BABYLONIAN CRESCENT OBSERVATION AND PTOLEMAIC-ROMAN LUNAR DATES

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ABSTRACT

This article considers three questions associated with Ptolemaic-Roman lunar chronology: did the temple service begin on Lunar Day 2; were lunar phases determined by observation and/or cyclically; how accurate were lunar observations? In the introduction, Babylonian and modern observations of old and new crescents are analyzed to obtain empirical visibility lines applicable to Egyptian lunar observations.

Introduction

The present study consists of two parts. The first considers visibility criteria for sighting new and old crescent on the basis of Babylonian and modern observation; the second concerns Ptolemaic-Roman lunar dates.

In Part 1, I deal with crescent visibility criteria which are applicable to John K. Fotheringham's azimuth-altitude method of determining new and old crescent. Paul Viktor Neugebauer adapted Fotheringham's method for use in his 'Astronomische Chronologie', a manual-like book for computational analysis of archaeo-astronomical problems. He developed a near-perfect tabular procedure for computing solar and lunar positions of new and old crescent (Neugebauer, 1929a: 80-82). Neugebauer complemented his procedure with a visibility line established by Karl Schoch for use in Fotheringham's azimuth-altitude diagram. The line applies to new and old crescents, since the physical circumstances are the same for sighting both.

Initially I used Neugebauer's procedure until the astronomer Hermann Mucke introduced me to astronomical computer software and post-Neugebauer astronomical parameters in the form of the program Uraniastar (Pietschnig & Vollmann, 1992-1995); however, I kept using Schoch's visibility line. Later the astronomer Bradley E. Schaefer put at my disposal a set of seasonally changing minimal altitudes for crescent visibility based on a theoretical model; I have applied the set in chronological studies which I published in the last decade (Krauss, 2006: 395-431).

Below I review the precursors of Schoch's line; Schoch's line per se; the modification of it by John A.R. Caldwell and C. David Laney; its use by Peter J. Huber, and by Bernard D. Yallop. I suggest a modification of Schoch's line on the basis of Babylonian, as well as modern crescent observations. After deducing empirical visibility criteria from Babylonian observations, I apply them to Ptolemaic-Roman lunar dates. Furthermore, I test Schaefer's set of visibility lines against Babylonian and modern observations. These procedures are possible for an archaeologist; theoretical analysis of crescent observation and the establishment of corresponding visibility lines remain, however, tasks for the astronomer.

Deduction of an Empirical Crescent Visibility Line

Basic Elements of Crescent Computation According to the Azimuth-Altitude Method and Van der Waerden's Mistaken Modification of It

In general new or old crescent becomes visible between sunset and moonset (new crescent), or between moonrise and sunrise (old crescent), when the sun is below the horizon as depicted in figure. 1. Sighting of the crescent is possible when the sun is above the horizon as, for example, in the case of the Babylonian new crescent of −194/6/7: "it was bright, earthshine, measured; it could be seen while the sun stood there; it was low to the sun" (*cf.* Appendix 2).

More than one method can be employed to determine first or last visibility of the moon. For a comprehensive bibliography on computing the visibility of the lunar crescent see Robert van Gent: <http://www.staff.science.uu.nl/~gent0113/islam/islam_lunvis.htm>.

None of the methods yields a definitive result, since the sighting of new and old crescent depends not only on the astronomical situation, but also on atmospheric conditions. Since astronomical extinction cannot be predicted, any computation of the date of a non-trivial new or old crescent results, in principle, in probability, rather than certainty.

The methods for computing new or old crescent make use of the three angles ARCL (arc of light; also called elongation), ARCV (Arc of Vision), and DAZ (Difference in Azimuth) which constitute a right spherical triangle and are defined as follows (*cf.* Yallop, 1997: 1):

ARCL is the angle subtended at the centre of the earth by the centre of the sun and the centre of the moon. ARCV is the geocentric difference in altitude between the centre of the sun and the centre of the moon for a given latitude and longitude, ignoring refraction.

ARCV can be expressed within the azimuth-altitude coordinate system as $|s| + h$, with s as geocentric negative solar altitude and h as geocentric lunar altitude.

DAZ is the difference in azimuth between the sun and the moon at a given latitude and longitude; geocentric and topocentric DAZ are approximately the same. Since the crescent can be to the south or north of the sun, DAZ can be positive or negative; I use in general the absolute value of DAZ. The angles ARCL, ARCV, and DAZ satisfy the equation cos ARCL = cos ARCV cos DAZ.

The azimuth-altitude method as introduced by Fotheringham explicitly uses the angles DAZ and ARCV at the moment when the sun is on the horizon (solar geocentric altitude 0°); thus, within the azimuth-altitude diagram, ARCV is to be understood as the geocentric altitude h of the moon. By contrast, Caldwell & Laney introduced topocentric lunar altitude to the azimuth-altitude method (*cf.* section 'Remarks on The Crescent Visibility Model of Caldwell & Laney').

In the early 1940s the mathematician Bartel L. van der Waerden erroneously corrected Neugebauer's azimuth-altitude based procedure for computing old crescent (Van der Waerden, 1942). In the 1980s Lee W. Casperson, a physicist, arrived at the same erroneous conclusion as had van der Waerden (Casperson, 1986); he was followed by John G. Read in two articles which cannot be taken seriously for various reasons (Read, 1995; 1996). Van der Waerden and Casperson assumed that the angle between horizon and ecliptic is, in general, the same for successive old and new crescents.

Uncertainty about the correct computation of old crescent implied by van der Waerden delayed work on my dissertation during the winter of 1980/81. For advice I contacted Fritz Hinderer, then professor emeritus of astronomy at the Free University, Berlin. Hinderer explained to me that the assumed symmetrical configuration of horizon and ecliptic exists only at the time of the solstices; during the rest of the year the angles at the intersection of the ecliptic with the eastern and western segments of the horizon differ by as much as 47° (for the latter *cf.* below Parker's mistaken computation of old crescent day in the case of No. 5; for the situation at the solstices see Schaefer's observations nos. 194 & 195, included in figure 3). For further clarification, I wrote to Van der Waerden, and when his reply was unsatisfactory, I asked Hinderer to write to him on my behalf. In Van der Waerden's reply to Hinderer, he did not concede the mistake which Hinderer had described in his letter. After publication of the letters (Krauss, 2009: 146-150), I donated them to the archive of the ETH-Bibliothek, Zurich, where Van der Waerden's papers are held.

Below, I compute a crescent date that refers to a specific place in two steps. The first step is the computation of solar and lunar coordinates for geocentric solar altitude 0°. The second step concerns crescent visibility; details are discussed in the following paragraphs. For step 1, I use Uraniastar 1.1 (Pietschnig & Vollmann, 1992-95) which computes lunar positions on the basis of Ernest Brown's lunar theory in the slightly abridged version of Jean Meeus (Pietschnig & Vollmann, 1992-1995: Handbuch 1-7). For the reliability of Uraniastar, see Firneis & Rode-Paunzen (2003: 48). As a control, I employ the more recent program Alcyone Ephemeris 4.3 (http://www.alcyone.de/ae/documentation/Index.html) which is based on Steve Moshier's analytical ephemeris and the lunar ephemeris of Michelle Chapront-Touzé and Jean Chapront, both adjusted to Jet Propulsion Laboratory's ephemeris DE 404 (Lange & Swerdlow, 2005). Both programs allow corrections for $\triangle$t (Delta T), the difference between Universal Time and Terrestrial Time that results from the slowing of the earth's rotation. The values of $\triangle$t that are programmed in Alcyone Ephemeris yield differences of ca. 3 arcminutes in lunar altitude for the Late Babylonian Period. Since ca. 3 arcminutes are negligible in determining crescent visibility, there are no further remarks about $\triangle$t below.

Remarks on Crescent Sighting Data Bases

Schoch's visibility lines rely upon his own and other modern observations – above all upon Julius Schmidt's observations made in Athens, and furthermore upon Babylonian observations. The Babylonian observations of the last centuries BC are documented in the so-called Astronomical Diaries, cuneiform tablets excavated at Babylon in the 1870s and 1880s and now, for the most part, in the British Museum (Sachs, 1955: VI; Stephenson, 1997: 107f). An edition of the Diaries was begun by Abraham Sachs. His work is about to be completed by Hermann Hunger who not only edited and translated, but also dated the texts astronomically. Five volumes have appeared so far (Hunger & Sachs: 1988-2006 = SH 1-3, 5-6); a final volume containing mostly non-datable texts is expected soon. By contrast, when Schoch analyzed the Babylonian crescent dates, this material was not fully at his disposal.

The original database of modern crescent observation consists of 76 sightings and non-sightings of new and old crescents, mostly made by the astronomer Schmidt at Athens in the 19th century. These observations form the core of the crescent sighting compilation of Schaefer and LeRoy E. Doggett (Schaefer, 1988a: 514-518; 1996: 762; Doggett & Schaefer, 1994: 107, 389-392). They compiled more data from the astronomical literature and observations from the moon-watches which they organized; their list comprises 295 observations from between 1859 and 1996. The SAAO (South African Astronomical Observatory) Lunar Crescent Visibility Database (Caldwell & Laney, 2000) comprises the non-trivial observations of the Schaefer-Doggett list and adds about 50 observations up to the year 2000. Furthermore, there are the databases INMS compiled by Roy Hoffman (2011) and ICOP cited by Mohammad S. Odeh (2004), respectively.

Motives for observing the crescent vary. Schaefer (1988a: 11-17) describes the observers whose data he used in his compilation as follows: "the majority of the reports were made by professional and amateur observers and are presented in the published astronomical literature". Schaefer's description also characterizes the majority of the observers who are cited in the SAAO list for the years 1996-2000. By contrast, the observers who contributed to the other compilations appear in general to have been motivated by ritual ends.

There is the basic question of the observer's reliability. Schaefer checked the data in his list against the original reports and additional pertinent information such as available meteorological information (Schaefer *et al.,* 1993); some of his comments can be corrected or modified. Caldwell and Laney checked the plausibility of the observations which they accepted for the SAAO list.

Nowadays, as a result of the availability of astronomical software, it is next to impossible to detect fabricated reports which refer to non-trivial crescents. The observations to which Odeh assigns numbers 341, 389, 433, and 455 may be cited as examples of suspect cases, because in each the crescent would have been observed decidedly below Schoch's line of 1929/30. The four observations were reported as naked-eye crescent sightings made by the same observer between 1999 and 2001 in Cape Town. Four exceptionable observations by one and the same observer within a short period is an improbable result. Odeh does not comment on the reports, but nos. 389 and 455 are rejected as unreliable by M. Shahid Qureshi (2010: 15). Qureshi's judgment qualifies the observer as not thoroughly reliable, making it therefore advisable to exclude all of his reports for the time being.

The Encoding of the Manner in which Observations Were Made

Table 1 presents my understanding of the Schaefer-Doggett code which appears to coincide with the interpretation by Caldwell & Laney; for the partially differing and apparently mistaken interpretation of Yallop see my comments below in section 'Remarks on Yallop's Crescent Visibility Test: The Concept of *Best Time*'.

Caldwell & Laney (2000) do not always apply 'E' according to the sources, as the following examples may show. On 1860/1/23 Schmidt did not sight the crescent visually, *i.e.* with the naked eye (Mommsen, 1883: 70), in so far, "E: no optical aid mentioned" is formally correct. On the other hand, the text which the authors cite implies without a doubt that no optical aid was used.

Schmidt's observation of 1864/5/6 is also coded 'E' by Caldwell & Laney (2000), but Mommsen's list states explicitly that Schmidt used a telescope (Fernrohr) and thus the observation should have been coded 'F'. Furthermore, they compute the same observation for Athens; actually the source states that it was made in Troy. Schaefer intended to compute the observation for Troy, although the coordinates he used and which are repeated by Yallop (1997: 11) refer to a site ca. 50 km south of Troy whether Schmidt observed at Hissarlik or Bunarbashi/Pinarbasi (Schaefer No. 20).

Caldwell & Laney (2000) classify all non-sightings of Maurice McPartlan as "E: not visually, no optical aid mentioned". Actually the observer states that all of his reports refer to naked eye observations, meaning that he did not use optical aid (McPartlan, 1985: 243).

Schmidt's Observations as Published by Mommsen: Debated Observations

The observations of Schmidt were published in 1883 by August Mommsen as excerpts from an article and a manuscript by Schmidt on crescent observation (Mommsen, 1883: 69-80); Mommsen's article included also a few observations of his own and an observation by Friedrich Schmidt. With the exception of an observation in Corinth and another in Troy, both by J. Schmidt, all other observations in Mommsen's list were made in Athens.

The historian Mommsen had hoped that an astronomer would use the observations for establishing a criterion based on the age of the moon for crescent visibility in Greek latitudes. An unsuccessful attempt was made by the astronomer Karl C. Bruhns (Mommsen, 1883: 69-80). It may be noted that in those days astronomers did not know how to define a useful criterion for crescent visibility. Thus, in the 1890s, the astronomer Walter F. Wislicenus (1895: 29) had to be satisfied with asserting: "if the sky is clear, but under otherwise differing astronomical conditions, the first appearance of the crescent can occur 1 to 3 days after conjunction".

Papers of Julius Schmidt were at one time in the possession of the former observatory (Sternwarte) at Potsdam. Most of these, though apparently not all, were transferred in the 1960s to what is today the archive of the Berlin-Brandenburgische Akademie der Wissenschaften, among them the observational diaries for 1860-1871 and 1879-1884. Other papers of Schmidt were kept at the Sternwarte in Bonn and are now in the Argelander-Institut für Astronomie in Bonn; the Athenian diaries of 1859 and 1872-1878 are not among them, as Hilmar Duerbeck who is in charge of Schmidt's papers informs me.

Julius Schmidt observed comets, planets, eclipses, halos, zodiacal light, and twilight appearances of bright stars and planets systematically. His Athenian observations cover about 25 years; he seems to have seen or recorded only every 3rd new crescent and very few old crescents. I have examined Schmidt's papers as far as they are available and have begun to excerpt the crescent observations.

There are a few reports in Mommsen's list which gave rise to discussion. Schaefer accepts the new crescent observed by Schmidt in Athens on 1859/10/27 and lists it as no. 2 in his compilation, although it would have been invisible according to his computational model, as outlined by Schaefer (1988b). Such a disparity does not mean that Schaefer rejects the reported observation as unreliable. There are about 27 such cases in Schaefer's first list which comprises 210 observations (Schaefer, 1988a) – among them, one of Schaefer's own observations (Schaefer No. 169).

By contrast to Schaefer, Yaacov Loewinger (1995: 450) "strongly doubts the correctness [of Schmidt's sighting], because of the unprecedented low 33 minute LAG value". He points out that "no other positive sighting in Schaefer's list has lower LAG than 45 min, for an optically unaided sighting". Note that Loewinger (1995: 448) checked the lag values in Schaefer's list which are "sometimes more than 100% off, in either direction, from the correct value".

Should Schmidt, "one of the greatest visual observers" (Schaefer, 1996: 761), have made a mistake when he observed the moon on 1859/10/27, describing it as "leicht kenntlich" which Mommsen rendered as "*leicht sichtbar*"? Schmidt's mistake would have had further consequences, since this particular observation was apparently chosen by Edward Walter Maunder and also by Schoch as one of the defining points of their respective visibility lines. Despite Loewinger's reservations, Schmidt's report can be accepted, since the Babylonians observed a new crescent with a lag of 36 minutes (–284/11/6; see Appendix 2), for example, and another one with a lag of 29 minutes (–264/9/26; see Appendix 2); finally, the compilers of the SAAO Database accept a new crescent observation made in Ashdod on 1990/9/20 with a lag of 29 minutes (Caldwell & Laney, 2000).

Furthermore, there is the problem of Mommsen no. 43 = Schaefer no. 44, an old crescent which was reported from Athens on 1871/9/14. At the time of observation, the age of the moon was –15.3 h; in 1993 Schaefer cited this report as a "record for unaided vision" (Schaefer *et al.*, 1993: 55f; see also Schaefer, 1988a: 514; 1993: 339), although it belongs to the observations which do not agree with his computational model. Later Schaefer (1996: 761) followed Loewinger (1995: 451) in rejecting it as an unreliable report made by Julius Schmidt's "unskilled gardener Friedrich Schmidt in a casual observation". The astronomer will have been aware of how exceptional the observation was; as he wrote in 1868 about his own observations since 1860 (Schmidt: 1868, 203f): *"Es ist nicht einmal die Wiederholung der merkwürdigen Beobachtung Mädler's gelungen, der 1834 Oct. 1 [noon epoch] zu Berlin die Sichel 18 Stunden vor dem Neumonde, aber nur am Fernrohr sehen konnte. In Mädler's Selenographie findet sich diese Angabe pag. 151, wozugleich erwähnt wird, dass Schröter den Mond nur einmal 29 Stunden vor der Conjunction gesehen habe"*.

Friedrich Schmidt's observation was cited by Mommsen (1883: 71) from a manuscript of Julius Schmidt: *"1871 Sept. 14 Abends 6 Uhr erzählte mir der Hofgärtner Friedrich Schmidt, dass er in der Frühe dieses Tages die sehr feine Sichel gesehen habe, 10 oder 20 Minuten bevor die Sonne hinter dem Hymettos aufstieg. Tags vorher, also Sept. 13 früh, sah er die Sichel höher und leichter. Eine mögliche Verwechselung des Datums wird nicht zugestanden"*.

The "*Hofgärtner*" Friedrich Schmidt was not the astronomer's gardener but rather, he was chief gardener – or director – of the "*Hofgarten*" (royal court garden) in Athens (Charkiolakis *et al.,* 2008: 926). Whether any gardener – and F. Schmidt in particular – was a skilled observer or not is immaterial. He may have had extraordinarily sharp eyesight – like the master tailor of Breslau who could see the first four moons of Jupiter with the naked eye, to cite yet another Schmidt, namely Arno (Schmidt, 1979: 25).

