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ENGINEERING A FEASIBLE RAMP FOR THE GREAT PYRAMID OF GIZA

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ABSTRACT

Although it is widely believed by archaeologists that the Great Pyramid was built using sleds hauled up ramps, no economically feasible ramp configuration has yet been found which would have permitted the placement of the 44 granite beams weighing up to 75 t and the 2.3 Mm^3 of limestone blocks of the pyramid, in a period corresponding to the 27 year reign of Pharaoh Khufu. This paper focuses on engineering considerations: it proposes a simple configuration which is structurally sound and consistent with the archaeological evidence and the principles of ergonomics, mechanics and materials engineering, with a volume of only 6% of that of the pyramid. It demonstrates how the blocks, beams, supporting capstones and pyramidion could have been placed using only the tools found at Giza which date from the 4th Dynasty or earlier, within the constraints imposed by the topography of the Giza Massif.

Abbreviations

h	hour
km	kilometre
m	metre
M	million
s	second
t	tonne

Note that the expression 'at level n' makes reference to a point or surface at a level of n metres above the foundation level of the Great Pyramid.

Ramp Configuration Hypotheses

Many authors have proposed alternative ramp configurations for the Great Pyramid, most of them combinations of straight and helical ramps. Nine examples of these are shown by Lehner (2007: 216). Generally, the volumes of the ramps are so large compared with the volume of the pyramid itself that they can be ruled out simply because of the extra work involved. Lehner's own proposal (1985) was for a ramp that would have had a volume one-and-a-half that of the pyramid. More recently, Houdin

(2006), an architect, proposed an internal ramp with a complicated counterweight device to raise heavier elements. None of these configurations appear to be simple or small enough to have been feasible.

In the last decade, a number of non-archaeologists, mostly professional engineers, have published studies that assert that a ramp/sled combination would not have been feasible, and instead have proposed alternative lifting mechanisms (Fonte, 2013; Hitchens, 2010; Isler, 1985; James, 2013; Massey, 2012; Parry, 2004). However, the configuration proposed in figure 1 demonstrates that such a combination is indeed feasible. As discussed later, we do not yet have conclusive evidence about how the Great Pyramid was built, and cannot therefore assess whether the hypothesis set out in this paper is correct. The purpose of this paper is to demonstrate that using ramps was possible with the resources of the time, and that there were a number of technical constraints that would have had to be respected, whatever solution was adopted.

General Applicability of the Hypothesis
It is unlikely that all pyramids were built in the same way. Their designs changed as they evolved over the 80-odd years from the start of the reign of Zoser to that of Khufu, and they continued to evolve under later pharaohs. No other previous pyramid was as complex as the Great Pyramid, and none other had the problems associated with the haulage of blocks as heavy as those found there (Lehner, 2007; Verner, 2002). If, as is shown here, a feasible hypothesis can be formulated for the construction of the Great Pyramid, then it is extremely likely that broadly similar hypotheses can be formulated for the construction of each of the other pyramids, modified to be consistent with the archaeological evidence available at each site.

Planning
Whatever ramp configuration was used, a project to construct the Great Pyramid would have required careful planning, possibly using clay models to assess the volume of work required at various stages and to explore alternative approaches.

The planning would have been undertaken by the principal architect, probably Hemiunu, and based on the experience gained by him and his forefathers, who had been architects on previous pyramids (Malek, 2000). They would have been aware of the maximum rate of work
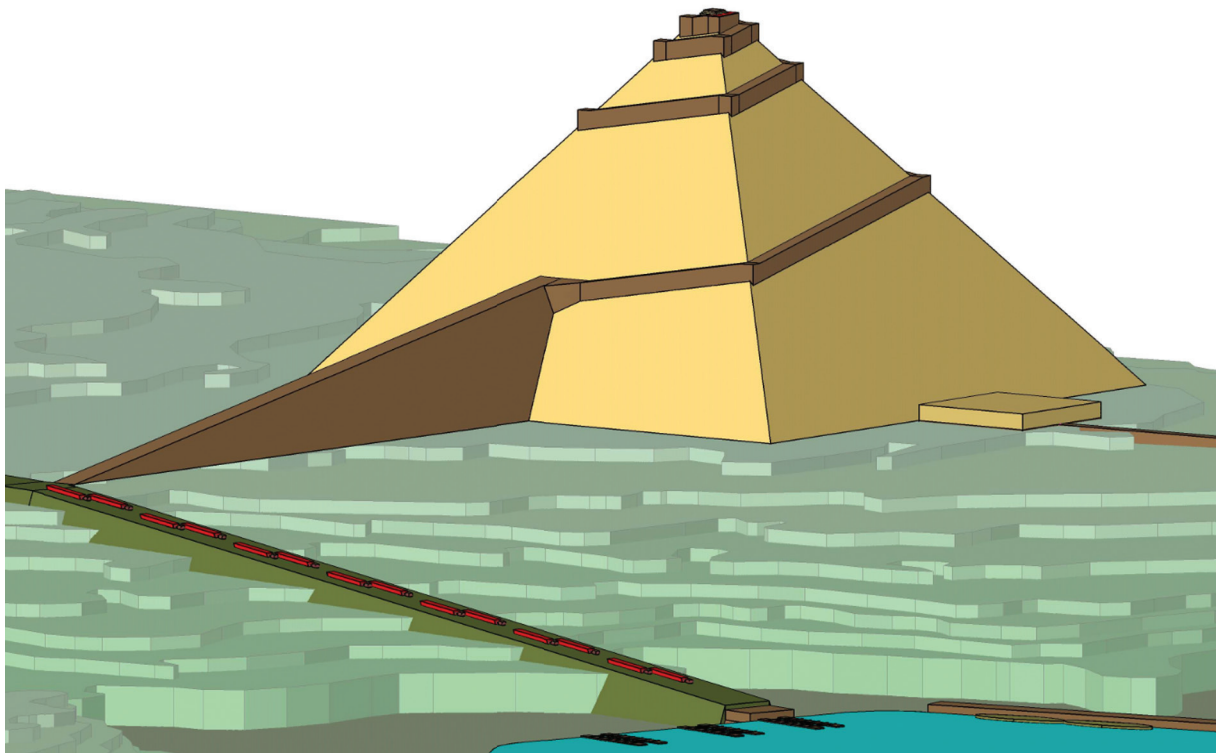

Figure 1. Proposed ramp configuration. © The author.

that could be achieved in quarrying and placing stone in previous pyramids. If Snefru had completed the Bent and Red pyramids within the 28-year period suggested by Stadelman (1997), the average rate of construction would have been 0.11 Mm³/year, and the peak rate possibly twice this figure. While we can only estimate this figure, Heminunu would have been in a position to know the maximum rates possible at different stages.

Without such planning, it would have been impossible to estimate the logistics, the size of workforce required, the location and size of workers camps, food preparation facilities, numbers of ships, harbour capacity, quantity of timber, rope and tools etc. Plans may have required periodic revision due to design changes and unforeseen delays due to technical and no doubt financing problems, but nevertheless planning would have been essential.

Construction Period

Planning on this basis, Hemiunu would not have needed to know how long Khufu would live. He could estimate how long it would take to complete the task, and like any prudent engineer would no doubt have added a contingency margin of 10-15% to reduce the problems that he might face if he were seen to be falling behind schedule. There is no evidence of unusual climatic conditions, floods or droughts or increased military activity during the construction period, so in all likelihood, his estimate was probably not far out. Working back from the actual construction time of 27 years, and adjusting for delays, it seems likely he would have planned to complete the works in around 23 years.

