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FUZZY SETS AND ITS APPLICATION IN DECISION MAKING PROBLEMS BY COMPARING THREE METHODS

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ABSTRACT:
Fuzzy sets are beneficial tool to detailed concept of uncertainty in decision making process. Making decisions is one of the most important activities of human beings. The field of decision making are the most effective and interesting area of application of fuzzy set theory. In this paper, we consider the theory of fuzzy sets to solve a real-world decision-making problem. The selection of the best one by defining linguistic value, Normalization and Weighted Normalized Decision matrix by using a Weighted sum method, Weighted Product method and Analytic Hierarchy Process. Comparing all and get a best one.

INTRODUCTION

In 1965, Prof. L.A. Zadeh proposed the fuzzy sets which is specified by a membership function with range in the unit interval [0, 1]. Fuzzy sets related to many fields such as medical diagnosis, engineering, etc. The fuzzy sets provide tools to deal with imprecision to many problems. The field of decision making is the most effective and relevant area of applications of fuzzy set theory. Making decisions is one of the most important activities of human beings. Decision making under incomplete or uncertain information allows fuzzy logic. Decision making is a tool to finding the best. There are many kinds of decision making. The decision is taken by a single person or a many

person called as an individual decision making and multi decision making, respectively. It is particularly useful to finding the incomplete solution. This paper has been synopsized as basic definitions, procedure for Weighted Sum method (WSM), Weighted Product method (WPM) and Analytic Hierarchy Process (AHP) and their examples.

PRELIMINARIES

Definition

A **fuzzy set** is characterized by its membership function taking values from the domain, space or the universe of discourse mapped into the unit interval [0,1]. A fuzzy set A in the universal set X is defined as $A = \{(x, \mu_A(x))/x \epsilon X\}$. Here $\mu_A(x): A \rightarrow [0,1]$ is the grade of the membership function and $\mu_A(x)$ is the grade value of $x \in X$ in the fuzzy set A.

Definition

A fuzzy set A is called **Normal** if there exists an element $x \in X$ whose membership value is one, i.e., $\mu_A(x) = 1$.

Definition

Here, we are using two criteria namely **Beneficial** and **Non-Beneficial**.

Beneficial criteria: $\frac{X_{ij}}{Max(X_{ij})}$

Non-Beneficial criteria: $\frac{Min(X_{ij})}{X_{ij}}$

(A) PROCEDURE FOR WEIGHTED SUM METHOD(WSM)

STEP 1: Construct the given problem in a table.
STEP 2: Determine the linguistic values to the given specification.
STEP 3: Tabulate the Normalization table using Beneficial and Non-Beneficial criteria.
STEP 4: Give the weightage to the above Normalized Decision matrix.
STEP 5: Construct the weighted sum Normalized decision matrix by using the formula called

$$(A_i)^{WSM} = \sum_{j=1}^{n} w_j x_{ij}$$

STEP 6: Selecting the best one.

(B) PROCEDURE FOR WEIGHTED PRODUCT METHOD(WPM)

STEP 1: Construct the given problem in a table.
STEP 2: Determine the linguistic values to the given specification.
STEP 3: Tabulate the Normalization table using Beneficial and Non-Beneficial criteria.
STEP 4: Give the weightage to the above Normalized Decision matrix.
STEP 5: Construct the weighted product Normalized decision matrix by using the formula Called

$$(A_i)^{WPM} = \prod_{j=1}^{n} {x_{ij}}^{w_i}$$

STEP 6: Selecting the best one.

(C) PROCEDURE FOR ANALYTIC HIERARCHY PROCESS (AHP)

STEP 1: Develop a model for the business.
STEP 2: Derive priorities (weights) for the criteria.
STEP 3: Check the consistency (weights assigned are correct or wrong?)
STEP 4: Calculate overall priorities (Model synthesis) and take a final decision by decisionMaker.

ILLUSTRATIVE EXAMPLE

Suppose a person wants to buy a mobile phone in a shop. In that shop, there are five types of mobile phones. Let $U = \{A, B, C, D, E\}$ be the set of mobiles. Let C be the parameters$C = \{C_1 = cost, C_2 = RAM, C_3 = Camera, C_4 = Batterybackup\}$. The problem is to find the best mobile by using WSM, WPM and AHP method.