The details of F. Schmidt's report fit the astronomical situation, including the *"sehr feine Sichel"*, corresponding to an illuminated fraction of 0.66% as computed by Loewinger. Nevertheless, the latter (1995: 451) pointed out that the sighting "is by far impossible by at least 4 modern and 2 medieval criteria ...". This may be so, but there are two comparable old crescents in the Babylonian data (see Appendix 1, Nos. 6 & 17, figure 6 and table 4 below for the crescents cited here). The point is that F. Schmidt had seen old crescent at −1.4° below Schoch's visibility line of 1929/30, whereas the Babylonian observers spotted crescents at −1.5° and −1.7° below Schoch's line; the illuminated fractions (Meeus, 1985: 145-147) amounted in both cases to ca. 0.6%. F. Schmidt's report concerns not only old crescent, but the moon on the preceding day as well; therefore the observation is not 'casual' in a strict sense. Presumably Loewinger overlooked the reference to the sighting on 1871/9/13.

Following Loewinger and Schaefer, Yallop (1997: 12) and the compilers of the SAAO list did not accept the observation of F. Schmidt. It was also not accepted for a study on the Danjon limit by Fatoohi, Stephenson & Al-Dargazelli (Fatoohi *et al.*, 1998). The latter study is based on 209 Babylonian new crescent observations as reported in the Babylonian Diaries (Hunger-Sachs, 1988-1996 = SH 1-3) and on 271 new crescents in Schaefer's compilation. Fatoohi *et al.* (1998: 72) found that "the crescent with the smallest elongation that has been seen by the unaided eye and whose detection did not include the use of optical aid nor watching from a high place is that of observation [no. 86 in Schaefer's list; observer Long; Cape Town 1913/11/28] which was 9.1° away from the sun at sunset".

Actually, F. Schmidt observed old crescent at an elongation or ARCL of 8.3° at sunrise; the Babylonian old crescent of −284/11/4 was observed at 8.9° and that of −248/10/27 at an elongation of 7.8° at sunrise (the values for elongation allow for parallax following Fatoohi *et al.* 1998: 70).

Two Questionable Observations Made by Mommsen

Schaefer (1988a, 513; *cf.* also 1988b, 11-17) discards Mommsen's own observations nos. 73 and 74 as "meaningless because the attempts were made through clouds". Here Schaefer relies on Fotheringham's (1910: 527) paraphrase of Mommsen's text which reads, in the case of no. 74 [1879/12/14]: "there was a gap in the clouds through which the Moon, if visible, might have been seen". Mommsen will have known what he was talking about, since he was an experienced observer; he mentioned 30 crescent observations which he made in Schleswig (54° 31' N, 9° 34' E) in the years 1877 to 1879, before his visit to Athens (Mommsen, 1883: 78 n. 1). As for no. 73 [1879/12/12] Fotheringham (1910: 527) wrote: "Mommsen's negative morning observation (No. 73) might, according to his own suggestion, be due to an obscuration of the moon by Hymettus, but Mommsen himself rejects this suggestion. The observation was made on a walk which extended till the disappearance of the stars. This raises a doubt whether, if the walk had been prolonged a few minutes longer, it might not have had a different effect".

Mommsen himself (1883: 71) remarked that the morning in question was *"ein recht klarer schöner Morgen"*; in other words, the visibility conditions will have been perfect. Mommsen argued that the moon was not visible on 1879/12/12 as follows: he observed new crescent first on 1879/12/15 when the moon had an age of "*etwas über 48 Stunden*" (actually 52 h); conjunction occurred on 1879/12/13 around noon and divided the interlunium into more or less equal halves; the first half began on 1879/12/11 around noon, and therefore the moon would have been invisible on 1879/12/12 around sunrise.

Thus, by contrast to Fotheringham's interpretation, Mommsen did not reject the possibility that the crescent was hidden by the mountain range of Hymettos; rather, he concluded astronomically that the moon was invisible on the morning of 1879/12/12. Mommsen's argument depends on the identification of 1879/12/15 as new crescent day, as he did not see new crescent on the evening of 1879/12/14, in spite of a gap in the clouds. In other words, on 1879/12/12 Hymettos probably hid old crescent from Mommsen, and, presumably because of clouds, he did not see new crescent on 1879/12/14. Thus, Mommsen's nos. 73 and 74 cannot be used, neither as non-sighted nor as sighted.

Fotheringham's Azimuth-Altitude Method

The azimuth-altitude method was originally devised by Fotheringham (1910) and subsequently taken up by Maunder (1911), Schoch (1927; 1928b), and Neugebauer (1929a). Fotheringham conceived of the idea to use lunar and solar azimuth and lunar altitude as empirical crescent visibility parameters. He analyzed the 76 Schmidt-Mommsen observations by computing DAZ and ARCV for the moment when the sun is on the horizon (when ARCV coincides with geocentric lunar altitude h; see figure 1);

he neglected refraction. Thus he arrived at an empirical principle that for old or new crescent to be considered visible, the moon at sunset or sunrise, respectively, must have a minimal geocentric altitude h which depends on DAZ. Fotheringham's principle reflects the fact that the brightness of the sky decreases and the moon is brighter at increasing azimuthal distances, resulting in decreases in lunar altitudes and vertical distances of moon and sun that are necessary for sighting the crescent. In 1910, he only summarized his method (Fotheringham, 1910: 530f.); in 1928, he then provided more details (Fotheringham, 1928: 45-48).

Fotheringham (1910, 528) thought it "unnecessary to complicate the calculation by introducing lunar parallax, because the parallax in altitude is practically constant for any given altitude, while the parallax in azimuth is small, and small changes in azimuth produce no perceptible difference in the result". Presumably, he had decided to do without parallax and topocentric lunar altitude to save computational work, but today this motive has become irrelevant, since computers do the work.

Figure 2 presents the 76 observations analyzed by Fotheringham. The diagram combines the western and eastern sky and also negative and positive azimuthal distances of sun and moon. The coordinates of the crescents are taken from Fotheringham's list. Compared to modern computation, there are occasionally differences of 0.1°. Presumably a typographical error has resulted in the old crescent of 1879/12/11; Fotheringham's DAZ = 25.9° is to be corrected to 23.9°. He lists three cases of sighting with a telescope as naked-eye non-sightings; but he does not differentiate between Schmidt's naked-eye non-sightings and the non-sightings with a telescope.

His visibility line satisfies "all the observations with the exception of Friedrich Schmidt's successful observation in the morning and one of Julius Schmidt's successful observations in the evening" (Fotheringham, 1928: 47; 1910: 530). In response to Fotheringham's article, Maunder (1911: 356) conceded that Fotheringham's dividing-line "very likely supplies a good criterion for the probability of a given phase of the Moon being observed". But he criticized Fotheringham for basing "his dividing line upon the negative observations, whereas it should have been based upon the positive. For the latter, if accepted, are definite and decisive, the former are not." Thus Fotheringham's dividing-line, according to Maunder (1911: 359) "gives us, not the actual limit of visibility, but an upper limit; we can say that above that limit the Moon ought not to be missed – it ought to be seen. But from time to time it will be seen below that limit, and occasionally much below it".

Maunder's Work on Crescent Visibility

Maunder (1911, 356) confirmed that Fotheringham's azimuth-altitude method "should apply to any place, for, as nothing except the relative positions of the Sun, Moon, and horizon are taken into account, it is independent of latitude". By contrast to Maunder's expectation, high latitudes may indeed have an effect on the azimuth-altitude-method (Caldwell & Laney, 2005: 6); such problems do not arise for the low latitudes of Athens, Babylon, and Egypt.

Maunder (1911: 357) omitted "considerably more than half of the 76 observations [of Mommsen's list]", since they referred to "the Moon when far advanced beyond the limit of visibility". Schmidt (1868: 205) was well aware of this, noting *"so war es doch in den meisten Fällen unzweifelhaft, dass unter den jedesmal gegebenen Umständen, der Mond mehr oder weniger früher hätte gesehen werden können"*. To the Schmidt-Mommsen list Maunder added nine crescent sightings made by various observers between 1865 and 1910 in England and Scotland, one in Belgium, and another in southern Spain. Two observations are remarkable: an old crescent reported from a site in Belgium and a new crescent reported from Tunbridge Wells (England).

The old crescent observation in Belgium was cited in a paper by William Frederic Denning (1909: 242): "Dr. Degroupet, of Belgium, saw the old Moon in 1889, November 22, from 6h 47m to 7h 22m G.M.T. At the latter time she was within 18h 22m of new Moon." The time difference to conjunction confirms the cited 7h 22m G.M.T. By contrast, 6h 47m is probably a typographical error for 6h 57m at which time the crescent would have been just above the horizon, refraction considered.

Denning provided no information on how he knew about the observation. Maunder listed the place of observation simply as 'Belgium' (as Denning, his source, had done) and cited the geographical coordinates of Brussels, presumably as a convenient stand-in for the unknown exact site; Schaefer-Doggett accept the observation without comment under no. 78. The observer was evidently Dr. Léon Decroupet (*sic*) of Soumagne (50.61° N, 5.74° E; elevation 250 m) about 100 km to the east of Brussels. Decroupet's observation of new crescent on 1885/12/7 in Soumagne is included in the Schaefer-Doggett list under no. 202 ('DeCroupet') and the SAAO list. Both lists follow Maunder in citing the coordinates of Brussels for the observation of 1889 whereas I presume, rather, that this observation was also made in Soumagne.

Schaefer discards the new crescent which Donald W. Horner reported as being sighted in Tunbridge Wells on 1910/2/10. The crescent would have been observed at an extraordinary –5.1° below Schoch's line of 1929/30. Maunder conceded that "the evidence for its correctness appears to be strong", but he seems to have retained some doubts (Maunder, 1911: 359). Note that he cites Tonbridge as place of observation; actually the observer was located at Tunbridge Wells which is some five miles to the south of Tonbridge (Horner, 1911: 163, 345). In the case of Horner's and certain other questionable crescents, Schaefer (1988a, 512f) found that "the moon was difficult to detect on the next night [1910/2/11] so that a simple error of date is indicated". Actually, sighting of the crescent would have been trivial on 1910/2/11, since at sunset in Tunbridge Wells the moon had an altitude of ca. 7° above Schoch's visibility line.

Schaefer had the weather reports checked; the National Meteorological Library in Exeter informed him that the sky was overcast or that it rained on the day in question in London, Clacton-on-Sea, and Dungeness; therefore Schaefer concludes "that the reported date was incorrect" (Schaefer *et al.*, 1993: 53f). Despite the meteorological reports, it is possible that it did not rain in Tunbridge Wells at sunset of 1910/2/10 and that there were no clouds in the relevant part of the western sky.

Horner (1911: 162) reported that he saw the crescent when searching the sky for "comet a 1910 [Great January comet of 1910] which was then fading". The latter remark confirms that Horner's observation refers indeed to mid-February in 1910. Horner mentioned no details, such as the observed altitude of the moon or whether he used an optical instrument. Charles T. Whitmell (1911: 375), who computed the circumstances of Horner's observation on the basis of Horner's letters to The Observatory Magazine, wrote that "Mr. Horner also first found the Moon when using a telescope to search for a comet". This seems to be mistaken; at least I have not found a remark by Horner about the use of a telescope.

Neither in naked-eye nor in optically aided observations is there a case of a crescent which would have been detected so far below Schoch's visibility line. The optically aided observations in Schaefer's list and the SAAO list which were made at elevations above 1000 m are presented in figure 3; Horner's crescent is indicated for comparison. Note that figure 3 includes observations made on the eastern and western horizon as well.

Figure 3 includes 'Pierce's crescent' (Schaefer Nos. 278-290), represented by three different observational results as reported by five observers (codes: V(V); I(V), I(I)). Pierce himself as the sixth observer saw the crescent without prior optical detection with the naked eye; his sighting is not indicated by a symbol in figure 3. The observations were published as second hand information by Schaefer (1996: 760) who states: "The report contains no anomalies, so I accept it". Note that there is some misunderstanding on Schaefer's part who writes: "John Pierce observed from Collins Gap in eastern Tennessee ...". The place of observation will have been Mount Collins in the Great Smoky Mountains rather than Collins Gap. Both are located in eastern Tennessee, but Collins Gap rises to less than 1200 feet whereas the elevation of Mount Collins is above 5000 feet, the height of the place of observation cited by Schaefer (Schaefer, 1996: Table 1); moreover, the geographic coordinates cited by Schaefer suit Mount Collins, rather than Collins Gap.

Next to the position of Horner's crescent are non-sightings by Schaefer of an old crescent and the subsequent new crescent, both searched for at an elevation of 2770 m (9100 f) in the very clear air of northern Chile (Schaefer Nos. 194 & 195); he could see neither crescent through binoculars. Horner's crescent had a magnitude of –4.98 and Schaefer's crescents of –4.92 and –4.86 (for the apparent magnitude of the moon as function of phase see Allen, 1963: 145; for the phase angle see Meeus, 1985: 145). Horner's crescent would have been about 1.05 and 1.11 times brighter than Schaefer's crescents (Meeus, 1985: 171f.). If the conditions of Schaefer's non-sightings are taken into account (high elevation, dry and clean air of northern Chile), then it seems to be excluded that Horner saw the crescent on February 10 in 1910, and this is so, despite the three witnesses Horner cited. If the report is not spurious, it will refer to the following evening when sighting of the crescent would have been unexceptional, if the weather allowed it all.

Figure 4 presents the crescents which were used by Maunder for establishing his visibility line. The situation illustrates his (1911: 359) assertion that "it is clear that no one line can be drawn so as to include all the Moons observed, and none of the Moons missed, on the same side of it; it is unreasonable to expect that it would be possible to do so". He deduced a visibility line which was markedly lower than Fotheringham's. Maunder decided in favor of a line which runs above Decroupet's old crescent and aims at J. Schmidt's new crescent of 1859/10/27. The line does not include all sighted crescents known to Maunder; it corresponds to (degrees) $11 - |DAZ/20| - DAZ^2$. In view of the unit fraction coefficients of DAZ, the formula appears to be idealized (Yallop, 1997: 2).

When Schaefer reviewed the merits of the different methods of predicting crescent visibility, he (1993: 339; *cf.* also 1996: 765) remarked that "a basic trouble with [the azimuth-altitude method] is that different workers [meaning Fotheringham and Maunder] have interpreted the same data with greatly different conclusions". He overlooked the fact that the differing interpretations do not refer to the same data. Furthermore, the reader misses an evaluation of Fotheringham's and Maunder's different approaches, regardless of the data they used.

Schoch's Work on Crescent Visibility: Inclusion of Predicted Babylonian Crescents

In 1921 Fotheringham (1921: 310) wrote that *"Herr Schoch has constructed tables, as yet unpublished, for the determination of the earliest visibility of the crescent to the naked eye"*. Schoch established a visibility line for 0° < DAZ < 19° "from more than 400 Babylonian observations from –500 to the year 0 confirmed by observations from –2095 to –1900", adding "I have made more than 100 observations of the crescent myself during 34 years" (Schoch, 1928b: 95). Neugebauer (1929b: 222) affirmed that *"Schoch [gab] der zuerst von Maunder und Fotheringham aufgestellten Regel aufgrund babylonischer und eigener Beobachtungen eine gesicherte Gestalt"*.

Schoch published the visibility line for 0° < DAZ < 19° twice, once in his '*Planetentafeln*' (Schoch, 1927: Table K) and also in the joint publication with Fotheringham on the 'Venus Tablets' (Schoch, 1928b: 95). Schoch (1927, XXXVIIf) stated *"die von mir aus so vielen praktischen Beispielen errechnete Tafel K zeigt, dass meine Werte von h sich sehr nähern den Werten von Maunder, während sich die von Fotheringham als bei weitem zu gross ergeben. Etwa 20 Neulichte, von den Babyloniern gesehen und von mir berechnet, waren nach Fotheringhams Werten von h unsichtbar"*.

This wording implies that Schoch aimed at a visibility curve with all observed crescents above or just touching it. A later version of Schoch's visibility line, which he himself corrected and extended to DAZ = 23°, was published and adopted by Neugebauer in the latter's 'Astronomische Chronologie' (1929a: 79, 82); Neugebauer describes Schoch's line explicitly as a minimum line ('*Mindesthöhe*'). Neugebauer also published the final version of Schoch's visibility line with an extension to DAZ = 24° (Neugebauer, 1930: B 17). Schoch had worked as a colleague of Neugebauer at the Astronomisches Recheninstitut in Berlin from 1926 until his sudden death in 1929 (Van Wijk, 1930: 3, 8).

Schoch's visibility lines are problematic insofar as his Babylonian data include "observations from –2095 to –1900" (Schoch, 1982b: 95), referring to the period between Rim-Sin of Larsa and Ammizaduqa of Babylon. Today it is an established fact that this dating was too early by at least 200 years (Mebert, 2009: 29f). The astronomical computation was handicapped since Schoch used a now outdated value of '$\triangle$t' (Krauss, 2003a: 51f.). Furthermore, he used new crescents which were calculated/predicted by the Babylonian observers, presuming that the Babylonian calculation/prediction of new crescent day was reliable and praising it as 'admirable'. I have found only one large error in them, *viz.* the crescent of –273 Nov 4" (Schoch, 1928b: 98). Here Schoch refers to date (h) in table 3 (below) which also figures in Stern's table 2 as 'predicted'. At the moment of sunrise the crescent of date (h) was situated –1.4° below Schoch's visibility line of 1927/28. If meteorological conditions were favorable, a sighting would not have been impossible, as the comparable cases of figure 6 demonstrate.

