PalArch's Journal of Archaeology of Egypt / Egyptology

FUZZY QUEUEING MODEL WITH AN UNRELIABLE SERVER USING ROBUST RANKING TECHNIQUE

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T.George, S. Santhi, K. Selvakumari.Fuzzy Queueing Model With An Unreliable Server Using Robust Ranking Technique--Palarch's Journal of Archaeology of Egypt/Egyptology 17(7), 4835-4840. ISSN 1567-214x

Keywords: Fuzzy Queues, Hexagonal Fuzzy Number, Heptagonal Fuzzy Number, Octagonal Fuzzy Number, Robust Ranking, Unreliable Server.

ABSTRACT:
This paper describes a method to find the variety of performance measures are in crisp values for fuzzy queueing model with an unreliable server where the service rate, entry rate, breakdown(collapse) rate and repair rate. These all rates are in some types of fuzzy numbers. After defuzzifying the fuzzy numbers apply the classical queueing performancemeasures formulas. Numerical examples are showing the implementation of the method.

INTRODUCTION

Queuing models have a vast application in managing associations. In most of the queueing models is always not possible to keep the operation of the server and service can be interrupted. An ordinary queueing system with interrupted service and weightage was first proposed by Gaver. Sengupta extended this system. Li et al and Wang developed the unreliable server in queueing models and recently investigate the controllable queueing systems with an unreliable server. Kao et al, Chen were scrutinising the fuzzy queues by Zadeh's principle. In this paper, our method is denoted by our notational convenience is FM/FM(FM)/I where the FM representing the exponential fuzzified arrival, service, repair, breakdown (collapse) rates, respectively. In other method M/M (M, M)/I represent the fuzzified exponential inter-arrival. i.e., Poisson, service, repair, breakdown rates, respectively. Here, fuzzy queueing model with an unreliable server with four fuzzy variables namely fuzzified exponential entry, service, repair, and breakdown (collapse) rate. The traditional queuing models in genuine circumstances are by fuzzy queuing models. The utilization of it is to develop a fuzzy number.Then transform the fuzzy queues is to defuzzified the fuzzy numbers by using Robust ranking technique.

PRELIMINARIES

Definition

A **fuzzy set** is characterized by its membership function taking values from the domain, space or the universe of discourse mapped into the unit interval [0,1]. A fuzzy set A in the universal set X is defined as $A = \{(x, \mu_A(x))/x\epsilon X\}$. Here $\mu_A(x): A \to [0,1]$ is the grade of the membership function and $\mu_A(x)$ is the grade value of $x \in X$in the fuzzy set A.

Definition

A fuzzy set A is called **Normal** if there exists an element $x \in X$ whose membership value is one, i.e., $\mu_A(x) = 1$.

Definition

A fuzzy set A' is **convex** if and only for any $z \in Z$ then the membership function of A' satisfies the condition $\mu_{A'}\{\lambda Z_1 + (1-\lambda)Z_2\} \geq \min\{\mu_{A'}(Z_1), \mu_{A'}(Z_2)\}$, $0 \leq \lambda \leq 1$.

Definition

A fuzzy number $\widehat{A_H}$ is a **Hexagonal fuzzy number**denoted by $\widehat{A_H} = (a_1, a_2, a_3, a_4, a_5, a_6)$ where $(a_1, a_2, a_3, a_4, a_5, a_6)$ are real numbers and its membership function $\mu_{\widehat{A_H}}(x)$ is given below.

$$\mu_{\widehat{A_H}}(x) = \begin{cases} 0 & , x < a_1 \\ \frac{1}{2}\frac{(x-a_1)}{(a_2-a_1)} & , a_1 \leq x \leq a_2 \\ \frac{1}{2} + \frac{1}{2}\frac{(x-a_2)}{(a_3-a_2)} & , a_2 \leq x \leq a_3 \\ 1 & , a_3 \leq x \leq a_4 \\ 1 - \frac{1}{2}\frac{(x-a_3)}{(a_4-a_3)} & , a_4 \leq x \leq a_5 \\ \frac{1}{2}\frac{(a_6-x)}{(a_6-a_5)} & , a_5 \leq x \leq a_6 \\ 0 & , x > a_6 \end{cases}$$

Definition

A fuzzy number $\widehat{A_{HE}}$ is a **Heptagonal fuzzy number**denoted by $\widehat{A_H} = (a_1, a_2, a_3, a_4, a_5, a_6, a_7)$ where $(a_1, a_2, a_3, a_4, a_5, a_6, a_7)$ are real numbers and its membership function $\mu_{\widehat{A_{HE}}}(x)$ is given below.

$$\mu_{\widehat{A_H}}(x) = \begin{cases} \frac{1}{3}\frac{(x-a_1)}{(a_2-a_1)} & , a_1 \le x \le a_2 \\ \frac{1}{3}+\frac{1}{3}\frac{(x-a_2)}{(a_3-a_2)} & , a_2 \le x \le a_3 \\ \frac{2}{3}+\frac{1}{3}\frac{(x-a_3)}{(a_4-a_3)} & , a_3 \le x \le a_4 \\ 1-\frac{1}{3}\frac{(x-a_4)}{(a_5-a_4)} & , a_4 \le x \le a_5 \\ \frac{2}{3}-\frac{1}{3}\frac{(x-a_5)}{(a_6-a_5)} & , a_5 \le x \le a_6 \\ \frac{1}{3}\frac{(a_6-x)}{(a_7-a_6)} & , a_6 \le x \le a_7 \\ 0 & , x > a_7 \end{cases}$$