Khufu may well have found this acceptable. Of the six or seven pharaohs of the 3rd and 4th Dynasty who had preceded him, Egyptologists estimate that four of them had reigned for similar periods. Typical estimates are 19-20 years for Zoser; 9-21 years for Sanakht/Nebka, 24 years for Huni and anything from 24-50 years for Snefru; estimates for the others are 5-8 years for Sekhemkhet and 3-24 years for Khaba (Allen, 1936; Arnold, 2003; Beckerath, 1999; Clayton, 1994; Dodson, 2000; Dodson & Hilton 2004; Krauss 1985; Lehner, 2007: Malek, 2003; Murnane, 1997; Redford, 2001; Shaw, 2000; Sitek, 2007; Verner 2002). These estimates are based on the limited available evidence from

King lists, seals and other archaeological findings as interpreted by different experts, and may be in error by significant margins. Until recently, there was widespread agreement among Egyptologists that Khufu reigned for 23 years, but recent evidence suggests that he may have done so for as long as 27 years (Tallet & Marouard, 2014).

Thus, in this article it is assumed Hemiunu planned for a construction period of 23 years. As discussed below, this period would have included time for preparatory works, and for finishing off once all the blocks in the structure had been placed.

Pyramid Structure

The Great Pyramid was built on the Giza Massif on the west bank of the Nile in Egypt between 2600 and 2500 BCE with its foundation 60 metres above sea level. It was originally built to a height of 147 m above its foundations on a square base with a side of 230 m, and had a notional volume, calculated as one-third of the product of the height and base area, of 2.6 Mm³. What now remains is a core of 203 stepped courses of grey limestone with thicknesses that diminish irregularly with elevation from 1.5 m to 0.6 m (Petrie, 2013: Plate 8). From the curve relating course thickness to elevation (figure 2), it can be deduced that there were originally a further ten courses each around 0.6 m thick, plus three courses of capstones.

The core was surrounded by a layer of white limestone casing blocks that were originally laid in the same stepped formation, but which were trimmed *in situ* from the top down to create a pure pyramid with a glacis of 53% i.e. 11 horizontal to 14 vertical (Cole, 1925: 6).

Within the lower courses of the pyramid is a knoll or hump of native rock, the remnants of a small hillock that originally occupied the site and which was terraced to receive core blocks. The maximum height of the hump is probably about 8 m above the foundation (Petrie, 2013: 211) and its volume therefore corresponds to about 10% of that of the notional volume of the pyramid.

Within the core are several passages, galleries and chambers lying along a vertical plane 7 m east of the north-south axis. The total volume enclosed within them is less than 0.1% of the pyramid volume, and they are mostly built
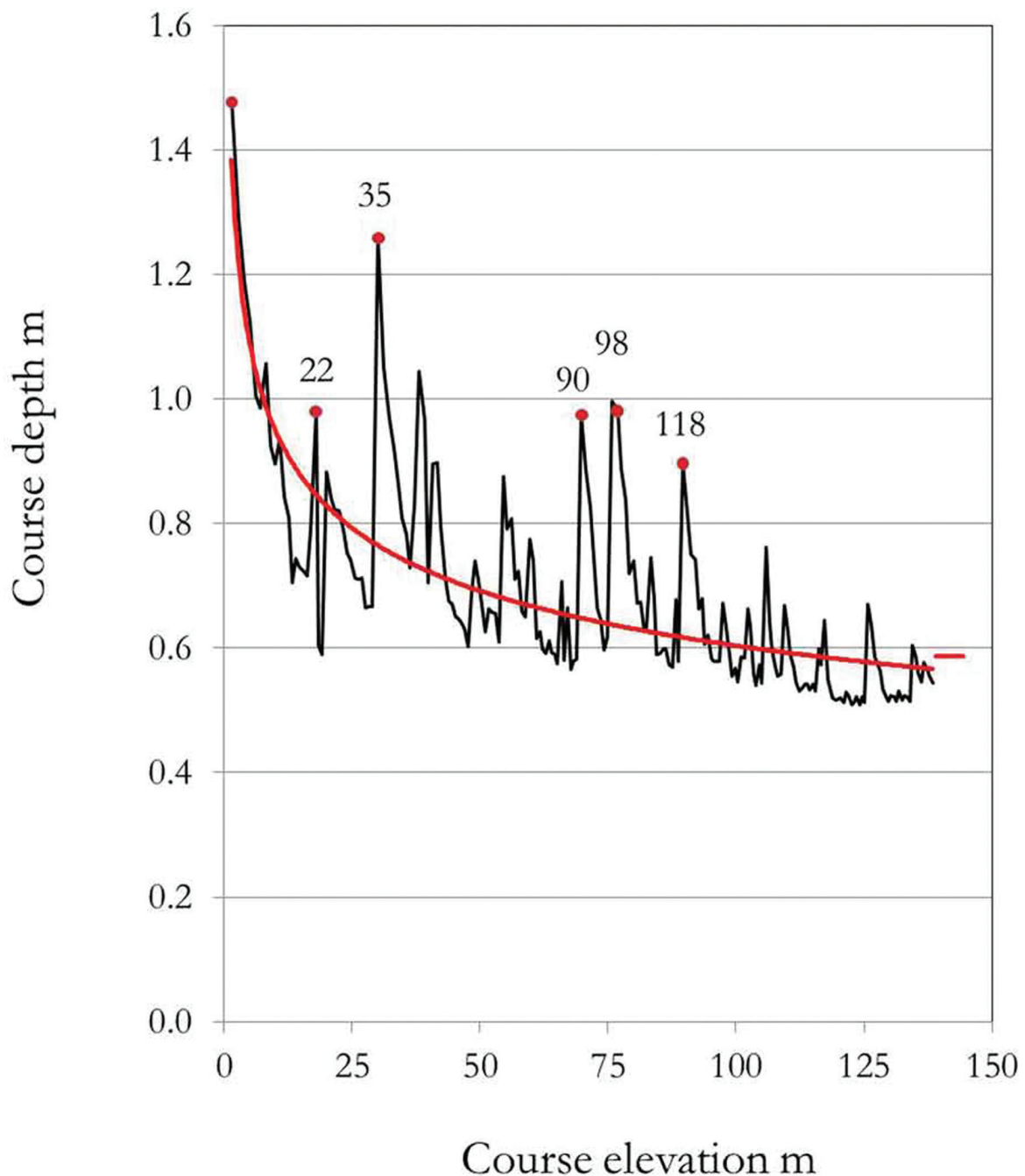
Figure 2. Variation of course thickness with elevation. © The author.

with limestone blocks. The exception is the King's Chamber, which has all four walls and a floor made from rose granite, and the four superimposed chambers, each floored with a layer of granite beams around 2 m deep. These were drawn in detail by Perring (1839: Plate IV) and from these drawings the total estimated weight of granite is 2,500 t. The weight of the largest individual beam, in the roof of the King's Chamber itself, is 75 t. This beam is oriented north-south, its base is at level 49 (i.e. 49 m above foundation level), and its centre of gravity lies on the axis of the King's Chamber roof, 11 m almost due south of the centre of the pyramid. The base of the highest granite floor is at level 60.

The adjacent faces of the remaining casing blocks fit very closely, but the core blocks fit far less well and there are many interstices between adjacent blocks, most of which are filled

with sand, limestone chippings or a sand and gypsum mortar (Lehner, 2007: 109). There is no evidence of a systematic tiling pattern in the orientation of the visible core blocks in each course, which exhibit a great range of widths and lengths. A number of core blocks intrude into other courses, and occasionally there are two superimposed blocks within a single course, but these cases are a small proportion of the whole. Where exposed in the interior spaces and in excavations made by intrusive explorers, such as the Robber's Tunnel and the gash excavated in the south face (Vyse, 1840: 166), the interior of the core appears to be similar in structure to the visible exterior.

Provenance of Construction Materials

The granite beams were quarried in Aswan, 930 km by river from Giza, and the white limestone facing blocks from the Tura quarries on the east bank 15 km to the south. Gypsum for the mortar and basalt for the pavement around the pyramid came from quarries to the north of the Fayoum, a distance of 250 km by river (Lehner, 2007: 202). Much of the core grey limestone material was quarried on site, but after allowing for the high wastage of rock (30-50%) in the production of blocks (Lehner, 2007: 206), it appears that a significant proportion of the total must have been imported from the 18 other Old Kingdom limestone quarries that bordered both banks of the Nile, up to 420 km upstream (Harrell, 2013). Stockpiling of blocks at the remote quarries could have started during the three years or so that the site at Giza was being prepared, thus reducing the high rate of quarrying otherwise required.