The Babylonian Astronomical Diaries contain more instances of wrong Babylonian predictions of new or old crescents, over and above the example cited by Schoch. Stern cites 7 out of 110 predicted new crescents which will have remained invisible to the observer (Stern, 2008: 30). Table 2 presents all of Stern's cases including those which he designates as "possibly early" or "early"; most would have been situated below Schoch's visibility line. "Early" designates those instances where the observation would have been early if the crescent were actually sighted.

Although I did not check the reports of predicted old crescents, I happened upon seven cases of the observer noting that he did not see a predicted old crescent (table 2) and when the moon indeed stood below Schoch's line. In –140/12/31, the observer had seen old crescent a day earlier, though he did not recognize it as such. These cases show that Schoch was too optimistic in his evaluation of predicted Babylonian crescents.

Crescent Observations Which Are Identifiable as Used by Schoch

Figure 5 presents all observations which I could identify as known to Schoch: the moons of Fotheringham's list as sorted out by Maunder; the crescents adduced by Maunder, except Horner's crescent; four observations by Schoch himself; and the Babylonian observations of table 3. Schoch did not publish a list of his own observations; only four of them are known, thanks to Fotheringham, and figure as nos. 79 and 87-89 in Schaefer's list. His papers, which Neugebauer mentioned (1929b: 222-224), may have included a list of his observations, but they are apparently lost. Herbert Hefele of the Astronomisches Rechen-Institut in Heidelberg informs me that the archive material of the Berlin Astronomisches Rechen-Institut, the predecessor of the Heidelberg ARI, was lost in the aftermath of World War II.

Schoch specified the Babylonian new crescents which he used only in those cases which I list in table 3 as visible or not visible. Crescent (g) was sighted, though not on Oct 13 in – 328, as Schoch presumed and Fatoohi *et al.* computed, but, according to Stern, on Oct 14. SH 1 reports the positions of the moon relative to Jupiter on the 6th and relative to Saturn on the 13th day of the respective lunar month; it follows that the 1st day was Oct 14. On the other hand, there was a medium chance that new crescent became visible on Oct 13th (see Appendix 2: –328/10/13+) which somehow alleviates Schoch's mistake.

Figure 5 indicates that Schoch defined his visibility line of 1929/30 by Decroupet's old crescent of 1889/11/22 and Schmidt's new crescent of 1859/10/27, since the line touches these two. By comparison, the line of 1927/28 has its course just below the sighted new crescents. It is curious that the earlier line ends without reaching the Athenian crescent of 1859/10/27; Friedrich Schmidt's old crescent of 1871/9/14 is below both lines.

Material for a Correction of Schoch's Line

Schoch's line of 1929/30 is too high if Friedrich Schmidt's old crescent is considered and also those Babylonian and modern crescent observations which are listed in table 4 and represented in figure 6. Table 4 includes Babylonian non-sightings which were explicitly reported as such.

As correction of Schoch's line, I suggest the line in figure 6 which runs below the sighted crescents; it corresponds to the quadratic polynom (degrees) $8.7462–0.0314 \ |DAZ| –0.0056 \ DAZ^2$. The line is defined by Babylonian new and old crescents, by F. Schmidt's old crescent of 1871/9/14, and J. Schmidt's new crescent of 1859/10/27, both observed at Athens (capitalized in table 4). Pierce's new crescent is an exceptional observation made at high elevation; I presume that the borderline of yet unknown unexceptionable crescent observations does not include it.

The course of the corrected line is very close to the empirical lower limit of crescent visibility as Frans Bruin defined it on the basis of the observations in Fotheringham's list (Bruin, 1977: Fig. 2). Bruin did not hesitate to accept F. Schmidt's old crescent of 1871/9/14 as a point of the lower limit; a second and third point are J. Schmidt's new crescents of 1859/10/27 and 1868/4/23.

Recent Critical Examinations of Schoch's Line

A critical examination of Schoch's visibility lines of 1927/28 and 1929/30 was undertaken by Fatoohi, Stephenson & Al-Dargazelli (Fatoohi *et al.*, 1999: 64-68). They incorrectly assumed that Schoch's criterion as published by Neugebauer in 1929 was Neugebauer's own, and they were unaware of Schoch's final version of the curve (Neugebauer, 1930). They analyzed the reports of 209 Babylonian new crescents contained in SH 1-3, painstakingly observing the distinction between observed and predicted crescents. According to the study, 8 of 209 new crescents fell below Schoch's visibility curves and should thus not have been observable (Fatoohi *et al.*, 1999: Figure 2). The team found a similar failing of Schoch's visibility criterion when they combined 209 Babylonian new crescents with 271 modern reports of new crescents from Schaefer's compilation. Therefore, they doubted the validity of Schoch's visibility lines and also expressed doubt about the independence of the azimuth-altitude method of geographical latitude. But Fatoohi *et al.* compared their topocentric referenced lunar positions to the geocentric referenced visibility criterion of Schoch (Krauss, 2009: 138). If the altitudes of the 209 Babylonian new crescents are corrected, *i.e.* if geocentric rather than topocentric altitudes are computed, only 2 of 209 crescents (Fatoohi *et al.* 1999: Nos. 54 and 63) are below Schoch's visibility line of 1929/30. Note that other astronomers have compared Schoch's geocentric criterion with topocentric lunar altitudes (De Jong & Van Soldt, 1987: 74; *cf.* Krauss, 2003a: 53f.).

Schoch's visibility lines were problematic even early on, since he left his database unpublished. Nevertheless, Schaefer's (1988a: 11-13) assertion that "in modern times, the only data set used [for an empirical crescent visibility rule] is that compiled by Fotheringham" does not do justice to Schoch's work.

There is the problem that Schoch's data include predicted crescents, while it cannot be known whether Schoch used any predicted crescents to define specific points of his visibility lines. Accepting the validity of Schoch's own observations does not solve the problem, since it is not known whether these observations define specific points of his visibility lines. Although this conundrum must have been obvious to anyone who has read Schoch's publications, his visibility line of 1929/30 became the standard gauge for visibility of new and old crescent, presumably under the influence of Neugebauer's 'Astronomische Chronologie'. Use of Schoch's visibility criteria of 1929/30 is now known as the 'Indian method' following the Indian Astronomical Ephemeris adoption of it in 1966 (Krauss, 2006: 397; Yallop, 1997: 2).

Schaefer (1992: 33) commented on Schoch's criterion of 1929/30 that it "has the serious fault that the seasonal variations in the extinction coefficients are not taken into account". He presumes from the outset that there is a general shift from clear skies in winter to hazy skies in summer, besides a daily random variation in extinction; furthermore, he expects that crescent visibility changes accordingly. Schoch (1928b: 96) was aware of seasonally changing visibility conditions when he "expected that the sky should be clearer on winter and spring nights than on summer nights with their heat mist". He assumed that the differences would influence the arcus visionis of first and last visibility of Venus, but did not reflect on a possible bearing on crescent visibility.

Furthermore, Schaefer maintains that observational crescent data collected in one place cannot or should not be applied in another place with different climatic conditions (Schaefer, 1988b: 11-13). By contrast, Schoch combined observations from different places (Babylon, Athens, and Central and Western Europe). Whereas Schaefer's objection appears to be justified in principle, it remains open to which degree the visibility lines of sites with different climatic conditions diverge.

It could be maintained that Schoch's original line and also a corrected version of it are no ideal tools for determining crescent visibility, since they are ambiguous. If, according to computation, a crescent is above Schoch's line it cannot be surmised that it will be visible; it might be invisible. As figures 5 and 6 demonstrate, there are sighted and non-sighted crescents above Schoch's line itself and also above the corrected version of it. To account for any actual distribution of sighted and non-sighted crescents, Schoch's lower limit for sighted crescents ought to be complemented by an upper limit for non-sighted crescents. Such an upper limit is equivalent to a line above which a crescent ought to be sighted, weather permitting. Crescent visibility appears to be determinable by a zone of uncertainty where sighted and non-sighted crescents mix, rather than by a single line.

A Crescent Visibility Line for Negative Solar Altitude

Finally it could be argued that a negative solar altitude is more appropriate for referencing crescent visibility than solar altitude 0°. Fotheringham introduced solar altitude 0° as the reference point for crescent visibility; Maunder and Schoch complied. Solar altitude 0° can be slightly misleading, notably in those cases in which ARCV = h + s (see figure 1) changes markedly between sunset (moonrise) and moonset (sunrise). Depending on their declinations, sun and moon set (rise) in different angles and thus take different times for setting (rising). For example, the setting of the Athenian new crescent of 1859/10/27 took more time than the setting of the sun, resulting in ARCV = h + s = 6.1° at solar altitude s = 0° and h + s = |–7.4°| at lunar altitude h = 0°. Since the vertical distance of sun and moon increased, the chances for sighting the crescent became more favorable between sunset and moonset. This is not accounted for by Schoch's visibility lines which assume solar altitude 0°.

By comparison, the rising of the Athenian old crescent of 1871/9/14 took more time than the rising of the sun, resulting in ARCV = h + s = |–9.2°| at lunar altitude h = 0° and h + s = +8.7° at solar altitude s = 0°. Thus in this case the chances for sighting the crescent became less favorable between moonrise and sunrise, unaccounted for by Schoch's lines.

A value for negative solar altitude which is suitable within the azimuth-altitude diagram might be derived from the times when a crescent was last sighted. Information about when a crescent was sighted, and for how long, is usually lacking not only in ancient, but also in modern reports. Table 4 inludes two observations in which sighting times were indicated, namely Schmidt's new crescent of 1868/4/23 and Decroupet's old crescent of 1889/11/22. The first one was last sighted at geocentric lunar altitude h = 4.2° and solar altitude s = –6.7°; the second, at h = 2.7° and s = –6.0°. In the case of Decroupet's observation, a lower lunar altitude and corresponding solar depression are feasible. By contrast, in the case of Schmidt's observation, the crescent was close to what Yallop designates as the "murk on the horizon" (Yallop, 1997: 4) which speaks against the supposition that the crescent might have been visible at lower altitude. Under these circumstances, I choose solar altitude s = –6.0° as reference point for the border cases listed in table 4; figure 7 presents the corresponding plot of the lunar positions. The minimal visibility lines for solar altitudes 0° and –6° are approximately parallel, although the relative positions of some crescents are not the same for different solar altitudes.

Remarks on the Crescent Visibility Model of Caldwell & Laney

Figure 8 is adapted from Caldwell & Laney and presents "the reputedly reliable reports of sightings and nonsightings" of the SAAO list within the azimuth-altitude diagram. DAZ and DALT refer to the time of sunset (or sunrise, respectively). The latter is defined as –0.83° of solar altitude; the altitude h' of the lower limb of the moon is topocentric, allowing for refraction. The observations that are included in the SAAO list are in general limited to DALT ≤ 10.5°, excluding trivial sightings. The symbols in figure 8 are to be understood as follows (Caldwell & Laney, 2005: 6): "Successful sightings by naked eye observers (class A) are represented by large filled circles; a few filled circles crossed by a short horizontal line represent marginal sightings (class B). Large open circles represent cases where the crescent was visible through telescopes or binoculars, but not visible to the naked eye (class C). A short horizontal line crossing the open circle denotes visibility in a telescope only (class D) and not in binoculars or by naked eye. Large 3-pointed delta symbols show the locations of crescents which were invisible both with optical aid and with the naked eye (class F). Small deltas represent unsuccessful sightings by naked eye observers without telescopes or binoculars. Points from events at latitudes greater than 45 degrees from the equator have a dotted halo surrounding the point".

Figure 8 indicates two visibility borders. The lower one (broken line) refers to crescents visible through telescopes or binoculars, but not visible to the naked eye. The upper one (solid line) refers to naked eye observations. For better orientation, I identify the naked eye observations on the visibility line. (1) in figure 8 represents Schaefer no. 2 and (8) represents the following three crescents: Schaefer no. 158; SAAO Islamabad 1991/2/15; Schaefer no. 86. The other crescents of figure 8 which refer to the visibility line are in order of decreasing DAZ: SAAO Ashdod 199/9/20; (x) = SAAO

Signal Hill 1997/2/8; SAAO Arad 1997/8/4; SAAO Signal Hill 1998/2/27; SAAO Ramlah 1996/10/13; Schaefer no. 147; Schaefer no. 162; these crescents are also represented in figure 9.

At least two reportedly sighted crescents are not indicated in figure 8 although they are included in the SAAO list. These are Ramlah 1997/5/7 and Pierce's new crescent of 1990/2/25 (see figure 3), since the SAAO authors 'doubt' or 'question' their validity. The Ramlah crescent appears indeed to be exceptional, as figure 9 demonstrates, but on closer scrutiny it can be accepted as shown below in the context of figure 25.

Figure 9 presents a comparison of the visibility lines of Schoch and Caldwell & Laney; both lines are expressed in terms of the Fotheringham-Maunder-Schoch diagram. Schoch's line is defined by the crescents Athens 1859/10/27 and [Soumagne] 1889/11/22. The visibility line of Caldwell & Laney follows a course below Schoch's line of 1929/30; this has been already recognized by Victor Reijs: <http://www.iol.ie/˜geniet/pic/compare2paramcriterions.gif> and <http://www.iol.ie/˜geniet/eng/benchmarking.htm#q-values>.

The authors (2005: 7) go a step further and express their visibility line by "taking advantage of the fact that at a larger arc of light, the moon is both brighter and necessarily located at an azimuth where the sky brightness is dimmer than it would be near the sun. The increase of the arc of light can then compensate for a decrease of altitude difference, and by experiment a factor of 3 seems to allow the effects to cancel over a considerable range of azimuth difference".

In other words, they express their visibility line of figure 9 as (h' + ARCL/3) = constant = 11.3°. Correspondingly, they designate visibility as "probable" if (h' + ARCL/3) ≥ 11.3°; below that value visibility is "improbable" and for (h' + ARCL/3) ≤ 9° it is "impossible". In what follows I test the practicability of the parameter by considering F. Schmidt's old crescent and the Babylonian crescents of table 4.

The transfer of the moons from table 4 to the Caldwell-Laney diagram results in sighted and not sighted crescents below and above the 11.3° line (see figure 10). The situation is compatible with Caldwell-Laney's description of the 11.3° line as border between "probable" and "improbable" crescent sighting. But if all the sightings that are indicated in figure 10 are accepted, then the line h' + ARCL/3 = 11.3° is not low enough to include all sightings. It would be possible to define a line h' + ARCL/x = constant on the basis of the lowest crescents in figure 10. On the other hand, the procedure suggested by Caldwell & Laney is in principle a detour which does not offer an advantage over the direct procedure, namely referring a crescent to a visibility line that changes its altitude depending on DAZ.

Huber's Work on Late Babylonian New Crescents

Around 1980 Huber took up the problem of dating the Venus Tablets of Ammizaduga. To choose between possible solutions of the Venus dates, Huber took into consideration Old Babylonian 30-day months which are cited in economic-administrative texts. He (1982: 3; Buja & Künsch, 2008: 13) expected that "being contemporary, the month-lengths should provide the most reliable data for astronomical dating purposes". Huber was aware that a scribe might possibly have used a schematic 30 day month, regardless of whether a month actually had only 29 days (Huber, 1982: 28). More decidedly, Stern (2008: 37) asserts that the administrative and economic documents "are less likely to provide reliable information about the calendar, because of the scribal tendency to assume 30-day months even when they actually had only 29 days". Such a shortcoming will have also affected the earlier analysis of Schoch who was the first to use contemporary month lengths to check the Venus dates of Ammizaduga (Schoch, 1928b: 42, 98).

To evaluate reported Old Babylonian month lengths Huber computed Neo- and Late Babylonian month lengths and new crescent reports. By counting "also crescents implied by month-length", he obtained "a total of 602 distinct crescents" (Huber, 1982: 25). He lists only the numbers of the diaries from which he excerpted the crescent reports without citing the individual dates of the 602 crescents (*Ibidem*).

Furthermore, the 602 cases include reports of predicted crescents. Huber (1982: 25) expected that the agreement with modern calculation "would be perhaps slightly inferior to that corresponding to observations made under ideal conditions, but superior to actual observations under imperfect conditions". Stern criticizes Huber for ignoring the distinction he considers the most significant, "i.e. between actual sightings of new moons and new moon pre/postdictions" (Stern 2008, 21). Under these circumstances, the results of Huber's analysis of the 602 crescents will be subject to some correction, if ever so slight.

Huber goes a step beyond Schoch. Without investigating Schoch's visibility line itself, he interprets it as an approximately 50:50 probability line for crescent visibility; the argument is based on the set of 602 new crescents cited above. Figure 11 presents Huber's plot of sighted and non-sighted crescents in a zone 1.3° below and 1° above Schoch's line of 1929/30. If Schoch's line of 1929/30 is applied, then the crescent is theoretically visible if above the x-axis; if the line of 1927/28 is applied, then the crescent is theoretically visible if above the broken line. Figure 11 shows that the actual distribution of sighted and non-sighted crescents does not conform to theoretical expectation, since there are sighted crescents below and non-sighted crescents above their theoretical visibility border.

Huber interprets Schoch's line of 1929/30 as the middle line of a zone where sighted and non-sighted crescents overlap. In 1982 he suggested a simple model for the probability that a crescent can be sighted within the zone. He (2011: 189) has now modified his model "taking into account that sightings can be missed because of poor atmospheric conditions...in view of the Babylonian evidence, I assume that a theoretically visible crescent ($\triangle h > 1°$) is seen with probability 0.9, but missed with probability 0.1. This modified probability model is:
- if $\triangle h < -1°$, the crescent is never seen;
- if $-1° < \triangle h < 1°$, the crescent is seen with probability $0.9(1 + \triangle h)/2$;
- if $\triangle h > 1°$, the crescent is seen with probability 0.9".

