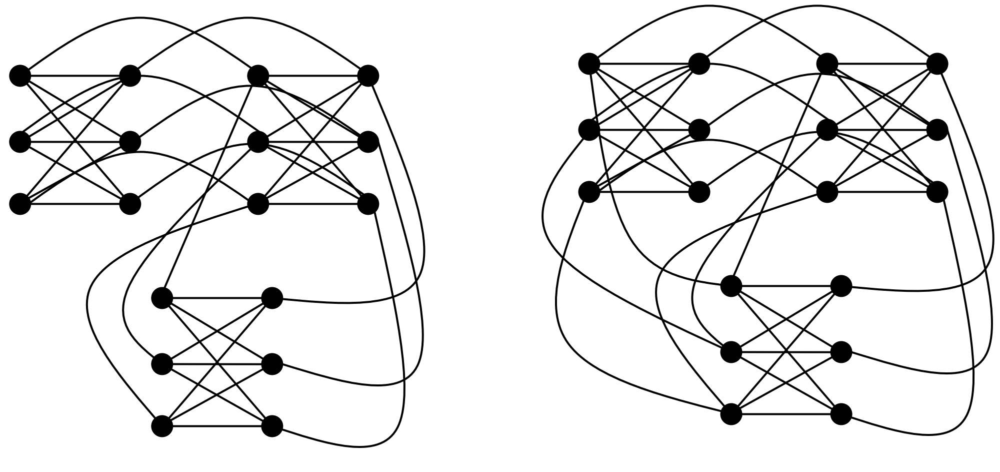

Figure 3.(a)$K_{3,3} \times P_3$(b).$K_{3,3} \times C_3$

Theorem 2.6. The Sanskruti index of tensor product for two graphs$K_{n,n}$and C_n is

$S\big(K_{n,n} \times C_n\big) = (n^3+2n^2)\left[\frac{n^4}{2n^2-2}\right]^3.$

Proof. Let$K_{n,n}$be the complete bipartite graph and $C_n$be the cycle graph. Let $G = K_{n,n} \times C_n$. We note that $|V(G)| = 2n^2$ and $|E(G)| = n^3 + 2n^2$ is shown in Figure 3(b).Vertices in this graph are of degree $n+2$. Using Table 6 and by the definition of $S(G)$ we get, $S\left(K_{n,n} \times C_n\right) = (n^3 + 2n^2)\left[\frac{n^4}{2n^2-2}\right]^3$. This result is true for all $n \geq 3$.

Table 6. The Edge Partition of $K_{n,n} \times C_n$ based on Degree Sum of Neighbourhood Vertices of End Vertices of Each Edge

Number of edges	$(S_u, S_v), uv \in E(G)$
$n^3 + 2n^2$	$((n+1)^2, (n+1)^2)$

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