    In Old Kingdom times, the Nile flowed within 100 to 200 m of the base of the Giza massif, where there is evidence that one or more harbours were constructed. These would have been connected to the Nile by navigation canals (Lehner, 2009: 97-151).

    Boats were in widespread use for both coastal and inland navigation in Old Kingdom times. They were used to ferry people and goods across and along the length of the Nile, and even for the transport of prisoners. The Palermo Stone records that Snefru built "100-cubit ships of meru wood" and "60 sixteen-barges" for his raids to the south, "bringing 7,000 prisoners and 200,000 large and small cattle" (Breasted,

1906: 66-67). This description implies boats were being built with large cargo capacities (Landström, 1970). Based on an analysis by the author of the flow regime of the Nile before it was altered by dam and barrage construction, cargo vessels of up to 500 t deadweight would have been able to navigate fully-laden on the Nile for around nine months a year, and with lesser cargoes for the other three months. Even if as much as half of all the core blocks had been brought in by boat, a modest fleet of no more than 30 vessels would have been required, so there would have been no barrier to such imports had they been needed. However, some stockpiling of blocks on the foreshore at Giza would have been required to cover construction in the low-flow season.

Haulage Challenges

Ramps would have been required to raise some of the core blocks and all the facing blocks and granite beams from the harbour wharves to the level of the foundation, and from there to their final resting place in the pyramid.

    The greatest single challenges would have been raising the 44 heavy granite beams to between level 49 and level 60. Whatever ramp was needed for these operations would have simply been widened for the placing of the core and facing blocks below this level, but alternative arrangements would have been needed for the higher blocks.

    No ramps would have been needed for the blocks in the lowest course, which were probably cut from the excess rock excavated around the hump, particularly on the west side. From observations of the visible blocks we can estimate that the maximum weight of blocks at level 2 was probably in the order of 10 t, and at level 60, no more than 6 t. This would have reduced to 2 t at the highest course of the core. If the top three courses of capstones had comprised 1, 4 and 9 blocks respectively, these 14 blocks would each have weighed 1.5 t.

Haulage Pathways

The haulage pathway from the harbour would have varied between 800 and 1200 m as the pyramid rose, made up of the following components:

- An approach ramp from the harbour to the pyramid foundation level, a lift of up to 50 m, depending on the river level in each season. This would have been linked at various points along its length to the working faces of different quarries on the Giza Massif by temporary roads.
- A main ramp from the top of the approach ramp to the King's Chamber roof axis, 11 m south of the pyramid east-west axis at level 60.
- A secondary ramp from the point of intersection of the main ramp on the south face of the pyramid to a level as close as possible to the apex.
- A working platform at the top of the secondary ramp for the placing of the last few courses and the capstones.
- Several temporary roads over the top surface of the course under construction from the top of the ramp in use to the delivery point of each block or granite beam.

The approach ramp and main ramp would have been designed for the passage of less than 100 individual loads weighing up to 75 t, and some 600,000 loads of limestone blocks weighing up to 10 t at a high delivery rate. The secondary ramp would have been designed for some 170,000 loads of between 2 and 6 t, and the apex platform designed to handle around 100 loads of between 1.5 and 2 t. All of them would have been subject to heavy wear. The rate of placement of blocks would have been set high enough to accomplish the task within the overall planned construction period of 23 years. Allowing a three-year period for preliminaries and a further two years for finishing off, this would leave an 18-year period of actual construction. The rate of placement would have been higher at lower levels, but reduced later on, and is calculated below.

Haulage Teams

The ramps would have been made wide enough to accommodate the maximum size of teams that would use them, typically in groups of 20 haulers (see below). Team capacity depends on the assumptions made about the coefficient of friction between the wooden sled runners and the ramp surface, the steepness of the slope, the body weight of the haulers and their grip.

Denys Stocks (2003) made experiments of the static friction of stone on stone, with both surfaces smoothed to a tolerance of 0.25 mm, and found values of 0.73 when dry and 0.14 when lubricated with liquid mortar. He noted that a wooden sled runner lubricated with mud has the same value. This and many other studies assume lubricants such as wet sand (Fall *et al.*, 2014), grease (Dunham, 1956) or Nile silts (Creasman & Doyle, 2010) were used to create a low coefficient of friction, but this does not take into account the need to ensure the haulers had a good grip underfoot when the previous team applied lubricants liberally.

Based on the image on a tomb at Deir el Bersha of 172 men hauling a statue of the 12th Dynasty nomarch Djehutihotep, whose discovery was described by Newberry (1895: 3-5), and experiments by Chevier and the NOVA team, Lehner (2006: 225) estimated that a 20-man team could haul a load of about 3.3 t on a lubricated slope of 10%. Such experiments do not take into account the sustainability of human power output. My own experience with 400-man labour gangs building airstrips in Zambia suggests that daily output over a six-month period is around half the maximum daily output (Brichieri-Colombi, 1970[1]). An analysis of studies of muscle-generated power suggest a sustainable power output of around 100 watts, very much less than the 250 watts produced by Bryan Allen, the pilot of the ultralight Gossamer Albatross in his three-hour pedal powered flight from England to France (Krendel, 2007).

Accordingly, a more conservative estimate has been adopted in this study, with a reduction of 20% for the steeper slope and of 25% for the sustained output.

I used measurements of Old Kingdom bones found in Giza cemeteries and corresponding estimates of male stature (Zakrzewski, 2003: 219-229) to estimate the average height of the heaviest 16% of workers (those with a height exceeding the mean by one standard deviation, who would have been the most suitable as haulers) to be 1.74 m. I assumed their Body Mass Index (BMI) was in the range 25-29, average 27, appropriate for someone engaged in heavy manual work for extensive periods, to arrive at an average bodyweight of 46 kg. With a realistic coefficient of friction of wood on stone of 0.25, as adopted by De Haan (2009: 6), each 20-man team of such workers would have been able to

haul a 2 t load on a 16% slope. Since the Egyptian population in Old Kingdom times is estimated at 1.6 million (Butzer, 1984), there would have been around 16,000 males of working age with this physique, so human resources would not have been a constraint.

An important part of the calculation of ramp sizes is to ensure the teams have adequate working room. Observations of tug-of war teams and measurement from photographs of teams in action, show that men in haul teams tend to stand closely together. I estimate that a 20-man team, with men two abreast, would have had a minimum footprint of around 6.5 by 1.6 m. In larger teams men would have been between four and eight abreast. Wide teams would have to have used wooden spreader bars to allow the haul ropes to remain parallel. In the aforementioned image of men hauling a statue of Djehutihotep, the haul ropes are shown attached to a single point on the sled without an intervening spreader bar, but this would not have been physically possible. When a large team of men hauls on a rope, it lies in a straight line between the team and the point of attachment unless passed around a spreader of some kind, and it exercises a force on the spreader that increases with the angle turned. Thus caution is required when drawing conclusions from scenes depicting Egyptian haulage activities.

Design of Approach Ramp and Main Ramp

For a sled loaded with an 11 m long beam weighing 75 t, a team of eight hundred men 80 m long and 6 m wide would have been required. If the ramp had not been made straight, it would have had to be very wide to allow the direction of the pull of the team to be closely aligned with the trajectory of the sled. To keep the ramp volume small, both the approach ramp and the main ramp would have had to be straight.

Table 1 lists the elevation, number and average weight of the 76 blocks and beams in the pyramid passages and chambers with weights in excess of 10 tonnes, based on detailed drawings prepared by Maragiolio & Rinaldi (1965: Plates 1-10).

Each of the three major pyramids at Giza had its own ceremonial causeway linking a valley temple and a pyramid temple. The one for Khufu's pyramid is 800 m long, with a pronounced kink, and if it had been built with a uniform slope, it would have risen to a maximum of 21 m above its foundations. This causeway would thus have been unsuitable as an approach ramp for construction, and was probably built for ceremonial purposes.