Definition

A fuzzy number $\widehat{A_O}$ is a **Octagonal fuzzy number**denoted by $\widehat{A_O} = (a_1, a_2, a_3, a_4, a_5, a_6, a_7, a_8)$ where $(a_1, a_2, a_3, a_4, a_5, a_6, a_7, a_8)$ are real numbers and its membership function $\mu_{\hat{O}}(x)$ is given below.

$$\mu_{\hat{O}}(x) = \begin{cases} 0 & , x < a_1 \\ k\left(\frac{x-a_1}{a_2-a_1}\right) & , a_1 \le x \le a_2 \\ k & , a_2 \le x \le a_3 \\ k+(1-k)\left(\frac{x-a_3}{a_4-a_3}\right) & , a_3 \le x \le a_4 \\ 1 & , a_4 \le x \le a_5 \\ k+(1-k)\frac{(a_6-x)}{(a_6-a_5)} & , a_5 \le x \le a_6 \\ k & , a_6 \le x \le a_7 \\ k\frac{(a_8-x)}{(a_8-a_7)} & , a_7 \le x \le a_8 \\ 0 & , x > a_8 \end{cases}$$

ROBUST RANKING TECHNIQUE

The robust ranking technique is given by

$R(a) = \int_0^1 (0.5)(a_\alpha^L + a_\alpha^U)\, d\alpha$

Where $(a_\alpha^L + a_\alpha^U)$ is the α-level cut.

FUZZY QUEUEING MODEL WITH AN UNRELIABLE SERVER

Let us take a fuzzy queueing system with an unreliable server and two types of breakdowns. In type I, there are no customers in the system even if the server is breakdown. In type II, there is at least one customer in the system even if the server is breakdown. Consider the customers arrive at a single server with fuzzy rate $\hat{\lambda}$ as a Poisson process, service time with fuzzy rate $\hat{\mu}$ as an exponential distribution, a breakdown with fuzzy rate $\hat{\alpha}$ as a Poisson process and the repair with fuzzy rate $\hat{\beta}$ as an exponential distribution respectively.

Let $\varphi_{\hat{\lambda}}(x), \varphi_{\hat{\mu}}(y), \varphi_{\hat{\alpha}}(s), \varphi_{\hat{\beta}}(t)$ be themembership functions of $\hat{\lambda}, \hat{\mu}, \alpha and \hat{\beta}$. Then the following fuzzy sets are:

$$\hat{\lambda} = \left\{\left(x, \varphi_{\hat{\lambda}}(x)\right) / x \in X\right\}$$
$$\hat{\mu} = \left\{\left(y, \varphi_{\hat{\mu}}(y)\right) / y \in Y\right\}$$
$$\hat{\alpha} = \left\{\left(s, \varphi_{\hat{\alpha}}(s)\right) / s \in S\right\}$$
$$\hat{\beta} = \left\{\left(t, \varphi_{\hat{\beta}}(x)\right) / t \in T\right\}$$

Where X,Y,S and T are the universal crisp sets of the entry, service, break down and repair rates, respectively.

Let f (x, y, s, t) be the system characteristic of interest. Since x, y, s, and t and f (x, y, s, t) are all fuzzy numbers.

Let A and B denotes the membership function of the expected time and the system is idle in type I and type II, respectively.

Type I

$$A = f(x, y, s, t) = \frac{ty - x(s+t)}{y(s+t)}$$

Type II

$$B = f(x, y, s, t) = \frac{ty - x(s+t)}{ty}$$

In steady state, it is required as$0 < \frac{ty - x(s+t)}{y(s+t)} < 1$ and $0 < \frac{ty - x(s+t)}{ty} < 1$

ILLUSTRATIVE EXAMPLES

Consider a hospital appointment system in which patients using a single-server reservation arrival according to a Poisson process. The time required for the reservation process may be interrupted (system, server, or printer problem) as a Poisson process. The recovery times of the problem interrupted as an exponential distribution and the reservation ends. To evaluate the single-server reservation, the manager of the system wishes to calculate the total hours that the system is idle. It follows FM/FM (FM, FM)/1/∞ and the expected time that the system is idle, derived by the proposed method.