Solution

(a) WSM Method

Table 1

Criteria→	Non-Beneficial	Beneficial	Beneficial	Beneficial
Alternative↓	Cost	RAM	Camera	Battery
Mobile A	350 $	32 GB	16 MP	Poor
Mobile B	250 $	16 GB	12 MP	Not Bad
Mobile C	200 $	16 GB	8 MP	Average
Mobile D	300 $	32 GB	8 MP	Good
Mobile E	240 $	16 GB	16 MP	Excellent

Table 2

Linguistic Terms	5 Point Scale
Poor	1
Not Bad	2
Average	3
Good	4
Excellent	5

Table 3

Criteria→	Non-Beneficial	Beneficial	Beneficial	Beneficial
Alternative↓	Cost	RAM	Camera	Examine
Mobile A	350 $	32 GB	16 MP	5
Mobile B	250 $	16 GB	12 MP	3
Mobile C	200 $	16 GB	8 MP	4
Mobile D	300 $	32 GB	8 MP	2
Mobile E	240 $	16 GB	16 MP	1

Table 4 Normalization Table

Criteria→	Non-Beneficial	Beneficial	Beneficial	Beneficial
Alternative↓	Cost	RAM	Camera	Examine
Mobile A	0.5714	1	1	0.2
Mobile B	0.8	0.5	0.75	0.6
Mobile C	1	0.5	0.5	0.4
Mobile D	0.6667	1	0.5	0.8
Mobile E	0.8333	0.5	1	1

Table 5 Weighted Sum Normalized Decision Matrix

Weightage	**25%**	**25%**	**25%**	**25%**	
Criteria→	**Non-Beneficial**	**Beneficial**	**Beneficial**	**Beneficial**	**Performance**
Alternative↓	**Cost**	**RAM**	**Camera**	**Examine**	
Mobile A	0.1428	0.25	0.25	0.05	0.6928
Mobile B	0.2	0.125	0.1875	0.15	0.6625
Mobile C	0.25	0.125	0.125	0.1	0.6
Mobile D	0.167	0.25	0.125	0.2	0.7417
Mobile E	0.2083	0.125	0.25	0.25	0.8333

Table 6 Rank Allocation Table

Alternative↓	**Overall Performance**	**Rank**
Mobile A	0.6928	3
Mobile B	0.6625	4
Mobile C	0.6	5
Mobile D	0.7417	2
Mobile E	0.8333	1

Finally, the person wants to purchase a best mobile that is **Mobile E.**

(b) WPM Method

Table 1

Criteria→	**Non-Beneficial**	**Beneficial**	**Beneficial**	**Beneficial**
Alternative↓	**Cost**	**RAM**	**Camera**	**Battery**
Mobile A	350 $	32 GB	16 MP	Poor
Mobile B	250 $	16 GB	12 MP	Not Bad
Mobile C	200 $	16 GB	8 MP	Average
Mobile D	300 $	32 GB	8 MP	Good
Mobile E	240 $	16 GB	16 MP	Excellent

Table 2

Linguistic Terms	**5 Point Scale**
Poor	1
Not Bad	2
Average	3
Good	4
Excellent	5

Table 3

Criteria→	**Non-Beneficial**	**Beneficial**	**Beneficial**	**Beneficial**
Alternative↓	**Cost**	**RAM**	**Camera**	**Examine**
Mobile A	350 $	32 GB	16 MP	5
Mobile B	250 $	16 GB	12 MP	3
Mobile C	200 $	16 GB	8 MP	4
Mobile D	300 $	32 GB	8 MP	2
Mobile E	240 $	16 GB	16 MP	1

Table 4 Weighted Product Normalized Decision Matrix

Weightage	**25%**	**25%**	**25%**	**25%**	
Criteria→	**Non-Beneficial**	**Beneficial**	**Beneficial**	**Beneficial**	**Performance**
Alternative↓	**Cost**	**RAM**	**Camera**	**Examine**	
Mobile A	0.8694	1	1	0.6687	0.5814
Mobile B	0.9457	0.8409	0.9306	0.8801	0.6513
Mobile C	1	0.8409	0.8409	0.7953	0.5623
Mobile D	0.9036	1	0.8409	0.9457	0.7186
Mobile E	0.9554	0.8409	1	1	0.8034

Table 5 Rank Allocation Table

Alternative↓	**Overall Performance**	**Rank**
Mobile A	0.5814	4
Mobile B	0.6513	3
Mobile C	0.5623	5
Mobile D	0.7186	2
Mobile E	0.8034	1

Finally, the person wants to purchase a best mobile that is **Mobile E.**

(c) AHP Method

Step 1: Developing a hierarchical structure with a goal at the top level. The attributes or criteria at the second level and the alternatives at the third level.
Step 2: Determine the relative importance of different criteria with respect to the goal.

Pair wise comparison matrix is created with the help of scale of relative importance.

