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## PIGEON HOLE PRINCIPLE AND MATHEMATICAL INDUCTION CONTRACTION

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**Abstract:** This paper tries to find the deficiencies and non-Universal validation of Pigeonhole Principle as well as established Peano's axiom as principle of Mathematical Induction. There is emergent need to search the applicability of Real number system in both of the cases. In the case of fractional particle which come out during reactions of different significant or non-significant needs based study for the industrial production process too may not be estimated by Pigeon hole principle or Principle of Mathematical Induction. Some other simulation process and treatments may not be limited to the scope of the set of Natural Number system that starts with unity and ends at infinity but what about Rational and irrational digits means a set of Real number system. Micro particles and their applicability and setting them to an extent level of assumption can make any Research task easy and significant for the further outcomes forecasting. Principle of Mathematical induction has been an important tool in proving any propositions or statements. Researchers have contributed more by saying that Pigeon hole principle may be proved with the help of principle of Mathematical induction and vice versa but none of these two established principles speak about the Rational and irrational digit taking case and analysis at these contexts, the enlistment rule is just an extraordinary instance of the categorize rule, and numerous combinatorial checking contentions lessen to the categorize rule and these two may have some lacuna in managing a few realities and structures of Analysis. Regardless of whether one can get rid of the feeble categorize rule, by demonstrating the presence of endlessly numerous primes inside  $\mathbb{N}$ .

**Keywords:** PMI, Real number system  $\mathbb{R}$ , Peano axiom, pigeonhole principle,

**Introduction:** In the study of Mathematical structures, it tends to be discovered that the enlistment standard is basically an extraordinary instance of the categorize guideline and

numerous combinatorial counting contentions decrease to the pigeonhole rule. Maybe as anyone might expect, at that point in addition or not one can shed the powerless categorize guideline, by demonstrating the presence of endlessly numerous primes inside  $\Delta O$  greyer the inherent difficulty of proving the pigeonhole principle is tightly connected to important question in proof theory and circuit complexity of another aspect of science and Technology. Thought has filled in as the exemplary hard model for evidence unpredictability and renditions of it had been utilized to get probably the most grounded lower bounds and partitions known date during various numerical treatments. For this we have a few models which incorporate Resolution, limited Cth Free frameworks, Cutting Planes, and relativized limited math existing. Presently a days, there are a few significant open issues associated with the intricacy of the more fragile types of the pigeonhole principle specifically with the weak pigeonhole principles which we characterize to be the situation in which  $n=m/2$ . During study it was, as at first saw by Macintyre on the setting of the presence of quadratic no buildups where the pigeonhole rule is personally associated with how much number they can be demonstrated in  $\Delta O$ , a powerless arrangement of math. Paris, Wilkie and Woods who had demonstrated that a significant piece of rudimentary number theory and furthermore the presence of endlessly numerous primes is conceivable and provable in  $\Delta O$  with the frail categorize rule for  $D_0$ -determinable fun particles added as an adage conspire that show towards a concealed lac an as there is no express. It is a longstanding open inquiry for all the specialists worry that whether one can get rid of the powerless pigeonhole principle, by demonstrating the presence of boundlessly numerous primes inside  $\Delta O$ . Author of this paper is under the feeling that all known confirmations of this reality red on the powerless categorize standard and its widespread application complexity. Interestingly, these outcomes interpret downwards as there are limited depth virgin polynomial-size circuits that can around include the quantity of 1's in a 0/1 cycle string during calculation. Although, indeed, all known evidences of accuracy require a lot higher verification hypothetical multifaceted nature work. Undoubtedly, this is exceptionally confounding and is it conceivable to demonstrate that little circuits exist for inexact checking, and furthermore to demonstrate that any accuracy verification for these little circuits is innately More unpredictable than these circuits, more modest or greater in any setting Researchers will find a positive solution and would follow on the off chance that one could demonstrate excessively polynomial lower limits on the size of limited profundity Frege evidences of the powerless categorize rule for the situation when  $n=m/2$  as a specific.

It appears to be that the unpredictability of the feeble pigeonhole principle is associated with the characteristic intricacy of demonstrating circuit lower limits additionally this standard has no explicit answer about the Real Number system as a whole.

### **Material and Method:**

Some Lacuna in the Principle of Induction:

Following are the facts asymptoms of misunderstanding in PI:

**Lack of Universality of testing** the rightness of the assertion for a pointless number of situations where Students will once in a while test the accuracy of the assertion for  $n=i$  and for  $n=2$  then  $n=m$  but whereas it would have sufficed to test it only for  $n=1$  One can analyze that here are many possible reasons why a

student would do so as regressive one fold '?'. Quick actuality is that an understudy's secondary teacher may have done as such, other understudy may have had a memory of his educator doing so consistently. May be cases, there are a few cases they do require testing the accuracy of the assertion for both  $n = 1$  and  $n = 2$ ,  $n=3$  and other natural numerical values but some where there should some extent hinting about the fractional position of a digit that equally important in the a sis of so many complex problems

**A. Selection of Basis of analysis of PI :** Selection of Basis of investigation of PI : Students are acclimated with demonstrating statements for any Natural  $n$  there isn't anything here to examine about the division part o number as a Rational or an Irrational number. Matter become part l during communication when understudies are approached to demonstrate an assertion and its approval, state for any normal  $n= 4$ , or for any considerably number, Here analyzers experience issues in choosing for themselves that what the premise of enlistment should be in the specific instance of conversation.