Thus Huber changes the definition of Schoch's minimum line to a 45% probability line within a zone where the probability for sightings increases from 0 to 0.9. In view of the actual distribution of sighted and non-sighted crescents, Huber's model is an idealization (*cf.* Huber, 1982: 27, Table 5.3). Note that Huber uses implied non-sighted crescents which refer to the moon a day before observed and reported new crescent. He does not seem to furnish support for his use of implied non-sighted crescents, but see below my remarks in section 'A Set of Visibility Lines Derived from Schaefer's Model of Crescent Visibility'.

Remarks on Yallop's Crescent Visibility Test: The Concept of Best Time

In the late 1990s Bernard D. Yallop introduced a crescent visibility test based on topocentric crescent width w' and geocentric ARCV. The reference line for the test is Schoch's visibility line of 1929/30 expressed in terms of ARCV and crescent width; he calls Schoch's line the Indian line. Yallop tests visibility at the moment of 'best time' for the definition of which he relies on the work of Bruin (Yallop, 1997: 4). According to Bruin the optimum situation for crescent sighting is given when the proportion of lunar altitude h (above horizon) and solar altitude s (below horizon) is $h/s = 5/4$ (Bruin, 1977: 339, fig. 9). Bruin (1977: 340) remarked that "while the sun and the moon are setting, ... h + s remains practically constant". His words imply that h + s does not remain constant; actually h +s changes depending on the declinations of sun and moon.

Moreover, one and the same optimum proportion h/s implies that the same h/s is valid for crescents of different DAZ. The implication follows from Bruin's "assumption of a western sky [at new crescent situation] of homogenous brightness" (Bruin, 1977: 338). The assumption contradicts Fotheringham's and Caldwell & Laney's basic premises that the brightness of the sky decreases and the moon is brighter at increasing azimuthal distances, resulting in decreasing lunar altitudes and vertical distances of moon and sun that are necessary for sighting the crescent (*cf.* 'Fotheringham's Azimuth-Altitude Method' & 'Remarks on the Crescent Visibility Model of Caldwell & Laney').

Yallop expresses Bruin's geometric optimum relation as temporal optimum by defining 'best time' Tb for new crescent sighting as time of sunset Ts plus 4/9 lag, the latter being the difference between the times of moonset Tm and Ts : $Tb = (5\ Ts + 4\ Tm)/9 = Ts + 4/9\ LAG$ (In the case of old crescent 'best time' results as Ts + 4/9 LAG, the latter defined as the difference between the times of moonrise and sunrise).

The correspondence between the geometric and the temporal relation is exact as long as ARCV = h + s at sunset (moonrise) is the same as later at moonset (sunrise). But if, for example, the sun sets quicker than the crescent, then $h/s = 5/4$ is attained earlier than in the case of uniform setting of both and thus 'best time' will occur after the time of optimal $h/s = 5/4$. For example, in the case of Schaefer no. 51 (Athens 1872/12/31) geocentric lunar altitude amounted to h = 12.2° at solar altitude 0°; at geocentric lunar altitude 0° solar altitude amounted to s = –15.0° and thus ARCV increased by 2.8° between sunset and moonset. The optimum proportion h/s = 5/4 = 7.5°/|–6.0°| was attained at 16h 45m 20s or 5m before 'best time' at which latter moment the proportion was h/s = 6.8°/|–6.9°| = 0.98. Thus it seems that the optimum proportion h/s = 5/4 does in general not coincide with 'best time' Tb.

Yallop's Problematical Use of Schaefer's Visibility Codes

Yallop applies the test to the 295 crescents in the Schaefer-Doggett list. There is a problem with his interpretation of the visibility codes in the Schaefer-Doggett list (*cf.* table 1). He interprets 'I B' as meaning invisible to the naked eye, but visible with binoculars, rather than as invisible even with binoculars (*cf.* table 1).

There are five cases of 'I B' in Schaefer's list (Nos. 120, 169, 172, 194 and 195); especially in the cases of Nos. 194 & 195 Yallop notes a contradiction between the results of his test and the reported observations (Yallop, 1997: 12f.). According to the test the crescents ought to have been invisible to the optically aided eye in both cases as indeed indicated by Schaefer's 'I B'. To solve this apparent problem, Yallop presumes that both crescents were observable with binoculars, due to the high elevation of the place of observation.

Yallop (1997: 5) understands Schaefer's 'V F' correctly as "optical aid was used to find the Moon, which was then spotted with the unaided eye". Schaefer used 'VF' in the first installment of his list (Nos. 1-201); in the 2nd and 3rd installments (Nos. 202-295) he replaced 'V F' with 'V(V)'. Yallop (*Ibidem*) interprets the latter as follows: "if the first character is followed by (V) it was visible with either binoculars or a telescope". Since 'V' is Schaefer's code for "sighted with the naked eye", Yallop seems to understand 'V(V)' as sighted with the naked eye and visible with optical aid. In any case, his code for visibility type B ("visible under perfect conditions") is 'V(V)', whereas it is 'VF' for type C ("may need optical aid to find the crescent"). As far as the cited cases are concerned, I cannot harmonize Schaefer's code with the use Yallop makes of it.

Representation of Yallop's Visibility Types in a Diagram

Figure 12 presents most of the crescents of the Schaefer-Doggett list within a diagram fitted to Yallop's terms and called 'Yallop's diagram' below. I omitted crescents with coinciding coordinates and also the first 15 entries and the 20th in Yallop's ordering of the Schaefer-Doggett list; the latter omissions refer to crescents with large ARCV or large w. The omissions lessen the lumping of the remaining crescent symbols in figure 12; the crescents which are omitted from the beginning of the list are in any case trivial sightings, if new or old crescents at all. Practically all naked-eye sightings of figure 12 are situated above Schoch's line. The exceptions are F. Schmidt's old crescent which is considered 'false' by Yallop (1997: 12) following Schaefer's verdict and Pierce's new crescent.

Figure 13 indicates the borderlines of Yallop's visibility types and Schoch's line as expressed by Yallop. As table 5 implies, he derives the values for geocentric w from DAZ and ARCV of Schoch's criterion by evaluating the equation w = d/2 (1 – cosARCV cosDAZ) for d/2 = 15 arcmin as the moon's semidiameter. Note that the equation for crescent width is approximative (Bruin, 1977: 337; Yallop, 1997: 3f.), since the moon's semidiameter ranges from 14.70 to 16.76 arcmin as a result of the range in horizontal equatorial lunar parallax from 61.4 to 54 arcminutes (Meeus, 1985: 169f). These ranges imply polynomials that are slightly different from Yallop's ARCV* cited below.

Using the data in Table 5 and interpreting ARCV as h + s (be s = 0° or not), Yallop fits a polynomial in w to ARCV by the method of least squares and thus transforms Schoch's DAZ-ARCV-line into: ARCV* = 11.8371 – 6.3226w + $0.7319w^2 - 0.1018w^3$.

His visibility types are numerically defined by the test parameter q, which expresses the difference between ARCV of a crescent at 'best time' and the corresponding ARCV*. If, for example, in the case of Schaefer no. 38 (Athens 1871/2/20) the chosen moment is 'best time', then DAZ = 8.45°, ARCV = 11.8° = h + s = 5.94° + |–5.86°| and topocentric crescent w' = 0.491 arcmin. If the formula for ARCV* is evaluated for w = 0.941 arcmin, the result is ARCV* = 8.91 and thus ARCV – ARCV* = 11.8° – 8.91° = 2.89° results, meaning that the crescent stood 2.89° above a certain point on Schoch's line as transposed to 'Yallop's diagram'. He (1997: 8) divides the differences ARCV – ARCV* by 10, "to confine it roughly to a range −1 to +1" so that his test for no. 38 results in q = +0.289.

However, within the azimuth-altitude diagram the difference between ARCV of No. 38 and the reference point on Schoch's line at ‚best time' amounts to only 2.3°. The same difference results if the reference point is transposed to 'Yallop's diagram' and the reference point's ARCV* is subtracted from the crescent's ARCV. The discrepancy arises because Yallop does not transpose the reference point on Schoch's line and operates with it; instead, he evaluates the formula for ARCV* with the topocentric crescent width as his argument. The procedure does not yield the point on Schoch's transposed line that defines the necessary altitude for sighting a specific crescent. I think that this part of Yallop's procedure is formally incorrect, since it is not a comparison between the lunar altitude, which is necessary for a sighting, and the actual altitude of the crescent. The procedure yields values of (ARCV-ARCV*) which are systematically enlarged in comparison to the differences between necessary and actual crescent altitudes. Since Yallop's q-values are not randomly, but systematically enlarged, they may be referred to, provided they are understood for what they actually are.

The q-based visibility types are as follows:

"(A) q ≥ 2.16 ; easily visible; ARCL ≥ 12°; 166 entries.

(B) +0.216 ≥ q > −0.014; visible under perfect conditions; 68 entries.

(C) −0.014 ≥ q > −0.160; may need optical aid to find the moon before it can be seen with the unaided eye; 26 entries

(D) −0.160 ≥ q > −0.232; will need optical aid to find the crescent; 14 entries.

(E) −0.232 ≥ q > −0.293; not visible with a telescope; ARCL ≤ 8.5°; 4 entries.

(F) −0.293 ≥ q; not visible, below Danjon limit; ARCL ≤ 8°; 17 entries."

Observations Which Do Not Conform to Yallop's Visibility Types

There is the question of how the visibility types and the crescents of table 6 match. The latter are in general not included in Schaefer's list and thus not tested by Yallop.

Figure 13 presents a detail of figure 12 with w' < 1.2 arcmin and 5.5° < ARCV < 14.0°; it combines the moons of figure 12 with the moons of table 6; the borderlines of the visibility types and Schoch's visibility line as expressed by Yallop are also indicated.

By contrast to Yallop's definition of visibility types, 12 of 20 naked-eye sightings in table 6 are located in types C and D. Eight crescents which are below Schoch's original line (*cf.* table 4), appear in figure 13 and table 6 above Schoch's line in 'Yallop's diagram', since their ARCV changed accordingly between sunset (sunrise) and 'best time'.

The difference in distribution of sighted and non-sighted crescents in figures 12 and 13 implies that the visibility types which fit the crescents of Schaefer's list do not apply if the moons of table 6 are included. For example, the border of types B/C is clearly defined in figure 12 by the lack of accepted sightings in C. By contrast, figure 13 shows a different distribution with naked eye sightings in C and below on the border of C/D.

The reference to possible optical aid in Yallop's definition of C is not relevant in the case of Babylonian or any other ancient crescents. Furthermore, except for a single border case, crescent sightings after optical detection (code: V F = V(V)) are not reported for C, but rather for the lower part of A and in all of B. Thus the definition "may need optical aid" seems to suit B and lower A, rather than C.

Since in C there are 10 naked-eye sightings and 11 naked-eye non-sightings, it would be feasible to define C by the probability p = 0.48 for sighting a crescent. Since in B there are about 28 naked-eye sightings and 18 naked-eye non-sightings, B could be defined by the probability p = 0.61 for the naked-eye sighting of a crescent.

Figure 13 includes the revised version of Schoch's line expressed in Yallop's terms as ARCV** = 9.7643 − 7.045*w' + 6.0672*w'^{2} − 3.002w'^{3}. If ARCV** were used as reference line, then all sighted crescents would be above it; due to its different course there would be changes in the q-values.

Methods of Fotheringham and Yallop Compared

There are three features in Yallop's method which are of advantage over the azimuth-altitude method in its traditional form: (a) ordering of sighted and non-sighted crescents into types of visibility and non-visibility; (b) determination of crescent visibility at 'best time' and thus close to actual sighting situations in border cases; (c) computational consideration of crescent width.

Feature (a) can be applied readily within the azimuth-altitude method. Feature (b) is also applicable, since visibility lines for negative solar altitudes can be established (see above). Feature (c) could be adapted if one should wish to do so. For example in the case of Schaefer no. 29 the critical altitude is h' = 5.3° and w = 1.3arcmin. The equation 1.3 arcmin = 15 arcmin (1 – cos 5.3° cos DAZ) yields DAZ = 23.47°. For the latter DAZ the lunar altitude which is necessary for crescent sighting is h' = 4.6°, according to Schoch. Since ARCV – h' = 9.4° – 4.6° = 4.8° follows, the result is the same as with Yallop's method.

A Set of Visibility Lines Derived from Schaefer's Model of Crescent Visibility

Schaefer discussed his computational model for crescent visibility among topics of historical astronomy in 1999 at the SEAC conference at La Laguna (Schaefer, 2000). On that occasion he explained to me various features of crescent observation; later he put at my disposal the set of crescent visibility lines in table 7. Schaefer's set expresses his "program results in terms of the 50% probability level" at the time of the solstices and the equinoxes and for four values of DAZ within the azimuth-altitude diagram. Table 7 gives "the critical altitudes of the moon with its 1σ uncertainty. The uncertainty regions are skewed, with it being quite unlikely that the altitude [of a sighted crescent] will be lower than the 1σ low value and there is a corresponding increase in the probability of excursions to a higher-than-one-sigma-up threshold due to bad haze" (Schaefer, letter to the author, 1999/11/23).

The critical altitudes of table 7 show seasonal variations: the 50% probability line varies between 10.2° in December and 11.6° in June while the uncertainty zones vary, dependent on the season and also on DAZ. As figure 14 shows, a problem is that three crescents (capitalized in table 8) are below either the line for October or November and thus not acounted for, whereas four crescents (italicized in table 8) are above either the line for October or November and thus accounted for by Schaefer's set.

Table 8 lists the difference △(h – h') between h of a certain crescent and the corresponding h' of a point on the lower border of the seasonal uncertainty zone according to Schaefer's set. The respective crescents are listed in table 4 above as crescents which were unknown to Schoch; only some are known to Schaefer.

The negative values of △(h – h') indicate that the uncertainty zones of Schaefer's set are too pessimistic for certain months and for certain values of DAZ, especially for DAZ < 10°.

Note that 'Schaefer's set' as tested here differs, properly speaking, from Schaefer's model. It has been remarked recently that in the absence of testing, it is not known how well the modern approaches to lunar visibility, including Schaefer's, perform with regard to observations made before the industrial revolution and resulting air pollution (Huber, 2011: 178). Such testing is the objective of the benchmarking of Victor Reijs's implementation of Schaefer's model (see Reijs's contribution, below).

Empirical Visibility Lines of Babylonian Crescent Sighting

I have described above how the visibility criteria relative to the azimuth-altitude method developed from Fotheringham, over Maunder and Schoch to Huber, and how Schoch's visibility line is applied by Yallop and modified by Caldwell & Laney. It is against this background that I deduce azimuth-altitude visibility criteria from the Babylonian crescent observations. Stern's magisterial study of the Astronomical Diaries comprises 331 observed new crescents and 110 predicted ones as reported in SH 1-3 and 5-6 (Stern, 2008). His list of observed new crescents is a reliable basis for the derivation of Babylonian visibility criteria. I add some additional reliable data in the form of about 160 old crescent observations from SH 1-3 and 5-6 (see Appendix 1).

Figure 15 presents the reported 6% of all Babylonian new and old crescents which occurred between the years –380 and –60. A few crescents are represented in figure 15 which are earlier than –380 and later than –60. Reported new and old crescents are plotted within the azimuth-altitude diagram for the moment when the sun is at geocentric altitude 0°; for the reports, see the Appendices. Sixteen non-sightings which were explicitly reported by the Babylonian observers are included (*cf.* table 4).

Furthermore, figure 15 presents the respective counterparts of the sighted crescents, namely the positions of the moon a day before sighted new crescent and a day after sighted old crescent. The use of an implied non-sighted moon as the complement of a specific sighted one seems justified to me, since the Babylonian observers watched the sky night after night: a report of seeing new (old) crescent on one particular evening (morning) implies in general that the observer did not see the moon on the evening before (following morning), although there will have been instances when the weather made observation impossible. For taking into account the implied non-sighted crescents, see also Huber's comparable procedure, above.

There are two implied non-sightings which are isolated within the field of sightings, viz. the non-sighting of a new crescent on –163/5/25 and of an old crescent on –210/4/3 (marked by □ in figure 15). Both implied non-sightings appear to have been caused by clouds, rather than by exceptionally high extinction (see the respective comments in Appendices 1 & 2). The corresponding sightings which were made a day after new crescent and a day before old crescent are marked as combined red and black symbols.

Above I have used the three Babylonian crescents with lowest geocentric altitude relative to DAZ (old crescents of –248/10/27 and –284/11/4; new crescent of –264/9/26; see Appendices 1 & 2) for the definition of a corrected version of Schoch's line. Thus the latter serves here as the borderline of the reported Babylonian crescent sightings.

Probability Zones in Babylonian Crescent Observation

Sighted and reportedly non-sighted crescents, as well as implied non-sighted ones, intersect in a zone represented in figure 16, a detail of figure 15. The intersecting zone can be described as an uncertainty zone in the following sense: if, according to computation, a crescent is located in the intersecting zone, then its sighting or non-sighting is uncertain; the probability of sighting a crescent is definable on the basis of the distribution of sighted and non-sighted crescents within the intersecting zone.

In other words, all sightings which are presented in figures 15 and 16 were made above the lower border of the uncertainty zone. The reportedly non-sighted and the implied non-sighted crescents were observed below the upper border of the uncertainty zone; the upper border is definable as a line parallel to the lower border. The reference points are the explicitly reported non-sighting of –234/11/22 (table 4; Appendix 1) and the implied non-sightings of –144/9/20 and –118/9/3 (Appendix 2).

Figure 16 demonstrates that the non-sighted Babylonian moons are distributed unevenly in the uncertainty zone as defined here. This uneven distribution is presumably coincidental, since there are modern non-sightings from places other than Babylon which are more evenly distributed (see figures 22 and 25).