Khafre's causeway is 400 m long and, from an examination of the 1:5000 survey maps (Institut Géographique Nationale, 1978: F17), it follows a natural bench of rock with a near-uniform slope of 9% from Khafre's valley temple to his pyramid temple. It passes through a point 390 m due south of the centre of Khufu's pyramid and thus would have been ideal as an approach ramp prior to its use as a causeway. As can be seen when walking up the causeway, the slope of the natural rock bench is so regular that the surface of the approach ramp would have been seldom more than a metre or two above ground level. Hence its volume would have been negligible in comparison with that of the pyramid.

To keep its intersection with the pyramid a simple right angle, the axis of the main ramp would have been oriented north-south. It was probably offset some 20 m west of the pyramid centre because, during construction, there would have been delicate survey control works at the centre, designed to ensure that the pyramid rose vertically from course to course. In this position, it would have avoided any of the tombs to the south of the pyramid, and the extension of the ramp across the pyramid would have been well to the west of the King's Chamber. This 20-m offset would have added only a few metres to the minimum necessary haul distance from the harbour. The 1:5000 survey map of the site shows that in this position, the main ramp would have crossed a narrow arm of one of the Giza quarries, but this could well have been opened up later, for the construction of Khafre's pyramid.

The toe (starting point) of the main ramp would have been located at the intersection of the ramp axis with that of the approach ramp, around 50 m north of the toe of the ramp that was proposed by Lehner (1985). The slope of a ramp from this point to a point at level 60 on the axis of the King's Chamber roof, 11 m south of the centre of the pyramid, would have been 16.4% (9.3°). This is well within the range of ramp slopes observed elsewhere (Stocks, 2003: 196-197) and, as discussed later, would not have been too steep for the haul team.

Location	Elevation (m)	No.	Average Weight (t)
Entrance gable	18	4	20
First girdle stone	7	1	40
Other girdle stones	17	3	27
Queen's Chamber roof	25	12	25
King's Chamber wall (largest)	43	1	33
King's Chamber roof	49	9	57
Davidson's Chamber roof	52	9	55
Wellington's Chamber roof	55	9	53
Nelson's Chamber roof	57	9	58
Lady Arbuthnot's Chamber roof	60	8	55
Campbell's Chamber roof	62	11	30
Total		76	

Table 1. Estimated average weights of blocks weighing over 10 t (based on detailed drawings prepared by Maragiolio & Rinaldi, 1965: Plates 1-10).

The main ramp would have intersected the south face of the pyramid at level 49 and, if it had been continued across the body of the pyramid, it would have intersected the north face at level 71. The maximum height above its foundations would have been 43 m, at the point where it crossed over the south edge of the pyramid.

Ramp Top Width
The top surface of the main ramp would have been made wide enough to allow teams to haul in parallel to achieve the rate of placement required. The total weight of material hauled to construct the pyramid was 5.5 Mt, of which 4.7 Mt was below level 71. If we allow 10 years for construction to this level, with teams working 300 days/year for 8.5 hours day, the required rate of placement would have been 186 t/h. This rate of working would have been around twice the average achieved by Snefru in the Bent and Red pyramids.

Assuming the sleds maintained a minimum spacing of five minutes to allow time to clear breakdowns or divert traffic into an emergency lane, the delivery rate would have been 12 loads per hour per lane. With loads averaging around eight tonnes each, made up of between one and three blocks of the larger blocks found at the lower levels, two working lanes would have been required, each wide enough to allow the passage of 100-man teams. Allowing 4 m per lane for each of these two lanes and an emergency lane, and a lateral safety margin each side of 1 m, the total top width required would have been 14 m.

On the higher ramp, only a single lane would have been necessary to deliver the remaining 0.8 Mt in eight years. At 12 loads an hour, the loads would have to average only 2 t each, an easy target with the smaller blocks at the top. A total width of 5.5 m would have been wide enough to allow for 60-man teams for the larger blocks in the vicinity of level 71, including room to overtake safely in case of a breakdown.

Ramp Material
Most archaeological studies (e.g. Dunham, 1956; Fall *et al.*, 2014; Hawass, 2000; Lehner, 2007) have suggested that the ramp would have been built of granular materials such as sand and rubble which generally have an angle of repose of between 30° and 45° to the horizontal. It would not have been possible to transport heavy loads close to the edge of a ramp made of such materials, as they would have been subject to slip failure of the kind seen in road embankments, especially if moistened by rain or the repeated application of lubricants under the sled runners.

The simplest solution would have been to build the ramp with limestone blocks similar to those that were used in the core, as no additional materials or supply chain would have been required. To avoid the danger of smaller blocks slipping out of the ramp side walls, these would have been built with a batter of around three vertical to one horizontal. With this batter, the base of the ramp would have extended a maximum of only 14 m each side of the top width,

much less than the minimum of 43 m required for a ramp of built of granular material.

If the lateral blocks had been fixed in position with gypsum mortar, the walls could have been made vertical and still have been stable. The base was wider and the maximum height would have been similar to the masonry towers built in the 13th Century in Siena, Italy (Giorgi & Matracchi, 2013: 648-654).

Ramp Construction

The ramp would have been laid in horizontal courses, each forming an extension of the course under construction, and with the same thickness as that course (figure 3). The toe of the ramp would always have been at the same point on the approach ramp, so no additional temporary roads would have been required to access it. The blocks in the initial 6 m or so of the length of each course of the ramp would have been trimmed to the overall slope of 16%, and the top surface would have been paved with limestone slabs to provide a smooth running surface for the sleds. These slabs would have been lifted and the majority laid again on the surface of the next course each time the ramp was raised.

Ramp Profile

After the intersection with the south face of the pyramid had been reached, the main ramp would have been continued at the same slope over the surface under construction, forming a trench with sides of the same 3:1 batter (figure 4). At level 49, the level of the heaviest granite beam, the top end of the main ramp would have been 154 m, measured horizontally, from the north face of the pyramid, just enough space to accommodate the 80 m long 800-man haul team needed. At level 61, the level of the floor of the highest chamber, the heaviest beam was only 43 t, and the 480-man team needed would have been only 48 m long, compared with the space available at this level of 66 m. The margins on these lengths are adequate, but not generous, which may suggest this was by design rather than accident.

From the ramp, the beams would have been hauled eastwards over the surface of the pyramid into their final positions by the same haulers, reconfigured into two or four smaller teams as required because of the limitations of space around the King's Chamber.

At level 71, the main ramp would have intersected the north face and so ended. By this time,