**C. Lack of Lemma and Misunderstanding of the Increment approximation.'**

At the point when an assertion is to be demonstrated for any Natural number  $N$ , the addition generally taken is 1 in the pattern circumstance. For different cases, in any case, other Increments can be suitable yet in any circumstance. Take an occurrence, if an understudy means to utilize enlistment to demonstrate an assertion for any event, or odd, common number, the addition will be 2 and afterward some time as regular and understudies learn, for instance, that when utilizing proof by acceptance for any even  $n$ , they should utilize an increment of 2, yet no celerity for a fragmentary arrangement However, after encountering an issue whereby they should demonstrate an assertion for each odd Natural Number  $N$ , students utilize an Addition of 1. When confronted with a difficult that requires the verification of an assertion for all  $n$  that are partitioned by certain another digits.

D. Just to find in specific cases it is smarter to utilize increases that are more

noteworthy than 1, state and augmentation of size  $k$ . For this situation, the accuracy of the assertion should be tried for  $n=i-1$ . Again, the redundancy of previous process.

### E. **Principle of Mathematical Induction**

Principle of Mathematical induction can be based for any set that is equal to the arrangement of Natural numbers, again here is characterized area of Natural Numbers just. For instance, the arrangement of negative numbers. For this situation, the premise of acceptance will be  $n \in \mathbb{I}$  and it should be demonstrated that the accuracy of the assertion for  $n \in \mathbb{I}$  follows from the assertion's rightness for  $n=1$ , as it the warrants of the investigation.

F. If there is a circumstance like as acceptance guideline can be likewise utilized on two factors. Let us guess we need to demonstrate that the assertion  $P(m, n)$  is valid for any regular  $m$  and  $n$ . For this situation, we start by demonstrating the premise of induction for one of the factors as  $n$ . For the accommodation, the rightness of  $P(m, 1)$  should be exhibited for any characteristic  $m$ . If conceivable, this can be indicated utilizing enlistment regarding  $m$ . At last, it should be demonstrated that the rightness of statement.  $P(m, n=1)$  follows from the rightness of  $P(m, n)$ , for any  $m$  and  $n$ .

G. There are cases in which acceptance can be utilized on a variable that is covered up in the issue. Understudies are normally acquainted with applying the rule of acceptance on factors that shows up in the issue, yet now and again a more prominent level of inventiveness is called for. On occasion, another variable should be characterized, and the guideline of acceptance applied to it. This sort of innovation somewhat takes after that which should be shown when one is needed to utilize helper development as the underlying stage in a mathematical verification.

### **Result and Discussion:**

A few inconsistencies in PMI: One approach to all the more likely sense where the holes in the understudies' Understanding of the acceptance standard is to introduce incorrect proofs by enlistment of mistaken proclamations and request that the understudies distinguish the slip-ups in the evidences. We don't propose that this activity be utilized with understudies who are

simply starting their excursion through the point; it may Cause them to lose confidence in the acceptance guideline, confidence that is now rather precarious. We will introduce two well known "paradox", with an alternate problematic point.

Result -/: For some random gathering of young men, the tallness of the entirety of the young men in the gathering is

Verification: Let us see Proof by acceptance on  $n$ , where  $n$  is the quantity of individuals in the gathering.

For  $n = 1$ , the assertion is clearly true, since in any gathering that contains just a single individual, the stature of the people in the gathering is uniform (and equivalent to the tallness of the said individual).

We expect the assertion is valid for any natural  $n$  and demonstrate it would be valid for additional  $n = 1$

Let  $A$  be a gathering of  $n$  young men for any natural  $n$  and prove it would be true for further  $n = 1$ . Presently, we need will demonstrate that the stature of the Boys in gathering  $A$  with  $n$  uniform boys. Allow us to eliminate someone in particular,  $x$ , from bunch  $A$ . Subsequently, we have now a gathering containing  $n$  young men, which we indicate by  $B$ . As indicated by the acceptance standard, the stature of the young men in gathering  $B$  is uniform. Allow us to expect that this stature is equivalent to  $x$  meter. Presently, let us take bunch  $A$  again and eliminate an alternate kid from this gathering, signified by  $y$ . The gathering acquired will be called bunch  $D$ . Since bunch  $D$  contains  $n$  individuals, the tallness of individuals might be assessed in the arrangement that comes up short on a choice based on PMI

**Conclusion:** Pigeon hole Principle and Principle of Mathematical Induction very much convergent to each other in so many cases a Mathematical structure both are unable to give some clear indication. Researchers will find a positive solution and would follow in the event that one could demonstrate excessively polynomial lower limits on the size of bounded-depth Frege evidences of the

powerless pigeonhole for the situation when  $n=m/2$  as a specific. The multifaceted nature of the powerless categorize standard is identified with the unpredictability of inexact tallying and there ought not to be any room of confusion in this. Both principles have some sorts of lacunas as became clear from the abovestudy.

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