As defined here, the uncertainty zone has a width of 3.5°. It comprises about 63 cases of the sighted Babylonian crescents or ca. 13 % of the ca. 490 reportedly sighted crescents plus 10 predicted but non-sighted ones, plus about 76 cases or ca. 16 % of the ca. 490 implied non-sighted crescents. The implied non-sightings are indispensable for a realistic estimate of the probability for a crescent to be sighted within the uncertainty zone; the 10 reportedly non-sighted crescents are not sufficient for this purpose.

The number of non-sighted crescents decreases towards the upper border of the uncertainty zone, while the number of sighted crescents increases; decrease and increase are not directly proportional. On the basis of the actual figures, the probability for a crescent being sighted in the uncertainty zone is $p = 63/63+10+76 = 0.42$.

Besides the uncertainty zone, the following other zones could be defined: the zone below the lower border of the uncertainty zone with $p \approx 0$ for sighting a crescent; the zone above the upper border with $p \approx 1$ for sighting a crescent; finally, the uncertainty zone and the zone above its upper border can be combined. There are about 490 sighted crescents above the lower border of the uncertainty zone plus 10 reportedly non-sighted crescents plus 76 implied non-sighted crescents, altogether 490 + 10 + 76 moons of which 490 were sighted. Thus a Babylonian observer sighted a new or old crescent above the lower border with probability $p = 490 / 576 \approx 0.85$.

These figures refer to reported actual crescent observations; the statistical consequences of cloudiness, rain, fog, or desert dust for crescent observation are discounted. Furthermore, any possible seasonal changes in altitude and width of the uncertainty zone remain unconsidered.

Seasonal Changes in Babylonian and Modern Crescent Visibility

The reported Babylonian crescents are unevenly distributed through the year as figure 17 shows. The number of reported crescents and the proportion of new to old crescents vary from month to month. There are relatively more preserved reports from June through November than from December through May. It is to be expected that the unevenness affects any analysis of the seasonal visibility conditions.

Figure 18, a month by month presentation of the upper and lower borderlines of the uncertainty zone of Babylonian crescent observation, also shows uneven distribution. In February, March, and April the fields of Babylonian sighted and non-sighted crescents are separated; the crescents do not mix in uncertainty zones. The Babylonian data, at least as far as they are preserved, contradict for once Maunder's (1911: 359) assertion that "no one line can be drawn so as to include all the Moons observed, and none of the Moons missed, on the same side of it" (see section above 'Maunder's work on crescent visibility').

To smooth the differences in the monthly reports, I have organized the Babylonian crescents according to the most basic seasonal division, namely warm season and cool season, corresponding to the periods of high and low astronomical extinction (*cf.* Schaefer, 1992: 33, cited above). The six months of the warm season incorporate slightly more than half of the total, namely 54% of the reported new crescents and 53% of the reported old crescents; the cold season includes the remainder.

Babylonian Crescent Visibility Lines in the Cool Season

Figure 19 presents the preserved Babylonian crescent observations of the cool season, the latter taken as the time between September 26 (inclusive) and March 24 (exclusive), the Julian calendar dates of the equinoxes in –200. The lower border of the uncertainty zone of the cool season is identical with the lower border of the seasonally undifferentiated uncertainty zone; the respective defining observations were all made in the cool season. The upper border of the uncertainty zone of the cool season is defined as a parallel line to the lower border, passing through the explicitly reported seasonal non-sighting with highest relative altitude.

The overall probability for a crescent to be sighted in the cool season is p = 224/224+40 = 0.82, since 224 crescents were sighted above the lower border of the uncertainty zone whereas 40 crescents were not sighted (*cf.* table 8). The uncertainty zone of the cool season has a width of 2.8° compared to 3.5° in the case of the yearly uncertainty zone. Note that sighted crescents which are dated to the cool season and are located between the upper border of the uncertainty zone of the cool season and the upper border of the yearly uncertainty zone – coinciding with the upper border of the warm season's uncertainty zone – do not belong to the uncertainty zone of the cool season.

As figure 19 shows, sighted and non-sighted crescents are unevenly distributed within the uncertainty zone of the cool season. There are 19 sighted crescents and 40 implied or reportedly non-sighted crescents resulting in p = 0.32 for the overall probability of a crescent being sighted within the seasonal uncertainty zone. Table 9 lists for each non-sighted or sighted crescent its altitude $\triangle$h above the lower borderline of the seasonal uncertainty zone.

The data in table 9 are organized in thirds (marked in color) of the width of the uncertainty zone. There are, for example, in the cool season 15 non-sighted crescents and 8 sighted ones in the lower third, resulting in p = 0.35 as probability of a crescent being sighted.

Table 10 combines the average values of $\triangle$h of the respective thirds and the corresponding probabilities of the cool season. Furthermore, table 10 includes the approximate sighting probabilities for $\triangle$h-values of –0.1° or +3.0°, corresponding to crescent positions just outside the seasonal uncertainty zone.

Figure 20 presents a quadratic polynomial fitted to the single point data of table 10 by the method of least squares. The polynomial indicates how the probability of crescent sighting develops within the uncertainty zone. For comparison, the probability curve for the values of table 9 as it results from Huber's model is also indicated; Huber's term takes here the form $0.9(1.4 + \triangle h)/2.8$, for $-1.4 < \triangle h < +1.4$ (*cf.* section above 'Huber's work on Late Babylonian new crescents').

Comparison of Babylonian and Modern Crescent Observations During the Cool Season

Here I compare the Babylonian observations of the cool season with the modern naked-eye observations of the same season in the lists of Doggett & Schaefer and Caldwell & Laney. The comparison is restricted by the unevenness in Babylonian as well as in modern reports. Figure 21 demonstrates that there are in general less modern than Babylonian naked-eye crescent reports; the month-wise distribution of the reports differs also.

Included here and below are naked-eye sightings after detection of the crescent with optical help. Fotheringham (1921: 311) has spoken out against such a procedure: "... if observations are to be of any use for the interpretation of references to the first appearance of the Moon or of dates dependent on that appearance, it is essential that the Moon should not be discovered either with a pointer or with any kind of optical glass, even if she is seen with the naked eye after being so discovered".

Contra Fotheringham's viewpoint, there exists the possibility that one of two observers at the same place might sight the crescent while the other did so after optical detection. Here I use such sightings that are included in the lists of Schaefer & Doggett and Caldwell & Laney; for the visibility codes see table 1.

As figure 22 demonstrates, the Babylonian visibility lines of the cool season do not coincide with the borders of the uncertainty zone of the modern reports. There are one sighting and three non-sightings outside the Babylonian uncertainty zone.

The single sighted crescent below the Babylonian uncertainty zone is Pierce's new crescent of 1990/2/25, an observation which was made at an elevation of 5000 f (1524 m). Presumably the clear air in high elevations provides an advantage over the Babylonian low observation altitude. Figure 22 indicates other observations made at elevations above 1000 m and which are in general not close to the lower borderline of the Babylonian uncertainty zone.

There are two modern non-sightings just below the upper borderline of the Babylonian uncertainty zone (Schaefer Nos. 99 & 100: MacKenzie, Cape Town 1921/12/30 & 1922/1/29). Three further non-sightings are markedly above it (Odeh No. 344: Isiaq, Lagos 1999/1/18; Schaefer Nos. 255 & 273: McPartlan, New Halfa 1983/12/5 & 1984/10/25).

McPartlan's non-sightings are problematic. At least 3 non-sightings out of his total of 11 are exceptionable. These observations seem to be improbable for one and the same observer at one and the same place between November 1983 and December 1984. McPartlan himself was aware of the problem that his non-sightings posed and suggested possible explanations like "observer error" or "adverse observational conditions" (McPartlan, 1985: 249 n. 48; 1984/10/25).

He communicated details about his non-sightings only in the case of 1984/10/25 when he sighted new crescent on 1984/10/26: "The sun disappeared at SS [sun set] –7 minutes and NC ["time at which the new crescent was first distinguished"; here 15h 05m UT, see McPartlan, 1985, Table on p. 246] was one minute after the disappearance of the sun" (McPartlan, 1985: 249). In other words, the sun disappeared 7 minutes before sunset, being still above the horizon though obscured by clouds or by dust; a minute later he observed the moon at a topocentric altitude of c. 23°. One day earlier he (1985: 249) noted, with reference to the non-sighting of the crescent: "The sun disappeared at SS [sun set] –10 minutes owing to dust or clouds on the horizon". If so, then not only the sun, but the new crescent might also have been obscured, although he does not mention the possibility. Note that he remained undecided whether clouds or dust had obscured the sun; he might have missed the crescent in similar circumstances without realizing that there was dust high above the horizon. Thus it appears that the observational conditions in New Halfa were exceptional, and therefore I disregard McPartlan's non-sightings for the definition of the upper border of the uncertainty zone of crescent visibility. On the other hand, I accept the possibility of comparable non-sightings and cite the report Odeh no. 344 (1999/1/18; Lagos; Nigeria); this non-sighting presumably resulted from Sahara dust, which reaches its maximum level over West Africa, including Nigeria, in mid-January (Nwofor, 2010: 549). For the time being, I suggest putting non-sightings that are presumably caused by desert dust into a class by themselves, comparable to non-sightings due to clouds, rain, or dense mist.

To sum up: Babylonian visibility lines of the cool season correspond to modern observations if the following reports are disregarded: Pierce's new crescent as a high elevation observation and three non-sightings from New Halfa and Lagos which were presumably caused by Sahara dust.

Babylonian Crescent Visibility Lines in the Warm Season

Figure 23 presents the Babylonian observations of the warm season defined as the time between Julian calendar dates March 24 (inclusive) and September 26 (exclusive) as dates of the equinoxes around −200. The lower border of the uncertainty zone of the warm season is defined as parallel to the lower border of the yearly uncertainty zone, passing through the sighting with lowest relative altitude; the upper border is parallel to the lower one, passing through the explicitly reported non-sighting with highest relative altitude; the zone has a width of 2.6°. The overall probability for a crescent to be sighted in the warm season is p = 267/267+31 = 0.88, since 267 crescents were sighted above the lower border of the uncertainty zone whereas 31 crescents were not sighted (*cf.* table 11).

Sighted and non-sighted crescents are unevenly distributed within the uncertainty zone of the warm season. There are 26 sighted crescents and 31 implied or reportedly non-sighted crescents within the uncertainty zone of figure 23, resulting in p = 0.46 for the probability of a crescent being sighted .

For each non-sighted or sighted crescent, table 11 lists its altitude $\triangle$h above the lower borderline of the uncertainty zone of the warm season.

As above in the case of the crescents of the cool season, I organize also the crescents of the warm season in thirds of the width of their uncertainty zone and append a crescent position for each one below and above the zone; table 12 combines the resulting data.

Figure 24 presents a quadratic polynomial fitted to the single point data of table 12 by the method of least squares. The polynomial indicates how the probability of crescent sighting develops within the uncertainty zone. For comparison, the probability curve for the values of table 11 as it results from Huber's model is also indicated; Huber's term takes here the form $0.9(1.3 + \triangle h)/2.6$, for $-1.3 < \triangle h < 1.3$ (*cf.* section above 'Huber's work on Late Babylonian new crescents'). Figure 24 implies that the development of the sighting probablities for the actually reported crescents during the warm season is approximately the same as in Huber's idealizing model.

Comparison of Babylonian and Modern Crescent Observations of the Warm Season

As figure 25 demonstrates, there are modern crescent observations of the warm season which were made outside the Babylonian seasonal uncertainty zone. Three modern naked-eye sightings and a naked-eye sighting after optical detection were made below the Babylonian seasonal uncertainty zone. Furthermore, there are three non-sightings above the Babylonian seasonal uncertainty zone. One of these is just above the Babylonian borderline (Schaefer No. 22: Athens, 1864/8/4); the other two are higher up, both being observations by McPartlan (Schaefer Nos. 262 & 269).

If the two non-sightings by McPartlan are disregarded, then the borderline for non-sightings is approximately the same for Babylonian and modern sightings, since the Athenian non-sighting Schaefer no. 22 is only 0.2° above the Babylonian line. By contrast, the Babylonian visibility border for sightings deviates by as much as 0.6° from modern observations. The naked-eye observation Athens 1871/9/14 (see table 4) is 0.7° below the Babylonian line, and Athens 1868/4/23, a sighting after optical detection, is 0.8° below. There are three other observations which are between 0.3° to 0.5° below the Babylonian line (SAAO: Ashdod 1990/9/20; Arad 1997/8/4; Ramlah 1997/5/7; see table 4). The reliability of Ramlah 1997/5/7 (see table 4) is doubted by Caldwell & Laney (see section above 'Remarks on the crescent visibility model of Caldwell & Laney'). If one accepts the other four observations which are below the Babylonian seasonal uncertainty zone, then the Ramlah observation can also be accepted.

The discordance between Babylonian and modern warm season observations results, in the first place, from the observations made in Athens. A possible explanation is that different climatic conditions in Athens and Babylon result in different visibility lines. For the time being, I propose to resolve this dilemma by presenting in table 13 two sets of visibility lines for the warm season, one based on Babylonian data and the other on Athenian data.

Seasonal Visibility Criteria on the Basis of Babylonian and Modern Crescent Observation

Finally, I present in table 13 visibility criteria for the warm and cool seasons. One example may suffice to explain the use of table 13. The entry DAZ = 0° and h* = 10.1° ± 1.5° for September through March is to be understood as follows: during the cool season and for DAZ = 0° the average of the Babylonian uncertainty zone lies at h* = 10.1°. If the altitude h of a crescent is close to h* = 10.1° – 1.5°, then the probability for a sighting is assumed to be slight; if close to h* = 10.1°, the probability is assumed to be medium; and if close to h* = 10.1° + 1.5°, probability is assumed to be sizeable. The probability degrees slight, medium, and sizeable correspond roughly to lower, middle, and upper third of the zone. I use these terms in order to harmonize the differences between the actual distribution of crescents in the uncertainty zone and the idealized model of it.

It ought to be clear that a table of visibility values such as presented here is not meant for pedantic, but rather for circumspect use. Thus I leave it to the eventual user of table 13 to extrapolate visibility values for the transition from one season to the other.

Below I use the Babylonian visibility lines h* for the computation of Egyptian lunar dates of the Ptolemaic-Roman Period. Babylonian visibility lines ought to be applicable to Egypt, since the climatic conditions in Babylon and Upper Egypt are similar. Baghdad lies in the alluvial plain, 100 km north of Babylon and can be substituted for the latter. Temperatures in Baghdad increase towards July and decrease towards December/January, whereas relative humidity decreases from its maximum in December/January towards its minimum in June and July (www.climatetemp.info/iraq/baghdad.html).

There are regional features which affect crescent observation in Iraq, such as the usually dusty winds Sharqi and Shamal, the first in April/May and again, if less strong, in September through November, the latter in May through August (Takahashi & Arakawa, 1981: 229).

For Egypt, a general seasonal shift in extinction is attested; extinction is high in the hot months and low in the cool months (Shaltout, 2001: 631-634). The spring wind Khamsin which is (or was, until Lake Nasser changed its impact) accompanied by dust is comparable to the Sharqi in Iraq.

Luxor can be considered typical of Upper Egypt. The yearly temperature curve for Luxor is quite similar to Baghdad's (www.climatetemp.info/egypt/luxor.html). Relative humidity takes a parallel course in Luxor and Baghdad in the first half of the year; the values are about the same in summer, but they diverge in the second half of the year. In antiquity, relative humidity in the Nile Valley will have been higher in September through October/November due to the inundation, which, since the completion of the first high dam at Aswan at beginning of the 20th century, no longer occurs.

Finally, there is the question of how to deal with possible differences between Upper Egypt and Lower Egypt in crescent observability which might follow from the climatic differences (Griffiths, 1972: 92). For the time being, I presume that the crescent observation conditions of LE lie between those of Babylon and Athens. The question to which degree the visibility lines of sites with different climatic conditions diverge remains open.

Comparing Empirical Criterions With the Implementation of an Analytical Criterion (by Victor Reijs)

The Schaefer criterion has been implemented by Victor Reijs (Extinction angle and heliacal events, http://www.iol.ie/˜geniet/eng/extinction.htm). Schaefer's criterion is based on the physical, atmospheric, and physiological modeling of Earth's atmosphere and observer properties at a certain location and time; using this model a prediction can be made about the visibility of a celestial object from a specific location, and this can be utilized for heliacal events (like new and old crescent). The prediction capability of the model has been verified with actual observations by Schaefer (2000: 127), and Reijs is continuing to benchmark his implementation (Benchmarking; http://www.iol.ie/˜geniet/eng/benchmarking.htm).

For the location Babylon, the atmospheric circumstances at sunset times seem to be quite constant through the year, giving an astronomical extinction coefficient of around 0.43 (using LunaCal Ver. 4.0, https://sites.google.com/site/

moonsoc/software). Using these meteorological parameters, new crescents have been computed for the period 570 BCE to 34 BCE.

The straight right hand border in figure 26 results from the extreme positions of new crescents around autumnal equinox (and old crescents around vernal equinox). These positions correspond to limiting values of a combination of DAZ and geocentric ARCV, which occurs when the moon has its maximum southern latitude during major standstill periods. Correspondingly, the left hand border indicates extreme positions of new crescents around vernal equinox (and old crescents around autumnal equinox). The computation of these border lines was based on astronomical theory for the relevant epoch (Reijs, Benchmarking of Schaefer criterion. <http://www.iol.ie/~geniet/eng/benchmarking.htm#Confining> Section: Confining boundaries due to Sun and Moon position). The reported Babylonian crescents are situated nicely between these computationally determined border lines. Note that the right and left hand borders would be less steep for observations in high geographic latitudes.