Core blocks as extension of second course
- prismatic
- trimmed to slope

Core blocks as extension of first course
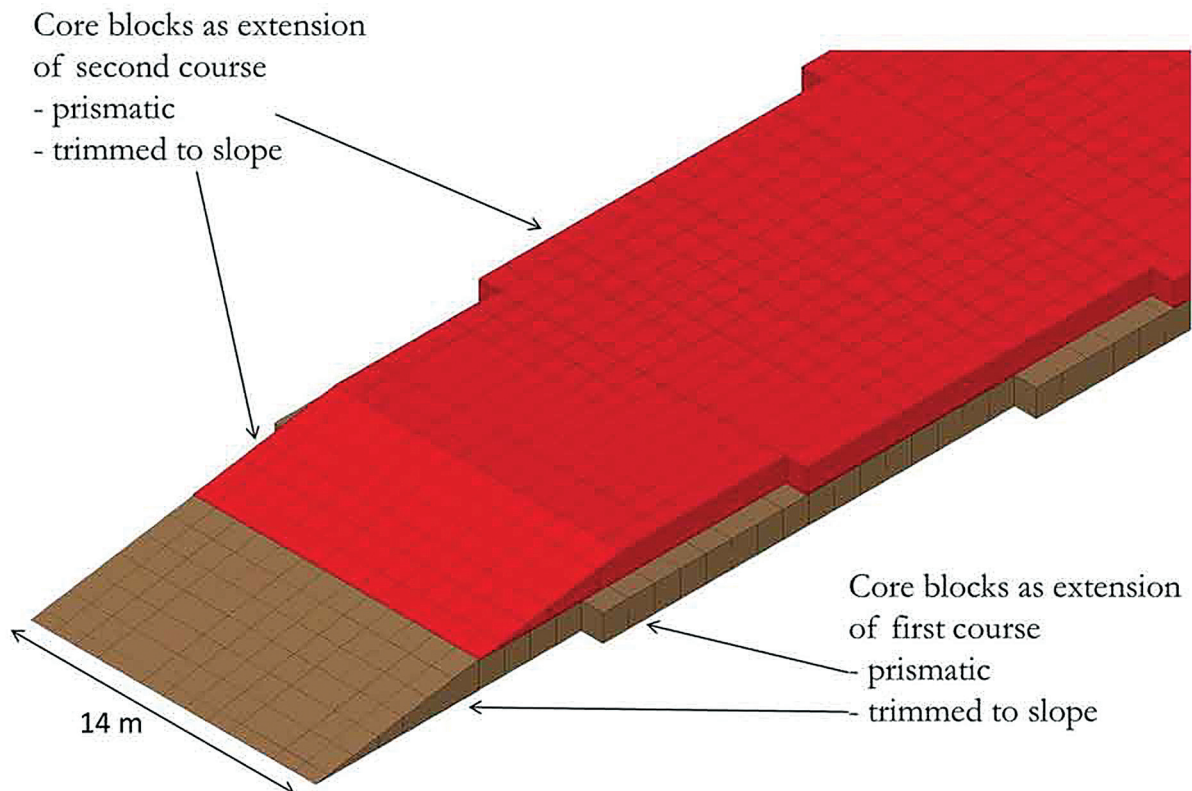- prismatic
- trimmed to slope

14 m

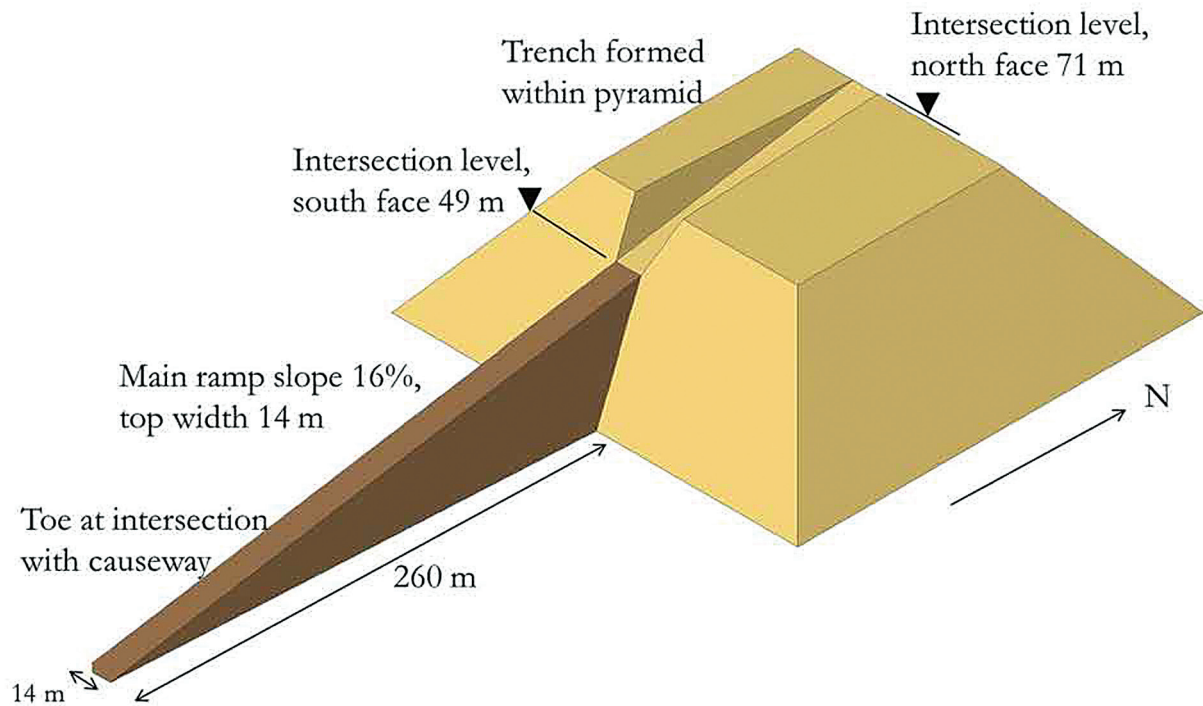Figure 3. Main ramp construction. © The author.

Figure 4. View of main ramp. © The author.

86% of the total volume of the pyramid and ramps would have been completed (figure 5). Some of the blocks used to backfill the main ramp trench below level 71 would have been hauled up the main ramp trench, others via the secondary ramp.

Tracks Over the Top Surface of the Pyramid

Except at the edges, where they supported the casing blocks of the course above, the top surfaces of the courses of the core were probably irregular, and on the surface of the course under construction, temporary tracks would have been provided for the sleds. Some 32 gangs of perhaps eight men each would have been needed to place the blocks as they arrived at the top of the main ramp every 2.5 minutes, giving the gangs around 80 minutes to orient and trim each block. To ensure a smooth workflow, well-defined work areas of approximately equal size in each quadrant of the surface would have been defined so that each sled could be directed immediately to the next available area.

To maintain the haul speed, semi-permanent tracks paved with reusable slabs laid on a bed of sand would have been provided from the top of the ramp to the centre of each work area (as

for the top surface of the ramp). From there, further tracks made with gravel and embedded transverse wooden logs of the kind demonstrated by Lehner in his experiment building a small pyramid at Giza (NOVA, 1997), would have led to the workface. In this case, sand and gravel could be used, as only a thin layer would have been required and the angle of repose would not have been an issue.

Design of the Secondary Ramp and Apex Platform

If the main ramp had been made higher, its volume would have increased rapidly and it would have become very uneconomic. A different design was therefore required for the secondary ramp, made possible by the fact that only 60-man teams would have been required for the blocks above level 71, and some of those needed to fill in the trench. The rate of work required for the remaining 14% of the pyramid volume could be reduced to one team every five minutes. Thus, only one working lane and one emergency lane would have been needed. With this maximum team size, the total ramp width required would have been only 5.5 m.

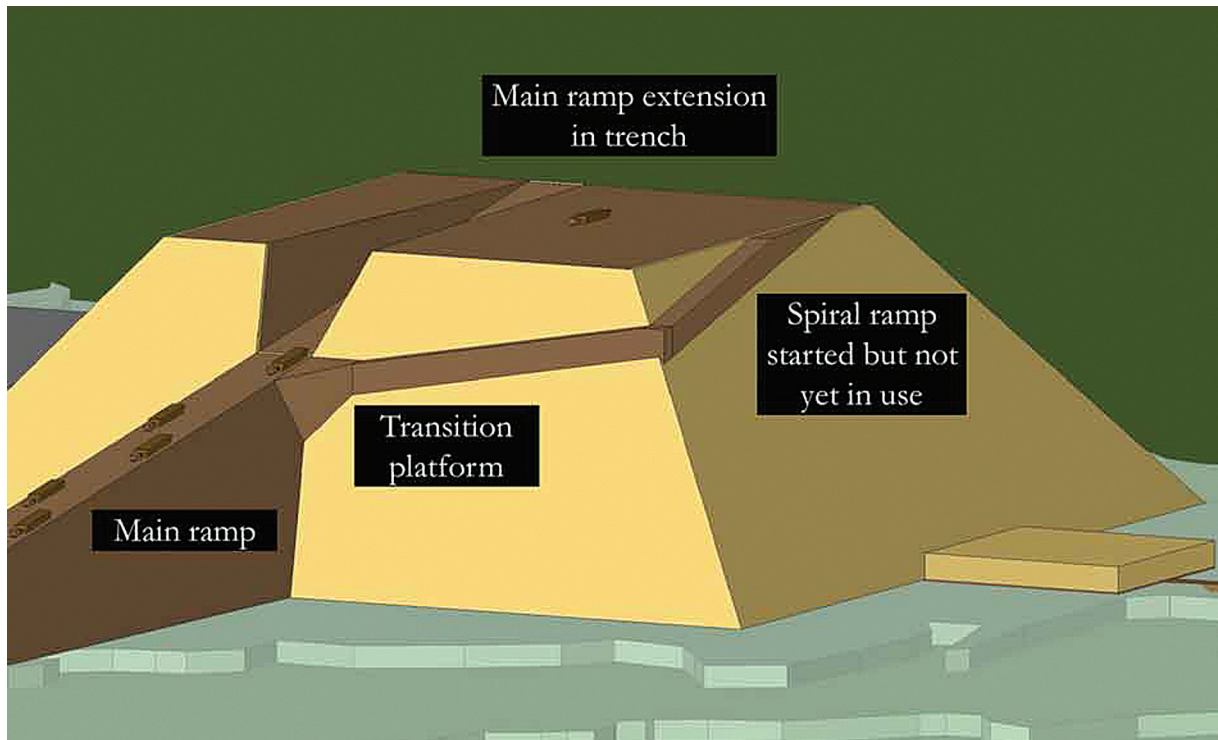The most economic form for the secondary ramp would have been a helix winding anti-