1	Equal importance
3	Moderate importance of one over another
5	Strong (or) Essential importance
7	Very Strong importance
9	Extreme importance
2,4,6,8	Intermediate values between the two adjacent judgements
1/3,1/5,1/7,1/9	Value for inverse comparison

Pairwise Comparison Matrix
A1:

	Cost	**RAM**	**Camera**	**Battery**
Cost	1	2	4	3
RAM	½	1	3	2
Camera	¼	1/3	1	2
Battery	1/3	1/2	1/2	1

$\text{Cost}(1 \times 2 \times 4 \times 3)^{1/4} =$ 2.2134
$\text{RAM}(0.5 \times 1 \times 3 \times 2)^{1/4} =$ 1.3161
$\text{Camera}(0.25 \times 0.3333 \times 1 \times 2)^{1/4} =$ 0.6390
$\text{Battery}(0.3333 \times 0.5 \times 0.5 \times 1)^{1/4} =$ 0.5372

Sum= **4.7057**

A2: $\begin{bmatrix} 0.4704 \\ 0.2797 \\ 0.1358 \\ 0.1142 \end{bmatrix}$

Step 3:A3=A1 x A2 **A4= A3/A2**

A3: $\begin{bmatrix} 1.9156 \\ 1.1507 \\ 0.5750 \\ 0.4787 \end{bmatrix}$ A4: $\begin{bmatrix} 4.0723 \\ 4.1141 \\ 4.2342 \\ 4.1918 \end{bmatrix}$

Measurement of Inconsistency

Average of A4 is $\lambda_{max} = 4.1531$

Consistency Index $= \frac{\lambda_{max} - n}{n-1}$

$= \frac{4.1531-4}{3} = 0.05$

C.I. = 0.05

Random Index (R.I.)

Attributes	3	4	5	6	7	8	9	10
(R.I.)	0.52	0.89	1.11	1.25	1.35	1.4	1.45	1.49

Consistency Ratio

Consistency Ratio (C.R.) $= \frac{Consistencyindex}{Randomindex}$

$= \frac{0.05}{0.9} = 0.05 < 0.1$

It is consistency.

Step *4*

Weights calculated by using AHP method is

$$\begin{bmatrix} Cost \\ RAM \\ Camera \\ Battery \end{bmatrix} = \begin{bmatrix} 0.4704 \\ 0.2797 \\ 0.1358 \\ 0.1142 \end{bmatrix}$$

Criteria→	**Non-Beneficial**	**Beneficial**	**Beneficial**	**Beneficial**
Alternative↓	**Cost**	**RAM**	**Camera**	**Battery**
Mobile A	350 $	32 GB	16 MP	Poor (1)
Mobile B	250 $	16 GB	12 MP	Not Bad (2)
Mobile C	200 $	16 GB	8 MP	Average (3)
Mobile D	300 $	32 GB	8 MP	Good (4)
Mobile E	240 $	16 GB	16 MP	Excellent (5)

Normalization Table

Criteria→	**Non-Beneficial**	**Beneficial**	**Beneficial**	**Beneficial**
Alternative↓	**Cost**	**RAM**	**Camera**	**Examine**
Mobile A	0.5714	1	1	0.2
Mobile B	0.8	0.5	0.75	0.6

Mobile C	1	0.5	0.5	0.4
Mobile D	0.6667	1	0.5	0.8
Mobile E	0.8333	0.5	1	1

Synthesis of Model

Weights	**0.4704**	**0.2797**	**0.1358**	**0.1142**	**Priority**
Alternative↓	**Cost**	**RAM**	**Camera**	**Examine**	
Mobile A	0.5714	1	1	0.2	0.7071
Mobile B	0.8	0.5	0.75	0.6	0.6865
Mobile C	1	0.5	0.5	0.4	0.7238
Mobile D	0.6667	1	0.5	0.8	0.7526
Mobile E	0.8333	0.5	1	1	0.7818

Rank Allocation Table

Alternative↓	**Overall Performance**	**Rank**
Mobile A	0.7071	4
Mobile B	0.6865	5
Mobile C	0.7238	3
Mobile D	0.7526	2
Mobile E	0.7818	1

Finally, the person wants to purchase a best mobile that is **Mobile E.**

Comparing WSM, WPM and AHP Method

Methods	**Best Mobile**
WSM	Mobile E
WPM	Mobile E
AHP	Mobile E

CONCLUSION

In fuzzy decision- making problem the person wants to buy a mobile in a shop. In that shop 5 types of mobiles and he considering four criteria. In this paper, we are comparing three methods. That is weighted sum method, weighted product method and Analytic Hierarchy method. Finally, mobile E is the best one according to the criteria.

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