An average visibility Schaefer line can be derived from the new crescents (blue dots) that have the lowest geocentric ARCV for a certain DAZ under an astronomical extinction coefficient of 0.43. This will produce a line just touching the underside of the blue dots. This average visibility Schaefer line is close to the empirical average line of Krauss (the yellow line). The agreement between Krauss's criterion and the implementation of Schaefer's criterion could indicate that the Schoch-Indian line is predicting an average ARCV while originally it was meant to be the minimum ARCV. [End of Contribution].

Previous Work on Egyptian Lunar Dates of the Ptolemaic and Roman Periods

Basic Elements in Astronomical Analysis of Egyptian Lunar Dates

The first step in astronomical computation of an Egyptian civil-lunar double date is the conversion of the Egyptian civil date into a Julian calendar date; the Julian calendar is used by astronomers for the period before 1582 AD. The standard tables for conversion are those of Neugebauer (1937).

The lunar component of the civil-lunar double date is generally one of the Lunar Days numbered 1 to 30. A meaningful astronomical computation of solar and lunar positions presupposes identifying of the lunar phase with which the count of the Egyptian lunar days began; another parameter of the computation is the beginning of the Egyptian calendar day. Egyptologists long discussed these issues; eventually a consensus was achieved, but nowadays there are still dissenters in some quarters.

In the 19th century, August Böckh could establish, on the basis of observation dates and times in Ptolemy's Almagest, that the Egyptian calendar day began at dawn before sunrise (Böckh, 1863: 298-308; Ginzel, 1906: 163). Richard A. Parker came to the same conclusion, independently of Böckh and without reference to Ptolemy (Parker, 1950: 32-35). Ptolemy and pharaonic sources which go back to the 2nd millennium BC provide sufficient evidence that the Egyptian calendar day lasted from first light (*i.e.*, quite some time before sunrise) until the next first light (Krauss, 2006:, 49-51 with literature, adding the different view of Luft, 2006).

Around 1920 Ludwig Borchardt realized that the Egyptian lunar month must have begun with the first day of invisibility after old or last crescent day, i.e. with an observable event (Borchardt, 1925: 620 n. 2; Borchardt, 1935: 19, 30 n. 10). Shortly thereafter, Karl Schoch came to the same conclusion independently (Schoch, 1928a). Subsequently Richard A. Parker argued in detail that the Egyptians counted the days of the lunar month from the first calendar day of the moon's invisibility (Parker, 1950: 25-48); more circumstantial evidence can be adduced (Krauss, 2006: 387-389; 2009: 136f). According to Parker, Lunar Day 1 (LD 1) coincided with the day of conjunction in ca. 88% of the cases, the day before conjunction in ca. 10.5%, and the day after conjunction in ca. 1.5% (Parker, 1950: 46).

Recently Peter J. Huber (2011: 176) cited a personal communication from Leo Depuydt that "there is evidence only that the month began earlier than first crescent, and that the available data alternatively would seem to be compatible with the assumption that the first day of the month (the *psḏntyw* day) aims to estimate the day when the moon catches up with the sun (*i.e.*, the conjunction of the two bodies, or syzygy, rather than first invisibility)". Below

I cite examples which could confirm Depuydt's opinion (first day of invisibility on the day of conjunction: Ptolemaic-Roman lunar dates Nos. 1, 4, 38, 39 and 40) and one example which is not a confirmation (conjunction on the day after first day of invisibility: No. 5; No. 19 may possibly be a second example). The single certain case tends to confirm that the first day of the lunar month was the first day of invisibility, without consideration of conjunction.

Parker used Schoch's age-of-the-moon method for the computation of Egyptian old crescents in his 'Calendars of Ancient Egypt' and in his later work on Egyptian chronology; he applied Neugebauer's procedure only in those cases which seemed to be doubtful. Schoch had developed his age-of-the-moon method for computing new crescent visibility for the latitude of Babylon in the mid-1920s, when he worked with John K. Fotheringham on the Venus Tablets of Ammizaduga (Schoch, 1928b: 94f). On this basis, Father Schaumberger computed the 8500 or so new crescent dates for the first edition of Parker's and Dubberstein's "Babylonian Chronology" (Parker & Dubberstein, 1956: v, 25). Sacha Stern (2008: 39) terms Schoch's age-of-the-moon method "out of date" and in some cases inaccurate.

Parker (1950,: 20) described his usual procedure for computing an Egyptian old crescent as follows: [Schoch's tables for calculating conjunction] "are for Babylon, but a correction for difference in longitude is easily made. His table G may be used for an approximation of the time from conjunction to new crescent or from old crescent to conjunction. Since all of Egypt lies south of Babylon, the time required in either case will not be greater than the figure derivable from Table G. Any doubtful case may be checked by the use of tables E 21-26 in Neugebauer's Astronomische Chronologie".

In at least one case, Parker's procedure resulted in a mistaken computation of old crescent (see below No. 5). In general Egyptologists either relied on the results of Parker's computations or were satisfied with the computation of conjunction rather than old crescent, presumably because old crescent does fall in most cases on the day before conjunction.

The Work of Borchardt and of Parker on Ptolemaic-Roman Lunar Dates

Here and below, Egyptian dates that are not designated 'civ.', 'alex.', or 'lunar' are to be understood as dates of the civil calendar.

Borchardt was the first to collect and analyze Saite, Ptolemaic, and Roman lunar dates (Borchardt, 1935: 39-43, 57-73). Apparently he did not compute old crescents on the basis of Neugebauer's 'Astronomische Chronologie'; rather he identified old crescent day with the day before conjunction. He postulated that the Egyptian lunar calendar was schematic so that there could be a difference of several days between calendric lunar dates and astronomical reference dates (Borchardt, 1935: 36f.). Borchardt's pioneering work was taken up by Parker (Parker, 1950: 17-23). It seems that when Parker worked on his 'Calendars' he was unaware of Borchardt's last book on Ptolemaic-Roman Egyptian chronology (Borchardt, 1938).

Ulrich Luft (1992: 33, 189-197) realized that at Illahun in the Middle Kingdom the lunar service of the temple phyles began on LD 2 (*ꜣbd*), thus correcting Parker, and others, who had assumed LD 1 (*psḏntyw*) as first day. Luft did not address the question of whether the lunar temple service began on LD 2 or 1 in other eras of Egyptian history. With reference to the Roman Period in Egypt, Sandra Lippert and Maren Schentuleit (Lippert & Schentuleit, 2006: 183) state rather sweepingly that "*der Phylenwechsel fand am ... zweiten Tag des Mondmonates (ꜣbdw) statt, wie bereits aus den Illahun-Papyri hervorgeht*".

When Parker wrote his book on Egyptian calendars, only two chronologically securely anchored examples of Ptolemaic-Roman dates for lunar temple service were available, *viz.* on the stela Moscow 145 and on oThebes D 31. According to the Moscow stela (see below, No. 5), in Nero's year 12 the first day of the *wrš* or temple service of a phyle in Coptos, coincided with IV prt 23 alex. = I *šmw* 10 civ. Borchardt (1935: 39 n. 4; Spiegelberg, 1931: 42f.) surmised that *wrš* meant 'lunar month', since the first day of the *wrš* on the Moscow stela coincided with a conjunction. Parker could demonstrate that in pDem Cairo 30801 *wrš* did indeed mean "service in the temple, by lunar months, of the various phyles" (Parker, 1950: 18-21). He mistakenly computed a LD 1 as the equivalent for I *šmw* 10 in year 12 of Nero; the observationally correct equivalent is a LD 2. Relying on his assumption that a lunar temple service began on the first day (*psḏntyw*) of the lunar month, Parker identified I *šmw* 10 as LD 1 and first day of the *wrš*.

Parker's mistaken computation is explainable, provided he proceeded in this instance according to his own rules (*cf.* 'Introduction'). Following Schoch he will first have computed conjunction for Babylon as AD 66, April 13, 10h 25m; he will have then corrected the time for the longitude of Coptos. Next he looked up the time when new crescent became visible in Babylon and which he thought to be in general, more or less, also the time from old crescent to conjunction. Since the required time was several hours less than the interval between conjunction and 6h in the evening of April 13 or 6h in the morning of April 12, he concluded that old crescent was visible in Babylon and Egypt (Coptos) on April 12.

New crescent was indeed visible in AD 66 on April 14 in Babylon and Egypt, weather permitting, but old crescent was not visible on April 12. If Parker had checked the situation within the azimuth-altitude diagram, he would have found from Neugebauer's tables that the moon stood at sunrise in Babylon 4.7° and in Coptos 3.6° below the necessary altitude. Invisibility of the moon on the morning of April 12 resulted from the small angle between ecliptic and horizon (Babylon: ca. 34°; Coptos: ca. 41°), combined with a large negative lunar latitude of –4.9°. By contrast, in the evening of April 14 the large angle between ecliptic and horizon (Babylon: ca. 79°; Coptos: ca. 80.1°) made the effect of the negative lunar latitude of –3.9° on visibility negligible. Considering van der Waerden's and Casperson's mistake (see section above 'Basic Elements of Crescent Computation According to the Azimuth-Altitude Method and van der Waerden's Mistaken Modification of it'), it seems possible that Parker was not aware that the angle between horizon and ecliptic is, in general, not the same for successive old and new crescents.

In the case of oThebes D 31 (see No. 24, below), Parker computed IV prt 28 alex. = II *šmw* 21 civ. = April 23, the first day of a temple service month in AD 190, as a LD 2; presumably, he followed the same procedure as described above when he computed the old crescent day in question. The result is not impossible; nevertheless, under average visibility conditions, the day was rather a LD 3. In the corresponding year 21 of the Carlsberg Cycle, II *šmw* 21 civ. coincided with a LD 1. In accordance with Parker's assumption that LD 1 was the first day of the lunar temple service, he decided in favor of the cyclical date of II Smw 21 civ. Note that the designation of the lunar monthly temple service in oThebes D 31 is *jbd* (*ꜣbd*, month), not *wrš* as on the Moscow stela.

In the 1970s Parker became aware of lunar phyle dates in Ptolemaic graffiti from Medinet Habu which Heinz-Josef Thissen studied (see below, Nos. 1-4 and also Nos. 14-17 and 21-22). The first days of phyle services coincided with a LD 2 in those Medinet Habu graffiti which can be assigned to specific reigns. The coincidences contradict Parker's assumption that LD 1 was the first day of the lunar temple service. Parker explained the 'anomaly' by presuming that the Macedonian lunar calendar, which began on new crescent day corresponding to Egyptian LD 2, had been used (Thissen, 1989: 181-183). He did not consider the fact that the Macedonian calendar was not in use in Egypt when the Medinet Habu graffiti were written (Bennett, 2008: 527 n. 9). When Parker's mistakes are corrected, the respective first days of Ptolemaic-Roman temple service (*ꜣbd*, *wrš*) coincide with LDs 2, not with LDs 1.

Chris Bennett's Recent Work on Ptolemaic-Roman Lunar Dates

Bennett presented an analysis of 40 lunar dates from the period between Amasis and Caracalla (Bennett, 2008). His material consists of four groups: Nos. 1-5 "Complete direct civil/temple service synchronisms"; Nos. 6-25 "incomplete direct civil/temple service synchronisms"; Nos. 26-32, "inferred direct civil/temple service synchronisms from Dime"; and Nos. 33-40, "other fixed Egyptian lunar synchronisms". Throughout below, I shall employ Bennett's numbering of the sources.

Bennett is interested in three issues: (1) whether the first day of the Ptolemaic-Roman lunar temple service was determined by observation or cyclically (in particular, by employing the cycle of pCarlsberg 9); (2) whether LD 2 was the first day of the Ptolemaic-Roman lunar temple service; and (3) the accuracy of Egyptian lunar observation (the hit rate of the observers). He (2008: 532) calculated crescent dates "using the program PLSV 3.0 which provides direct estimates of first and last crescent visibility using an empirical formula derived by Caldwell and Laney. The default parameter settings of the Caldwell-Laney formula appear to make it a good estimator, correctly predicting 193 of the 209 Babylonian first crescent observations studied by Fatoohi *et al.* (1999) – a success rate of 92.3%. However, it does occasionally miss observable crescents, or predict unobservable ones".

The Caldwell-Laney parameter is a better estimator than cited, since actually 98.5% of the 209 Babylonian new crescents are covered by it. I find only three cases which yield values below h' + ARCL/3 = 11.3° (*cf.* table 4). Apparently there is a mistake, either on Bennett's part or in the program; the latter cannot be tested, since it no longer seems to be available. According to Bennett (20008: 532f): "More significantly, the default settings are optimistic in two respects. First, they assume that the observer is experienced, trained, and has good vision. While these are reasonable assumptions for the observers at the E-Sagila temple in Babylon, we do not know a priori whether they are valid for observers in contemporary Egypt".

The question addressed here is whether the default settings of the PSLV program which work for Babylon are valid for Egypt. In Babylonia and Egypt, professionals made astronomical observations (Steele, 2009: 32-34; Fissolo, 2001: 15-24; Bonnet, 1952: 307 s.v. Horologe). The work of the Babylonian observers is directly preserved in the Astronomical Diaries. The results of the Egyptian observers' work have come down to us indirectly in the form of lunar and Sothic (Sirius) dates used by scribes and others. In the worst-case scenario, the Egyptian observers were inexperienced, untrained, and had poor eyesight. The latter possibility must be given weight considering that Egypt is the classical land of eye diseases. Bennett (2008: 533): "Second, the formula was developed for observations of first crescent visibility. Wells noted that last crescent visibility presents greater psychophysical challenges to the observer".

Wells (2002: 463) himself states that determination of last crescent visibility is considerably more difficult than first crescent visibility because the former involves detection of a rising object while for the latter, the Moon is generally 10° or more above the horizon when the sky becomes dark enough after sunset for the crescent to be apparent. Because of the large extinction factor on the order of 14 magnitudes at the horizon, it is easier to miss the thin rising crescent which will soon disappear because of impending sunrise".

Wells's assertions are not reliable throughout (*cf.* Krauss, 2003a). Extinction affects new and old crescent in the same way. The rising old crescent is as close to the horizon as the setting new crescent.

Extinction on the order of 14 magnitudes results when light has to travel through 40 airmasses at an extinction factor of 0.35; the latter factor was determined by Schaefer for the Nile valley in ancient times (Wells, 2002: 462 n. 19). By contrast, De Jong deduced an extinction factor of 0.27 for arid Upper Egypt; he allows an extinction factor of 0.35 for humid Alexandria (De Jong, 2006: 436f). Considering that there are not 40, but rather 38 air masses at the astronomical horizon (Kasten & Young, 1989: 4735ff), an extinction of 10.26 magnitudes follows from De Jong's parameter. Thus an old crescent with a magnitude of ca. –6 cannot be sighted on the astronomical horizon. However, the situation at the horizon is irrelevant for actual crescent observation. The point is that an observer who is stationed in the Nile valley does not see the astronomical horizon; old crescent becomes observable only after the moon has risen above the local horizon. If the latter has an altitude of say 2°, the light of the crescent has to travel through 19.4 air masses, resulting in an extinction of 5.24 magnitudes.

According to Yallop's concept of 'best time' old crescent is observed on average far above an altitude of 2°. In the case of the 153 reported Babylonian old crescents, the average lag between moonrise and sunrise amounted to 74 minutes; in the case of 332 reported new crescents the average lag also amounted to 74 minutes. Thus at 'best time' the average old crescent as well as the average new crescent had a topocentric altitude of ca. 8°, whereas the sun had an altitude of ca. –7.5°.At a topocentric altitude of ca. 8° the light of the crescent has to travel through 6.8 air masses, resulting in an extinction of 1.8 magnitude which compares favorably to a magnitude of 14. Such are the astronomical facts – that is, if I did not doctor my numbers, for which possibility compare Schneider (2008: 292) and Gautschy (2011: 15f.).

According to Bennett (2008, 533) "In order to detect whether a PLSV result may be sensitive to the visibility threshold, the program was run with baseline lunar altitude settings of 10.8° and 12.3° in addition to the Caldwell-Laney setting of 11.3° (n.: 32)."

In other words, when he computed Ptolemaic-Roman dates, Bennett set the default parameters at 10.8° < h' + ARCL/3 < 12.3°. He does not explain the reasons for his choice of parameters. I presume that 12.3° is to be understood as a schematic addition of 1° to 11.3°. By contrast, 10.8° is perhaps derived from h' + ARCL/3 = 10.73° for the Babylonian new crescent of –264/9/26 which has the smallest value of h' + ARCL/3 of the 209 Babylonian new crescents. As I explained above, the Caldwell-Laney default parameter might be set at still smaller values (see section 'Remarks on the Crescent Visibility Model of Caldwell & Laney' above).

Thus Bennett computed 209 Babylonian new crescents employing an astronomical program with parameters developed on the basis of worldwide crescent observations; he expresses reservations about applying the results to Egyptian crescent observation out of concern for various possible shortcomings on the part of the Egyptians.

The Lunar Cycle of pCarlsberg 9

Reconstructions of the Carlsberg Cycle

Whereas the Babylonians used computational models to reckon new crescent, the Egyptians during the Ptolemaic and Roman Periods used lunar cycles, at least according to Egyptologists; in any case, schematic lunar cycles have been preserved. An Egyptian 25 year lunar cycle from the reign of Ptolemy VI is documented in pRylands IV 589 (Turner & Neugebauer, 1949/1950: 80-96). The cycle is fragmentary, but enough remains to show that it differs from the Carlsberg Cycle which is presented in table 14.