Figure 5. Ramp configuration at level 71. © The author.

clockwise around the pyramid, starting from level 49 where the main ramp met the south face. It would have been triangular in cross-section, with a top width of 5.5 metres and a vertical outer wall 7 m high, not high enough to require a batter (figure 6). Like the main ramp, it would have been built from the same core blocks as a lateral extension of the course under construction, laid outside the line of the casing blocks, the initial blocks on each layer trimmed to the same 16% slope.

Cornering

At each corner, successive flights of the helical ramp would have been joined by a horizontal 5.5 m square corner platform. Turning the corner with a 60-man team 10 m long would have required a special manoeuvre to ensure the sled did not jam against the blocks of the pyramid.

When making a documentary about constructing a 7 m high pyramid at Giza, Lehner's team passed their haul ropes around a vertical post mounted on the outer edge of the corner platform, and then pulled back the way they had come (NOVA, 1997). This worked with 20-man teams hauling only light loads, but would not have worked with a longer team if another team had been coming up the ramp behind them.

To find a solution, I built a 1:20 scale model of a ramp corner platform to demonstrate that it is perfectly feasible to mount the vertical post on the inner corner of the platform, and with square knots of the kind widely used on Egyptian boats, attach to it two timber spars laid horizontally and linked together with a rope to subtend a maximum angle of about 110° between them (figure 7).

In order to protect their outer ends, the spars would have been tipped with stone rope guides of the kind found at Giza by Hassan (1960), and considered by him to be an ancient form of pulley. Lehner (2007: 211) has suggested these 'mystery tools' were fixed into triangular arrangements of spars so that loads could be lifted by ropes that could slide over them. However, it was neither necessary nor desirable for the haul rope to slide in the grooves of the guides, as this would have added friction losses. The team simply passed the haul ropes around the ends of the spars and hauled from a position on the upper flight of the ramp. The spars rotated around the post, guiding the sled so it cleared the corner without taking it too close to the outer edge of the ramp. This cornering operation would have taken less than a minute, and have ensured that the team did not interfere with the one following less than 60 m behind.

◄ Figure 6. Section on secondary ramp.
▼ Figure 7. Model of corner manoeuvre.
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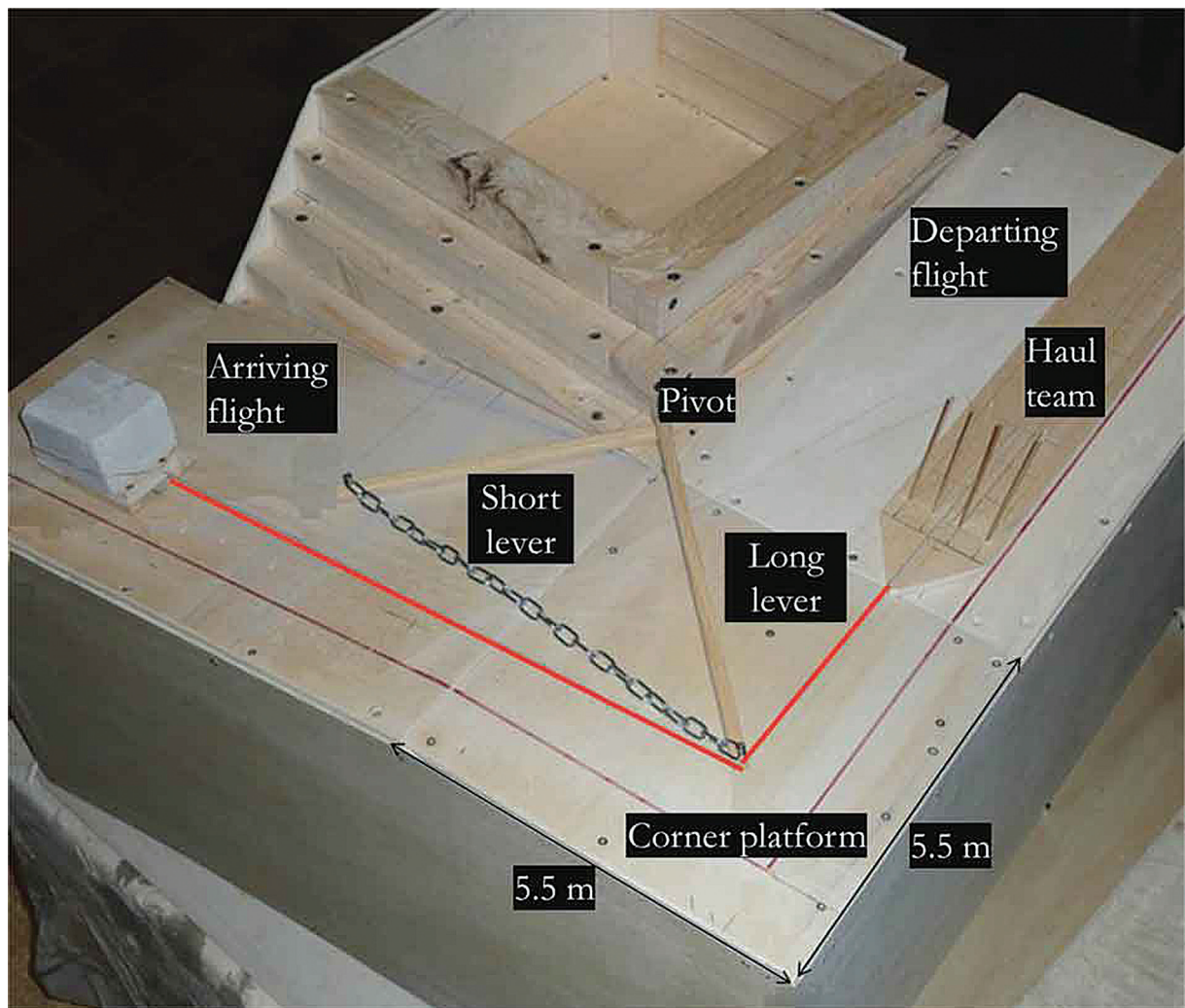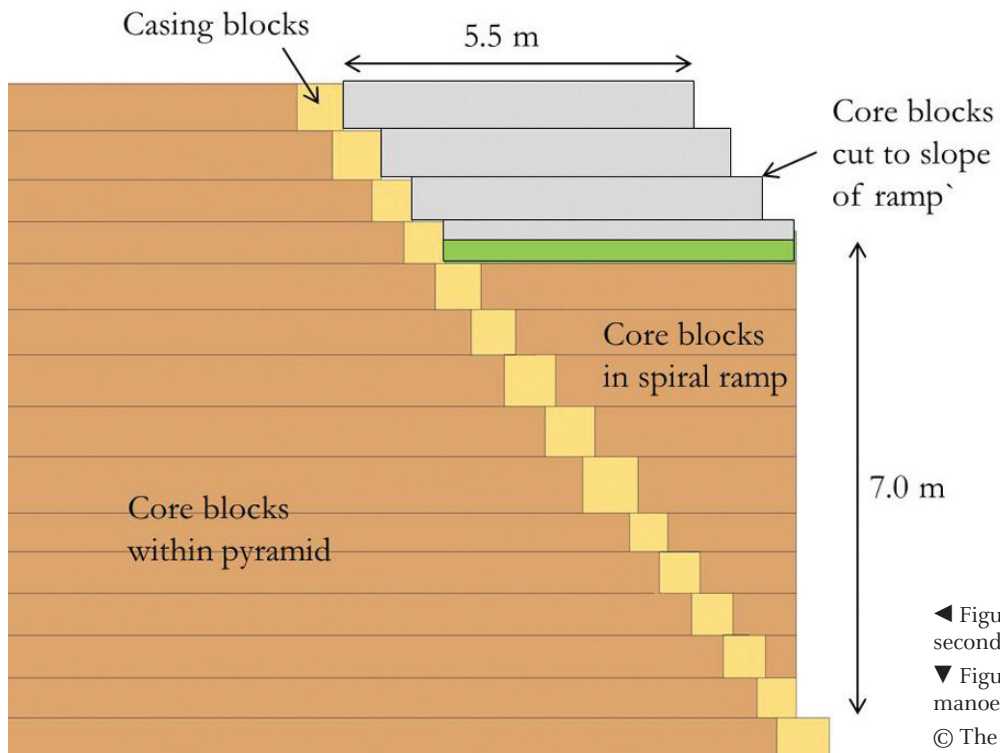
Placing the Pyramidion