For Hexagonal Fuzzy Number

Suppose the entry, service, breakdown, repair rates are Hexagonal fuzzy number represented by $\hat{\lambda} = [1,2,3,4,5,6], \hat{\mu} = [2,3,4,5,6,7], \hat{\alpha} = [0.05,0.06,0.07,0.08,0.09,0.1], \hat{\beta} = [1,3,5,7,9,11]$

According to the Robust Ranking Technique,

$R(\hat{\lambda})= (1 + \alpha, 6 - \alpha) =\int_0^1 7(0.5)\, d\alpha = 3.5$

Similarly, we get,

$R(\hat{\mu})= (2 + \alpha, 7 - \alpha) =\int_0^1 9(0.5)\, d\alpha = 4.5$

$R(\hat{\alpha})$= $(0.05+\alpha, 0.1-\alpha)$=$\int_0^1 0.15(0.5)\, d\alpha = 0.075$
$R(\hat{\beta})$= $(1+\alpha, 11-2\alpha)$=$\int_0^1 11.5(0.5)\, d\alpha = 5.75$
Let us take x = 3.5, y =4.5, s =0.075 and t = 5.75
Applying the values x, y, s and t in type I and type II
Type I = $\frac{ty-x(s+t)}{y(s+t)}$
=$\frac{5.75(4.5)-3.5(0.075+5.75)}{4.5(0.075+5.75)}$
= 0.2093
Type II = $\frac{ty-x(s+t)}{ty}$
= $\frac{5.75(4.5)-3.5(0.075+5.75)}{(5.75)4.5}$
= 0.2121

For Heptagonal Fuzzy Number

Suppose the entry, service, breakdown, repair rates are Heptagonal fuzzy number represented by $\hat{\lambda} = [1,3,5,7,9,11,13], \hat{\mu} = [2,4,6,8,10,12,14]$, $\hat{\alpha} = [0.03,0.04,0.05,0.06,0.07,0.08,0.09], \hat{\beta} = [2,3,4,5,6,7,8]$
According to the Robust Ranking Technique,
$R(\hat{\lambda})$= $(1+\alpha, 13-2\alpha)$ =$\int_0^1 0.5(14-\alpha)\, d\alpha = 6.75$
Similarly, we get
$R(\hat{\mu})$= $(2+\alpha, 14-2\alpha)$ =$\int_0^1 0.5(16-\alpha)\, d\alpha = 7.75$
$R(\hat{\alpha})$= $(0.03+\alpha, 0.09-\alpha)$=$\int_0^1 0.5(0.12)\, d\alpha = 0.06$
$R(\hat{\beta})$= $(2+\alpha, 8-\alpha)$=$\int_0^1 10(0.5)\, d\alpha = 5$
Let us take x = 6.75, y =7.75, s =0.06 and t = 5
Applying the values x, y, s and t in type I and type II
Type I = $\frac{ty-x(s+t)}{y(s+t)}$
=$\frac{5(7.75)-6.75(0.06+5)}{7.75(0.06+5)}$
= 0.1172
Type II = $\frac{ty-x(s+t)}{ty}$
=$\frac{5(7.75)-6.75(0.06+5)}{5(7.75)}$
= 0.1186

For Octagonal Fuzzy Number

Suppose the entry, service, breakdown, repair rates are Octagonal fuzzy number represented by $\hat{\lambda} = [3,4,5,6,7,8,9,10], \hat{\mu} = [2,4,6,8,10,12,14,16]$, $\hat{\alpha} = [0.03,0.04,0.05,0.06,0.07,0.08,0.09,0.1], \hat{\beta} = [3,5,7,9,11,13,15,17]$
According to the Robust Ranking Technique,
$R(\hat{\lambda})$= $(3+\alpha, 10-\alpha)$ =$\int_0^1 0.5(13)\, d\alpha = 6.5$
Similarly, we get
$R(\hat{\mu})$= $(2+\alpha, 16-2\alpha)$ =$\int_0^1 0.5(18-\alpha)\, d\alpha = 8.75$
$R(\hat{\alpha})$= $(0.03+\alpha, 0.1-\alpha)$=$\int_0^1 0.13(0.5)\, d\alpha = 0.065$
$R(\hat{\beta})$= $(3+\alpha, 17-2\alpha)$=$\int_0^1 0.5(20-\alpha)\, d\alpha = 9.75$

Let us take x = 6.5, y =8.75, s =0.065 and t = 9.75

Applying the values x, y, s and t in type I and type II

Type I = $\frac{ty-x(s+t)}{y(s+t)}$

$=\frac{9.75(8.75)-6.5(0.065+9.75)}{8.75(0.065+9.75)}$

= 0.2505

Type II = $\frac{ty-x(s+t)}{ty}$

$= \frac{9.75(8.75)-6.5(0.065+9.75)}{9.75(8.75)}$

= 0.2522

COMPARISON TABLE

Comparing the fuzzy numbers Hexagonal, Heptagonal and Octagonal fuzzy number of Unreliable server of Type I and Type II.

Fuzzy Number	Type I	Type II
Hexagonal Fuzzy Number	0.2093	0.2121
Heptagonal Fuzzy Number	0.1172	0.1186
Octagonal Fuzzy Number	0.2505	0.2522

CONCLUSION

In this paper, FM/FM (FM, FM)/1/∞ queueing model is studied utilizing Robust Ranking technique with an unreliable server. It is used in operations and service mechanism for calculating system performance. The fuzzy number difuzzified by the robust ranking indices. Comparing the solved three fuzzy numbers. Finally, the manager can take the best optimum decisions.

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