Papyrus Carlsberg 9 was written in AD 144 or later (Neugebauer & Volten, 1938: 383-406). It contains a cyclical lunar calendar spanning 25 Egyptian years or 309 lunar months. Thus both the Carlsberg Cycle and the pRylands cycle are based on the fact that there are 9125 days in 25 Egyptian civil years, and that 309 mean lunar months amount to 9124.95231 days (Parker, 1950: 50). The text of the Carlsberg Cycle does not indicate a date for every month, but lists only six dates for every year, *viz*. those falling in the second and fourth month of each season or, in other words, the even numbered months. Of 309 implied dates, only 150 are therefore actually given. Parker (1950: 15) notes that "It also indicates the years of 13 months ('great years') according to the following scheme: 1st, 3rd, 6th, 9th, 12th, 14, 17th, 20th and 23d year of each cycle" (capitalized in table 14).

After studying the dates for the year AD 144, Otto Neugebauer and Axel Volten, the editors of pCarlsberg 9, concluded that the cycle months began with new crescent visibility. Parker took into account that a lag of 1.14 hours develops within one cycle and also that in AD 144 at least five cycles had passed. The papyrus lists the beginnings of five successive cycles; the earliest began in year 6 of Tiberius or AD 19/20, and the last, in year 8 of Antoninus or AD 144/145 (Lippert, 2009: 189f.). The lag is the difference between 9125 days = 25 Egyptian years and 309 mean lunar months of 29.53059 days each; thus the resulting lag is 9125 days minus 309 x 29.53059 days = 1 hour 8.6 minutes.

Under the premise that the dates of the cycle represent the first days of lunar months, Parker argued (1950: 60), "the closer in time the cycle approaches AD 144, the less frequently will the months start with invisibility and the more frequently with visibility". He concluded that the Carlsberg Cycle was introduced in the 4th century BC and presented the civil dates of first invisibility days or LDs 1.

Parker did not doubt that the Carlsberg Cycle was actually used for determining lunar days. Under this presupposition he reconstructed eight of the missing cycle dates on the basis of Ptolemaic-Roman civil-lunar double dates; he chose the wrong cycle year in one case (see Nos. 7-13bis below). For the reconstruction of the remaining 151 dates, he developed a set of rules and completed the cycle accordingly. He also extrapolated a series of cycles back to the 4th century BC on the basis of the recorded cycles in pCarlsberg 9.

According to Neugebauer, it seems doubtful a priori whether there were fixed rules for the months which are not recorded in pCarlsberg 9, since in that case it is hard to see why the papyrus should give only half of the dates (Neugebauer, 1975: 563). Alexander Jones emphasizes that the number of attested lunar dates Parker used to reconstruct the scheme is insufficient to establish a regular pattern (Jones, 1997: 162).

More recently Leo Depuydt (1998) completed the Carlsberg scheme in the sense "of an arithmetical game seeking to fit 309 months into 25 Egyptian years according to as simple a rule as possible". In his (1998: 1293) opinion, "it is possible that a cycle according to which a lunar Day 1 coincided with civil I Axt 1 of 144 C.E. was used during much of the Greco-Roman period in Egypt". On the other hand, he (1998, 1294) mentions the possibility that the Carlsberg cycle was "a mere arithmetical *Spielerei* [that] was never used". In any case, some of the dates that are reconstructed by Depuydt do not correspond to cycle dates deducible from the Ptolemaic-Roman double dates (see below Nos. 10, 13bis, 27, 34/35 and 36).

Instead of using either Parker's or Depuydt's reconstruction of the Carlsberg Cycle, I shall rely on what I call the interpolated form of the cycle. Since a lunar month has either 29 or 30 days, there are, generally speaking, two possibilities for each missing date in the Carlsberg cycle. By contrast to Parker and Depuydt, I accept the two possible dates which are implied by the recorded form of the Carlsberg Cycle and ascribe to each the probability 1/2. Perhaps the ancient user was free to choose between a 29 or 30 day month to fill in the gaps in the recorded scheme (which may have been Neugebauer's idea of how the cycle was to be used).

Regardless, the question remains whether the Carlsberg Cycle was actually used or not. Below I follow Bennett's lead and compare the Ptolemaic and Roman lunar dates not only with observationally determined dates, but also with the dates of the Carlsberg Cycle. All assertions about coincidences with cyclical dates are made under the assumption that the actual use of the cycle has not yet been confirmed beyond doubt.

Ideal and Actual Repetition of Cyclical Dates

Parker cited the difference of 1 hour and 8 minutes between 9125 days and 309 mean synodic months of 29.53059 days each, as the only cause for the Carlsberg Cycle's incomplete repetition over longer periods. But in actuality, there are no mean lunar months of 29.53059 days; rather, there are lunar months of full 29 or 30 days. The 25 year cycle presumes 309 LM = 9125 d, composed of 164 LM of 30 days + 145 LM of 29 days. Since lunar motion is irregular, it can happen that in a sequence of 309 lunar months there are 165 months with 30 days each and 144 months with 29 days. In that case, a lunar day does not coincide with the same civil calendar day after exactly 25 Egyptian years, but it falls rather on the next day. If, by contrast, the number of lunar months with 29 days were 145 and the number of lunar months with 30 days were 164, then after 25 Egyptian years the given lunar day fell a day earlier (for an example of a repetition after 9126 days, see Krauss, 2003b: 190-192).

It is erroneous to presume that Egyptian lunar dates repeat regularly every 25 years, as for example Kenneth A. Kitchen (1991: 204) asserted: "these moon-risings occur in the ancient calendar every twenty-five years". By contrast, Parker stated that on average only about 70% in a given set of lunar dates repeat on the same civil day after a single 25 year shift (Parker, 1950: 25f). The repetition of lunar phases depends not only on the synodic month, but also on the anomalistic and dracontic (or: draconitic) month of 27.55 and 27.21 respectively; these periods do not share a common duration of 25 years (Krauss, 2005a: Excursus 2). The interplay of these and other factors results in a complicated pattern, if Egyptian lunar dates are shifted consecutively in 25 year intervals (Krauss, 2006: 405f.).

Presumably Parker understood the lag of 1 hour and 8 minutes as a mean value. But Depuydt does not (1998: 1281f.): "If sun, moon, and earth, in that order, position themselves in a single plane, that is, are in conjunction, in the morning of I *3ḫt* 1 or New Year's Day of a given Egyptian wandering year, they will again be found in this position on I *3ḫt* 1 25 Egyptian years or 9125 days later, though about an hour earlier in the day". As table 15 shows, there are, in general, no one hour differences between conjunctions that are 9125 days apart.

Ptolemaic and Roman Lunar Dates

Ptolemaic and Roman Lunar Dates Nos. 1-5: "Complete Direct Civil/Temple Service Synchronisms"

Nos. 1-4 (Medinet Habu Graffiti 43 [2 dates], 44 and 47) – The Medinet Habu graffiti were published in facsimile by William F. Edgerton (1938) and studied by Thissen (1989). Michel Chauveau (1995) has commented on Thissen's readings and interpretations. Parker analyzed the civil-lunar double dates that are found in certain graffiti (Thissen, 1989: 181-183). Nos. 1–4 refer to phyle services in the small temple of Medinet Habu. Nos. 1–2 are consecutive, whereas Nos. 3 and 4 are separated by several years; all four dates lie within the extrapolated Carlsberg Cycle 57/32 BC. The scribes cited already established civil-lunar double dates and did not refer to expected dates.

MH Graffito 43 (Thissen, 1989: 15-18; Chauveau, 1995: 252f.) refers to an increase in priestly income by the strategos. The graffito comprises two civil-lunar double dates (Nos. 1 and 2) and was written at the earliest on the date of No. 2. According to No. 1, day 12 of a phyle-service (*wrš*) and I *prt* 1 coincided in 56 BC = year 26 of Ptolemy XII, corresponding to year 3 of Ptolemy XII & Berenice IV (Bennett, 2008: 533 n. 35); thus service day 1 fell on IV *3ḫt* 20 = –55 Dec 24. As tables 16 & 17 imply, IV *3ḫt* 20 coincided observationally with a LD 2, since last visibility fell on IV *3ḫt* 18. Furthermore, service day 1 coincided also with a LD 2 in year 2 of the recorded Carlsberg Cycle corresponding to 56 BC. The reader may note that the coincidence of service day 1 and LD 2 in the Carlsberg Cycle is implied by Bennett's Table 1 (Nos. 1-4), since it demonstrates a difference of 1 day between Carlsberg Cycle day 1 and Service day 1. Bennett does not comment on the coincidence.

No. 1: Service day 1 coincided observationally and cyclically with a LD 2.

According to date No. 2, the phyle-service (*wrš*) following that of No. 1 began on I prt 19 = –54 January 22. Bennett considered January 20 or 21 in –54 as old crescent day and decided in favor of January 20, thus identifying the first service day as LD 2 by observation. I concur, since as table 18 implies, observational invisibility of the moon was on I prt 18 more likely than visibility. As table 19 shows, the first service day could have been a cyclical LD 1 or LD 2, since in Carlsberg Cycle year 2 the interpolated LD 1 fell either on I prt 18 or 19. Parker chose I *prt* 19 as LD 2 in his reconstruction of the Carlsberg Cycle; I *prt* 19 is also a LD 2 in Depuydt's reconstruction.

No. 2: It remains open whether service day 1 was determined observationally or cyclically as a *Ld̠* 2; both are possible.

No. 3 (MH Graffito 44: Thissen, 1989: 18-29; Chauveau, 1995: 253) refers to a visit of the strategos; the graffito could not have been written before the visit. According to No. 3, day 20 of the phyle-service (*wrš*) and I *3ḫt* 14 coincided in 48 BC, corresponding to 5 Ptolemy [XIII] & Cleopatra [VII]. Bennett (2008: 534 n. 36) follows Chauveau whose assignment of the graffito allows a lunar dating. By contrast, Parker could not achieve a lunar match, since he presumed that the graffito dated to Ptolemy [XII] and Cleopatra [V] (Thissen 1989: 182). Following Bennett's premises, day 1 of the phyle-service fell on IV Smw 30 = –47 August 29. As Table 20 implies, there was a better than 50:50 possibility that last visibility was observable on –47 Aug 27 = IV S 28, resulting in a LD 2 as service day 1. As Table 21 shows, service day 1 coincided with a LD 2 in year 9 of the recorded Carlsberg Cycle.

No. 3: The first service day possibly coincided observationally with a LD 2 and certainly with a LD 2 in year 9 of the recorded Carlsberg Cycle.

No. 4 (MH Graffito 47: Thissen, 1989: 41-44; Chauveau, 1995: 253) refers to coinciding service months of two priests. The graffito seems to have been written on II prt 21 which coincided with a phyle service (*wrš*) day 17 in 37 BC, the latter year corresponding to 15 Cleopatra VII; thus service day 1 fell on II prt 5 = –36 Feb 3. As tables 22 & 23 indicate, last visibility fell observationally on II prt 3, resulting in service day 1 as LD 2; furthermore, service day 1 also coincided cyclically with an LD 2.

No. 4: The first service day coincided with a LD 2, be it determined in the Carlsberg Cycle or by observation.

No. 5 (building inscription from Coptos; stela Moscow 145; Spiegelberg, 1931: 42f.; Parker, 1950: 69-71) – The text notes that IV prt 23 alex. (= I *šmw* 15 civ.) coincided with *p3 hrw n mḥ 6 n p3 wrš* in year 12 of Nero, corresponding to AD 66. Parker relied on his postulate that the monthly lunar service began on a LD 1; as explained above, he mistakenly computed IV prt 18 alex. (= I *šmw* 10 civ.) = April 13 in AD 66, the first day *n p3 wrš*, as a LD 1 (Parker, 1950: 71). Borchardt (1935, 39) computed (mean?) conjunction as occurring on April 13 at 10 h 11 m (modern value for Egyptian time zone: 9 h 33 m; Borchardt calculated for Babylonian time). He concluded that April 18 was a 'calendric' LD 6 and thus also determined April 13 as first day of the *wrš*. The recent analysis of Depuydt is based on Parker's premise that the lunar temple service began on LD 1; furthermore, he computes conjunction rather than last visibility.

Thus his result is inconclusive (Depuydt, 1997: 184ff). As table 24 indicates, the observationally correct day of invisibility, or LD 1, was IV prt 17 alex. (= I *šmw* 9 civ.) = April 12 in AD 66. It follows that April 13 as first day *n pꜣ wrš* was observationally a LD 2, coinciding with IV prt 18 alex. (= I *šmw* 10 civ.). Furthermore, Parker argued that the first day of the *wrš* also coincided cyclically with a LD 1. Again relying on his postulate, he asserted: "*Psḏntyw* must fall on IV prt 18 [alex.]. For A.D. 66 this should be in cycle year 22, but there is no agreement. In the civil calendar, however, the date would be I *šmw* 10; and this fits nicely in the correct cycle year between IV *prt* 11 and II *šmw* 10." In year 22 of the recorded Carlsberg Cycle, IV *prt* 11 and II *šmw* 10 are cited as LDs 1, so that I *šmw* 10 or 11 might be interpolated between them as LD 1. The interpolation would result in I *šmw* 11 or 12 as LD 2 and LD 30 or LD 1 for I *šmw* 10 civ. as first service day (see table 25). Parker chose I *šmw* 10 civ. as LD 1 on the wrong assumption that it coincided observationally with a LD 1 in AD 66 and that it also coincided with a LD 1 as first day of the *wrš*. For different reasons, Depuydt also reconstructs the missing first lunar day in year 22 of the Carlsberg Cycle as I *šmw* 10.

No. 5: Provided the first day of the *wrš* on I Smw 10 civ. was meant to be a LD 2, then the date was determined observationally and not cyclically.

Ptolemaic and Roman Lunar Dates Nos. 6-25: "Incomplete Direct Civil/Temple Service Synchronisms"

No. 6 (pDem Ox. Griffith 41, from Soknopaiu Nesos; year 40 of [Ptolemy VIII]) – Edda Bresciani (Bresciani, 1975: 52f.) translates the relevant passage: "*Era il giorno 19 di Paofi, la veglia; noi andammo al tempio per fare le aspersioni del nostro mese di aspersioni.*" Bennett (2008, 535) understands II *ꜣḫt* 19 in year 40 of Ptolemy VIII "as the date of the start of the phyle service given by the papyrus", although he sees "grounds for supposing that the correct date is in fact II *ꜣḫt* 20". He cites a new reading by Karl-Theodor Zauzich which proposes "that the phyle came to the temple on the day before the start of the service [*wrš*], *i.e.* that the correct date for the start of the service is II *ꜣḫt* 20".

Furthermore, according to Sandra Lippert, "the term *wrš* [*la veglia*] could refer to the *wrš* feast in this context rather than the month of phyle service" (Bennett, 2008: 534f. The calendric timing of the *wrš*-feast is implied by a passage in the Demotic Chronicle. According to Heinz Felber (2002: 76f.), Chronicle II, 9 provides the information that "*das Asche(?)-Fest das Ende des Monats ist*". Joachim Quack suggests emendation of the otherwise unknown *ꜥše* in *wrš*. In Chronicle II, 10, we read "*das Nebti-Fest der Anfang des Monats ist*". According to another late source, the *Nbtj*-feast is identical with the feast of LD 2 (Osing, 1998: 110f.). Thus within the Demotic Chronicle the beginning of the 'month' (*ꜣbd*) seems to be shifted by one position, relative to the pharaonic enumeration of lunar days; the same should be true for *ꜥše* > *wrš*-feast as the end of the month (Krauss, 2005b: 45f). As one possible solution of the problem, I propose that the term 'month' in the Chronicle does not refer to the calendric lunar month that begins on its first day and ends on a 29th or 30th, but rather that the lunar temple service month is meant, in which the *Nbtj*-day figured as the first and the *wrš*-day as the last day of phyle service.

According to Zauzich and/or Lippert, the phyle service of No. 6 probably began on II *ꜣḫt* 20 = –130 November 13. As tables 26 & 27 indicate, observable last visibility occurred on –130 November 11, resulting in II *ꜣḫt* 20 as LD 2, whereas cyclically II *ꜣḫt* 20 fell on a LD 1.

No. 6: Provided the phyle service did indeed begin on II *ꜣḫt* 20 = LD 2, the date was determined observationally and not cyclically.

Nos. 7 - 13bis (pDemot Cairo 30801 from Gebelein; on the Verso: year 41 [Ptolemy VIII]) – The recto cites a series of consecutive *wrš*-months in connection with several priestly accounts of grain and other commodities (Parker, 1950: 89-91); the monthly sequence of dates is interrupted by the earlier account no. 7 (table 28): Taking III *šmw* 16 as last day of a *wrš*-month in No. 13, Parker did not hesitate to conclude that the next *wrš* began on [III *šmw* 17]. I count the extrapolated date as No. 13bis; it is equivalent to III *šmw* 16 as the concluding date of No. 13 in the following sense: if the initial dates of Nos. 8 to 13bis were LDs 1 (Parker's premise), then the concluding dates would be 30th or 29th LDs; but if the initial days were LDs 2, then the concluding dates would be LDs 1. Bennett uses only Nos. 8 to 13.

Parker assigned the series to 26 [Ptolemy VIII] when four dates coincided observationally, and in the extrapolated Carlsberg Cycle as well, with LDs 1 (*sic*). By contrast, Bennett dates the recto and thus Nos. 7-13 in 40 [Ptolemy VIII] or 131/130 BC, arguing that the verso is dated to 41 [Ptolemy VIII]. Table 29 presents the circumstances of last visibilities for Nos. 7-13bis, according to Bennett's premise that the set is dated to 40 [Ptolemy VIII].

As table 29 indicates there can be no doubt about the date of last visibility in the cases of Nos. 7, 8, 10, 12, and 13, whereas the remaining cases of Nos. 9, 11, and 13bis allow for two possible dates.