The vertical interval between successive spirals of a helical ramp on a pyramid reduces as it approaches the apex. From its starting point, the ramp would have made three complete circuits of the pyramid before it reached level 142, just 4 m from the apex. At this level, the outer wall of the uppermost flight would have impinged on the previous circuit. The ramp would have been continued horizontally to form a working apex platform 14 by 6 m on the south side of the pyramid. The remaining 100 or so blocks could then have been lifted into position from the sled using a pair sheerlegs made from two 7 m long wooden spars.

For the placing of the pyramidion, the sheerlegs would have been mounted on the casing blocks at level 144, straddling the next core course and the two lower courses of capstones. The haulers would have been at a mechanical disadvantage in this position, and so the sheerlegs would have been mounted in two different positions in notches at this level. The first lift would have raised the pyramidion from the sled onto a 50 cm high intermediary wooden platform, and the second from there to its final resting place. The final lift would have required 70 workers standing on the top flight of the ramp and hauling down on the sheerleg ropes (figure 8).

Total Ramp Volume

When the helical ramp was in use, only one of the two 4-m wide working lanes would have been needed in the main ramp, and its top width could have been reduced accordingly. The blocks removed would have been much closer and more convenient to use than freshly quarried blocks, and therefore could have been reshaped and reused in the construction of the upper courses. This would have reduced the total additional volume of the ramps to 136,000 m³, just 5.4% of the volume of the pyramid.

If the sides of the ramp had been made vertical rather than battered, as discussed above, the final ramp volume would have been just 2.7% of the volume of the pyramid.

The blocks forming the ramps would have been removed course-by-course as the work of trimming the casing blocks to their final profile proceeded. They would have been hauled back down the ramp and reused in the many other minor pyramids and tombs on the site. Unfortunately, no trace of their existence would have been left.

Conclusion

Papyri that show the ramp configuration that was used to build the Great Pyramid have yet
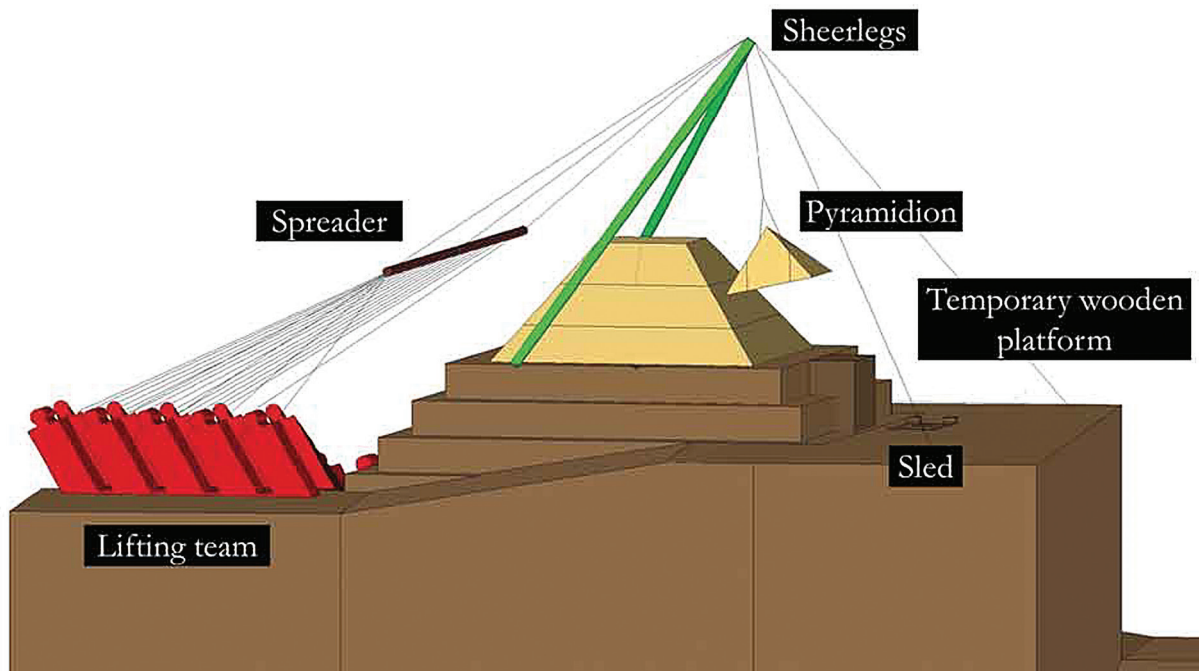

Figure 8. Raising the pyramidion. © The author.

to be discovered, although those recently found at Wadi al-Jarf raises the possibility that some might exist (Tallet & Marouard, 2014). Until that time, we can only speculate.

The above hypothesis describes a ramp configuration that fits with the current state of archaeological and other scientific knowledge, and therefore provides a benchmark against which alternative configurations can be assessed. It may not be the correct answer, but it does demonstrate that, contrary to assertions frequently made by the non-archaeologists referred too earlier, there exists a ramp configuration with a very small volume that could have been used to build the pyramids using only the resources known to have been available.

Research is now in progress to incorporate this ramp configuration into a benchmark hypothesis that shows that the Great Pyramid could have been built within 23 years by a team at Giza of around 6,000 men and women, including those engaged in hauling, quarrying, placing stone and providing support services.

Notes

1   Unfortunately, this paper is no longer available. In it, I describe my efforts to motivate workers to undertake earthworks building airfields, which included the removal of up to 30 termite hills, 10 m high and 40 m around the base and which had to be excavated to a depth of 0.6 m. When paid on a day-work basis, workers would initially earn high wages, but after a few weeks they were unable to sustain the effort, and their take-home pay dropped. This led to domestic strife, with wives accusing their spouses of laziness, and a further decrease in productivity. The eventual solution was to set (by personal example, causing some merriment to the assembled community) a standard day's work, close to the workers' maximum production, and for the men to produce half this amount each day and then go home. The workers were able to keep up this rate of work for month after month, and so produce a reliable income.

Cited Literature

Allen, G. 1936. Egyptian Stelae in Field Museum of Natural History. – Chicago, Field Museum Press.

Arnold, D. 2003. The Encyclopedia of Ancient Egyptian Architecture. – New York, Princetown University Press.

Beckerath, von, J. 1999. Handbuch der ägyptischen Königsnamen. – Mainz am Rhein, Philipp von Zabern.

Breasted, J.H. 1906. Ancient Records of Egypt. Vol. I. Sections 76-167. – Chicago, University of Chicago Press.

Brichieri-Colombi, J.S.A. 1970. Use of Labour in Developing Countries. – Presentation to a Conference at the Construction Industry Research and Information Association (CIRIA), UK.

Butzer, K.W. 1984. Early Hydraulic Civilization in Egypt: A study in Cultural Ecology. – Chicago, University of Chicago Press.

Clayton, P.A. 1994. Chronicle of the Pharoahs. – New York, Thames & Hudson.

Cole, J. 1925. Determination of the Exact Size and Orientation of the Great Pyramid of Giza. – Cairo, Ministry of Finance: 6 (online at: http://www.ronaldbirdsall.com/gizeh/errata/Cole%20Survey.pdf).

Creasman, P.P. & N. Doyle. 2010. Overland Boat Transportation during the Pharaonic Period: Archaeology and Iconography. – Journal of Ancient Egyptian Interconnections. Vol 2:3 14–30 (online at: http://jaei.library.arizona.edu).

Dodson, A. 2000. Monarchs of the Nile. – Cairo, The American University in Cairo Press.

Dodson, A & D. Hilton. 2004. The Complete Royal Families of Ancient Egypt. – New York, Thames & Hudson.

Dunham, D. 1956. Building an Egyptian Pyramid. – Archaeology, 9.3: 159-165.

Fall, A., B. Weber, M. Pakpour, N. Lenoir, N. Shahidzadeh, J. Fiscina, C. Wagner & D. Bonn. 2014. Sliding Friction on Wet and Dry Sand. – American Physical Society: Physical Review Letters 112 (week ending 2 MAY 2014) 175502 1-4.

Fonte, G.C.A. 2013. Energy Management Reduces Great Pyramid Build Effort by More Than 98%. – ASCE Construction Engineering Management, 1392: 249-251.

Giorgi, L. & P. Matracchi. 2013. The Towers of San Gimignano. Architectural Diagnostic for Knowledge and Conservation. In: Boriani, M. Ed. Built Heritage 2013: Monitoring Conservation Management. Online proceedings of the cConference. - Milan, Italy 18-20 November 2013: Millan, Politecnico di Milano 648-654.

Haan, de, H.J. 2009. Building the Great Pyramid by Levering. A Mathematical Model. – PalArch's Journal of Archaeology of Egypt/Egyptology 6, 2: 1-22 (online at: http://www.

palarch.nl/2009/05/building-the-great-pyramid-by-levering-a-mathematical-model-j-dehaan/).

Harrell, J.A 2013. Ancient Egyptian Quarries and Mines. Table 2: Ancient Egyptian Softstone Quarries. – http://www.eeescience.utoledo.edu/Faculty/Harrell/ Egypt/ Quarries/Softst_Quar.html (Last updated 17 July 2013).

Hassan, S. 1960. The Great Pyramid of Khufu and Its Mortuary Temple. – Cairo, General Organisation for Government Printing Offices.

Hawass, Z. 1998. Pyramid Construction: New Evidence Discovered at Giza. In: Guksch H. & D. Polz. Eds. Stationen. Beiträge zur Kulturgeschichte Ägyptens. Rainer Standelmann gewidmet. – Mainz am Rhein, Philipp von Zabern

Hitchins, D.K. 2010. Pyramid Builder's Handbook. – London, Lulu Enterprises UK Ltd.

Houdin, J.-P. 2006. Khufu: The Secrets Behind the Building of the Great Pyramid. – Cairo, Farid Atiya Press.

Institute Géographique Nationale. 1997. Le Caire 1:5000 series map. – Cairo, Ministère de l'Habitat et de la Reconstruction: Sheet F18.

Isler, M. 1985. On Pyramid Building. – Journal of the American Research Centre in Egypt XXII: 129-142.

James, P. 2014. The Rise and Demise of Egypt's Largest pyramids: A builders View. – Structure Magazine April 2014 (online at: http://www.structuremag.org/?p=1860).

Krauss, R. 1985. Sothis und Mondaten. – Hildesheim, Gerstenberg Verlag (Hildescheimer Äegyptologische Beitrage).

Krendel, E.S. 2007. Muscle Generated Power. In: Avallone, E., T. Baumeister & A. Sadegh. Eds. Marks' Standard Handbook for Mechanical Engineers 9.4. – New York, McGraw Hill.

Landström, B. 1970. Ships of the Pharaohs. – London, Allen Lane.

Lehner, M. 1985. The Development of the Giza Necropolis: The Khufu Project. – Mitteilungen des Deutschen Archaologischen Institut Abteilung Kairo, 1: 141-146.

Lehner, M. 2007 [1997]. The Complete Pyramids. – London, Thames & Hudson.

Lehner, M. 2009. Capital Zone Walk-About 2006: Spot Heights on the Third Millennium Landscape. In: Lehner, M., M. Kamel & A. Tavares. Eds. Giza Occasional Papers 3: Giza Plateau Mapping Project Seasons 2006-2007:

Preliminary Report. – Boston, Ancient Egypt Research Association Ltd: 97-151.

Malek J. 2000. The Old Kingdom. In: Shaw, I. Ed. The Oxford History of Ancient Egypt. – Oxford, Oxford University Press.

Malek, J. 2003. Egypt. 4000 Years of Art. – New York, Phaidon Press.

Maragioglio, V. & C.A. Rinaldi. 1965. Complesso di Cheops a Ghisa. – Turin, Centro per le Antichità e la Storia dell'Arte del Vicino Oriente.

Massey, C. 2012. The Pyramids of Egypt: How Were They Really Built. – Hove, Book Guild Ltd.

Murnane, W.J. 1997. Three kingdoms and 34 Dynasties. In: Silverman, D. Ed. Ancient Egypt. – London, Duncan Baird.

Newberry, P.E. 1895. El Bersheh. Part 1: The Tomb of Tehuti–hetep. In: Griffith, F.L. Ed. Archaelogical Suvey of Egypt. – London, Egypt Exploration Fund.

NOVA. 1997. "This Old Pyramid" (film, online at: https://www.youtube.com/watch?v=8SiHWCTHs8g).

Parry, D. 2004. Engineering the Pyramids. – Stroud, Sutton Publishing.

Perring, J.S. 1839. The Pyramids of Gizeh from Actual Survey and Measurement. – London, James Frazer.

Petrie, W.M.F. 2013 [1883]. The Pyramids and Temples of Gizeh. – Cambridge, Cambridge University Press.

Redford, D.B. 2001. The Oxford Encyclopedia of Ancient Egypt, – Oxford, Oxford University Press.

Shaw, I. 2000. The Oxford History of Ancient Egypt. – Oxford, Oxford University Press.

Sitek, D. 2007. Ancient Egypt: History and Chronology (online at: http://www.narmer.pl/indexen.htm).

Stadelmann, R. 1997. Die ägyptischen Pyramiden, vom Ziegelbau zum Weltwunder. – Mainz, Philipp von Zabern.

Stocks, D. 2003. Experiments in Egyptian Archaeology. – London & New York, Routledge.

Tallet, P. & G. Marouard. 2014. The Harbour of Khufu on the Red Sea Coast at Wadi al-Jarf, Egypt. – Near Eastern Archaeology 77, 1: 4-14.

Verner, M. 2002. The Pyramids: The Mystery, Culture, and Science of Egypt's Great Monuments. – New York, Grove Press.

Vyse, R.W. 1840. Operations Carried on at the Pyramids of Gizeh in 1837. – London, Weale & Nikkison.

Zakrzewski, S.R. 2003. Variation in Ancient Egyptian Stature and Body Proportions. – American Journal of Physical Anthropology 121: 219-229.