Table 30 indicates that as a set, the dates Nos. 7-13bis are not observationally determined, whereas the dates Nos. 7-13bis as a set are possibly determined by the Carlsberg Cycle. By contrast, Bennett (2008: 534) states "that the Carlsberg cycle was not used to regulate temple service". This creates a problem insofar as he indicates in his table 2 the constant difference of one day between the first service days of Nos. 7-9 and 11-13 and the corresponding first days of the Carlsberg Cycle in Depuydt's reconstruction. The difference of 1 day is tantamount to the coincidence of the first service days with the corresponding LDs 2 of the Carlsberg Cycle, implying that the cycle was used to regulate the temple service. Bennett does not comment on the coincidence of observational and cyclical dates.

Nos. 7 -13bis were originally utilized by Parker for the completion of cycle year 16 (*sic*) corresponding to year 26 of Ptolemy VIII. If these dates are assigned instead to year 40 of Ptolemy VIII or Carlsberg Cycle year 2, the same dates would be available for completion of cycle year 2. Note that Nos. 10 and 13bis do not match Depuydt's reconstruction of cycle year 2 or 16;

Nos. 7-13bis: The set does not constitute a series of observational LDs 2, but corresponds instead to a series of recorded or interpolated LDs 2 in the Carlsberg Cycle, provided the series was determined uniformly.

Nos. 14-15 (Medinet Habu Graffito 48; Thissen, 1989: 44-46; Chauveau, 1995: 253) – According to Thissen, the text refers to a service month (*wrš*) of phyle 1. The reading of regnal years, seasons, and day numbers cannot be established with certainty. Thissen proposed to read and to restore the date as II *ꜣḫt* (?) 1[5] to III [*ꜣḫt*] 14. According to Bennett (2008: 536), "the traces of the month name permit reading either of the other two seasons; the only certain day number is 14".

The identification of the rulers – a Ptolemy and a Cleopatra – remains open. With some reservations, Thissen made the following suggestion: "Year [21?] Ptolemy (VI?) and Cleopatra (II?) = Year 10 (Ptolemy VIII?)". However, secure attestations of the joint reigns of Ptolemy VI, Cleopatra II, and Ptolemy VIII end in year 7 of Ptolemy VIII (Bennett, 2008: 536 [citing Thissen]). For this reason, Bennett (2008: 536-538) considers "Year [1 ?] Ptolemy (XV ?) and Cleopatra (VII ?) Year 11 (*sic*)" as yet another possibility. He further argues that "year 11 is actually a partially erased year 1<2>, even though Edgerton failed to notice any erasure in his facsimile. This suggests two possibilities: year 15 of Cleopatra III = year 12 of Ptolemy X or year 9 Ptolemy X = year 12 of Cleopatra III. The surviving traces of the first number could be compatible with 9, although it would be an unusual form of the numeral, but not with 15. However, the double date year 9 = 12 is compatible with Ptolemy X being named first. More importantly, with this reading there is a reasonable lunar solution: year 9 = year 12 of Ptolemy X and Cleopatra III, II *šmw* 16 to III *šmw* 14."

If Thissen's reading be accepted, year 21 of Ptolemy VI or 161/160 BC corresponded to an extrapolated Carlsberg Cycle year 22 when a LD 2 fell on either of the interpolated dates III *ꜣḫt* 14 or III *ꜣḫt* 15; observationally III *ꜣḫt* 14 fell on a LD 2, last visibility was on December 12 with probability ~1. Like Thissen, I note that the result is surprising; nevertheless, the coincidence of III *ꜣḫt* 14 and a LD 2 cannot be taken as proof that the Egyptians dated documents in years 8 to 25 of Ptolemy VIII.

Nos. 14-15: In view of the contradictory diversity of the options, I have omitted both dates from chronological consideration.

Nos. 16-17–17bis (Medinet Habu Graffito No. 51; Thissen, 1989: 51-55; Chauveau, 1995: 253f.) – The text refers to the *wrš*-service of a priest in phyle 4; it was written in year 11 of Cleopatra VII, corresponding to 41/42 BC, on I prt 11, according to Thissen, but on I *šmw* 11, according to Chauveau. The dates of the *wrš*-service were read by Thissen (1989: 51-55) as *ibd 4 ꜣḫ.t sw 19 r tpi pr.t sw* 19. Chauveau (1995: 254) comments "*tpi pr.t sw 19 semble paléographiquement im-*

possible, le fac-similé imposant de lire tpỉ šmw sw 15 … pour le début, il faudrait pouvoir lire ỉbd-4 pr.t sw 17 (17 Pharmouthi), mais si pr.t au lieu de ꜣḫ.t est plausible, sw 17 est en revanche douteux du point de vue paléographique, mais je ne vois aucun autre moyen de concilier écriture et tables calendaires."

Bennett takes into account other possible readings for the day numbers and concludes that "the most likely dates for the temple service month are: Year 11 of Cleopatra VII, IV *prt* 15 to I *šmw* 15".

There is further information to be gained from MH graffito no. 52. According to Chauveau it dates from July to August in year 11 of Cleopatra VII (Chauveau, 1995: 254). Line 26 of the graffito informs us that it was written in year 11 of [Cleopatra] and her son Ptole[maios] in *ỉbd* 3… ? … (Thissen, 1989: 56). In line 1, Thissen reads the date as "*ḥꜣ.t-sp* [x *ỉbd* x *n …? … sw* x *r ỉbd* x *n … ? …*] *sw 14 wrš n sꜣ 2.nw* (*?*): *Regierungsjahr [x, von Tag x des Monats x] bis zum 14.* [……] *Wachdienst der 2. (?) Phyle*". As Chauveau realized, the day number 14 of line 1 and the month number 3 in line 26 imply that in year 11 of Cleopatra VII, corresponding to Carlsberg Cycle year 16, the lunar service month of phyle 2 (?) ended on [IV *šmw*]14. Provided that phyle 2 is indeed meant in line 1 and that the phyle rotation was regular, then the service month of phyle 4 that is documented in Nos. 16-17 lasted from IV prt to I *šmw*. In other words, Chauveau's reading *tpỉ šmw* (*sw* 15) in MH 51 is confirmed. Furthermore, he identified [IV *šmw*]14 as a last service day. I accept [IV *šmw*] 14 in 11 Cleopatra VII = –40 August 11 of MH graffito No. 52 as a last service day that coincided observationally (see table 31) and also in Carlsberg Cycle year 16 with a LD 1 and count it as No. 17bis.

As tables 31 & 32 indicate, neither IV prt 15(?) (Bennett) nor IV prt 17 (Chauveau) as suggested readings for No. 16 coincides with an observational LD 2. Bennett's date does not coincide with a LD 2 in Carlsberg Cycle year 16, but Chauveau's date does, since it was chosen to coincide with a cyclical LD 2.

Tpỉ šmw sw 15, the end date of No. 17, was observationally a LD 1; within year 16 of the Carlsberg Cycle the date can be interpolated as a LD 1 or a last LD. Since *tpỉ šmw sw* 15 was mentioned in advance when the graffito was written on I *šmw* 11, it will have been determined cyclically. Furthermore, since it is possible to interpolate *tpỉ šmw sw* 15 or 16 as LD 1, the cyclical coincidence of *tpỉ šmw sw* 15 with a LD 1 is a 50:50 possibility.

Nos. 16-17: The analysis of the dates remains inconclusive; therefore I omit both from chronological consideration.

No. 17bis: The reported last service day was cyclically and also observationally a LD 1. Since the date was mentioned in advance when the graffito was written, it will have been determined by the cycle.

No. 18 (oDem Zauzich 20; Kaplony-Heckel, 2004: 300f.) – The ostracon refers to a priest of phyle 1 leasing two months of temple service in Medamod, from IV *ꜣḫt* 1 to I *prt* 30 (Bennett, 2008: 538 mistakenly writes II *prt* 30) in year 35 of an unnamed pharaoh. The contract was written on IV *ꜣḫt* [1] as the first day of the respective monthly service, according to Ursula Kaplony-Heckel. Although she (2004: 300) admits it would be possible to assign the year 35 to Augustus, she favors 35 Ptolemy IX, corresponding to 83 BC, arguing "*die Personennamen [passen] besser in die späteste Ptolemäerzeit*". Bennett (2008: 538) realized that the dates IV *ꜣḫt* 1 to I *prt* 30 correspond to a lunar phyle service only in 35 [Augustus] = AD 5, not in 35 [Ptolemy IX (or II, VI or VIII)]. Table 33 presents the dates of the last visibilities before IV *ꜣḫt* 1 and I *prt* 30 in AD 5 and 6; the service dates are listed as 18a and 18b.

If IV *ꜣḫt* [1] was counted as a LD 2 when the contract was signed, then old crescent of November 19 = III *ꜣḫt* 30 had not been observed. Reporting or declaring old crescent on November 18 by mistake would have implied an acceptable 29 day lunar month. On the other hand, since the crescent reached a topocentrical altitude of 27° at sunrise on November 18, an observer should have expected old crescent on the following morning.

In year 12 of the Carlsberg Cycle, corresponding to 35 Augustus, a LD 2 did not fall on IV *ꜣḫt* 1, but rather on IV *ꜣḫt* 2. Thus neither a cyclical nor an observational LD 2 matches the initial date cited in the source.

There is another problem which results from I *prt* 30 as last date of the 2-month lease. In 35 Augustus, I *prt* 30 was observationally a LD 1 (see table 33), but this could not have been known in advance when the contract was signed. Furthermore, we do not know whether the concluding date was counted inclusively or exclusively. As an interpolated date in the Carlsberg Cycle, I *prt* 30 could have been a LD 1 or LD 2.

Since the initial date, IV *3ḫt* 1, is neither cyclically nor observationally correct, it makes no sense to acknowledge the concluding date, I *prt* 30, as determined by the Carlsberg Cycle or by observation. In other words, neither date seems to have been determined cyclically or observationally. For this reason, the alternative – that No. 18 concerns year 35 of Ptolemy IX – ought to be tested. IV *3ḫt* 1 as the first date in No. 18 corresponds to December 12 in 83 BC as 35 Ptolemy IX; the date coincides with a LD 21. The second date, I *prt* 30, corresponds to February 9 in 82 BC as 35 Ptolemy IX; this date also fell on a LD 21. LDs 1 occurred between these dates on IV *3ḫt* 11 = December 22 in –82 and on January 20 in –81.

In year 35 of Ptolemy IX, the two dates of No. 18 are distant from LDs 2 as the expected start days of a lunar phyle service. This circumstance prompted Bennett to assign the document to the time of Augustus. Otherwise he followed Kaplony-Heckel's interpretation that the dates IV *3ḫt* 1 to I *prt* 30 refer to a 2-month lease (Kaplony-Heckel, 2004: 301). This interpretation is problematical since it implies two consecutive service months of one and the same phyle in contradiction of the rotation principle.

No. 18: To resolve these difficulties, I suggest that the lease concerns only one service month of phyle 1 and that the dates which are cited in the text are not to be understood as lunar dates, but rather as starting and ending dates of the two civil months which include the lunar service month in question. The contract would not have been signed on the first day of the lease. If my interpretation be correct, then the text of No. 18 does not contain any specific lunar date at all.

No. 19 (oDem Zauzich 23; Kaplony-Heckel, 2004: 303f.) – The ostracon refers to a sub-lease agreement at the temple of Medamod covering 15 days within a main lease during the two civil months IV *prt* and I *šmw*. The contract is dated to IV *prt* 8 in year 39 of Augustus, corresponding to AD 10. The lessee was to pay part of the rent – in silver and oil – by IV prt 15; lessee and leaser agreed to divide the leaser's temple income for 15 days.

Kaplony-Heckel supposes that the contract was dated according to the Alexandrian calendar. Bennett (2008: 538f.) objects that in AD 10 IV *prt* 15 alex. = April 10 as date for the payment "is not near to either the new moon or the full moon". Furthermore, he argues that, by contrast, IV *prt* 15 civ. "was 2 April A.D. 10, one day after last crescent visibility. Thus, the lease was for the first half of a service month. Since IV *prt* 15 [civ.] was also the last day of the payments, it may be accepted as the start date of the lease: year 39 of Augustus, IV *prt* 15 [civ.] However, since the document is a lease agreed in advance, the service month may have started on IV *prt* 16 [civ.]."

By contrast to Bennett's arguments, it may be emphasized that, according to the text, the date IV *prt* 15 is the last day of certain payments in advance of the 15 days temple service; the text does not mention IV *prt* 15 as the first of the 15 service days. Since IV *prt* 16 cannot be deduced from the source, it can only be postulated that the 15 days service began on IV *prt* 16 [civ.] which was an observational LD 2 (see tables 34 & 35). If it is admissible to guess at IV *prt* 16 as the observational starting date, it may also be presumed that the 15 days service began on cyclical LD 2 = IV *prt* 17 [civ.] in Carlsberg cycle year 16, corresponding to AD 10 (table 35).

In my view, the dates of No. 19 fit more easily into the Alexandrian, rather than into the civil calendar. If the contract was signed on IV *prt* 8 alex. [= IV *prt* 16 civ.] = April 3 in AD 10, it seems just possible that the temple service and the main lease started on that same day, since it coincided observationally, though not cyclically, with an LD 2 (see table 35). Presumably Bennett overlooked the coincidence of an LD 2 with IV *prt* 8 alex. = IV *prt* 16 civ. in AD 10.

Alternatively, it also seems possible that the service was determined cyclically and that the contract was signed one day before the start of the service, since cyclical LD 2 fell on IV *prt* 9 alex. = [IV *prt* 17 civ.] (see table 35). The 15 days of the sub-lease may have started on the same day as the main lease. The payments in advance would have been due on IV *prt* 15 alex. [=IV *prt* 23 civ.], seven days after the signing of the contract; the date IV *prt* 15 alex. would then not refer to any point within the temple service;

No. 19: Apparently the text is susceptible to different interpretations. I prefer the alternative which relates No. 19 to the Alexandrian calendar when the chances of observationally or cyclically determined dates are 50:50.

No. 20 (oThebes D235 = oDem Zauzich 25; Kaplony-Heckel, 2004: 306) – Herbert Thompson (1913: 55f., Pl. X) understood the text to record the lease of temple service in three different Theban temples, and, by implication, to cover a period of eight months, beginning on I *prt* 4 alex. in [year 2 of Vespasian] and ending on Thoth [1] of the following year, corresponding to the interval between December 30 in AD 69 and August 29 in AD 70. As table 36 shows, the date I *prt* 4 alex. = I *prt* 27 civ. = December 30 in year 2 of Vespasian or AD 69, corresponded observationally either to the day before old crescent day or to old crescent day itself. On December 31 the chances for seeing or not seeing the crescent were more or less the same.

Within Carlsberg cycle year 1, the date I *prt* 27 civ. was a LD 28 in a 29 or 30 day lunar month, depending on the interpolated LDs 1 for I *prt*. As Bennett (2008: 539) noted, the observational or cyclical correspondences of I *prt* 4 alex. would result in "the nominal start of service two or three days early", relative to LD 2 as the expected initial day. If so, No. 20 would be the only example of a temple service beginning on one of the last days of a lunar month. Perhaps the leases included the last two or three days of a preceding service month, notwithstanding that Kaplony-Heckel (2004: 288, 293) cites only 15 day leases covering less than a month.

Finally, it may be noted that the interval between December 30 in AD 69 and August 29 in AD 70 amounts to 242 days, *i.e.* more than 8 civil months and 5 to 6 days more than 8 lunar months. Observationally, the last day (invisibility) of the eighth lunar month after I *prt* 4 alex. in AD 69 would have fallen on August 23 or IV *šmw* 30 alex. = I A 18 civ. in AD 70. Within the Carlsberg Cycle, the corresponding last day of a lunar month would have fallen on Epagomene 1 alex. or Epagomene 2 alex., corresponding to I *3ḫt* 19 civ. or I *3ḫt* 20 civ. in AD 70. Since a certain disregard of the Epagomena is known (Leitz, 1989: 5f.), it is feasible that the text cites Thot [1] instead of an epagomenal day.

No. 20: The ostracon was known to Parker who did not use it. Bennett presumes that Parker judged it a "mismatch" (Bennett, 2008: 539 n. 53). In view of the uncertainties I follow Parker in omitting the text from chronological consideration.

Nos. 21 - 22 (Graffito MH 228; Thissen, 1989: 134-138) – The graffito mentions temple service (*wrš*) in a year 10 between I *šmw* 16 and II *šmw* 16. No assignment to any regnal year 10 seems to work. Bennett considered various possibilities, though he did not include the results in his final analysis; I also omit both dates.

No. 23 (oDem Zauzich 28; Kaplony-Heckel, 2004: 309-311) – The Theban ostracon preserves a lease and exchange contract for two service months, one spanning the civil months I and II *3ḫt*, and the other II and III *3ḫt*. The contract was signed in advance on I *3ḫt* 8 alex. = September 6 in AD 147 or 11 Antoninus.

Bennett (2008: 539) argues: "the body of the text notes that one portion of the 60 days starts on II *3ḫt* 17 [alex.] ... Hence we have a temple service date of: Year 11 of Antoninus, II *3ḫt* 17 [alex.]". In his table 2, he mistakenly equates II *3ḫt* 17 alex. and III *3ḫt* 29 civ. instead of III *3ḫt* 30 civ; therefore, he cites a difference of three, instead of four, days between II *3ḫt* 17 alex. and "last crescent visibility on 11 October A.D. 147 [= II *3ḫt* 13 alex.]".

As tables 37 & 38 imply, II *3ḫt* 17 alex. = III *3ḫt* 30 civ. = October 15 in AD 147 coincided observationally with a LD 4. A discrepancy of +1 or +2 days also results if the dates are interpreted as cyclical. Thus II *3ḫt* 17 alex. is not compatible with the expected beginning of a lunar phyle service on a LD 2.

Bennett (2008: 549f) suggests that "the service month may have started on II *3ḫt* 18 [alex.] ... since the document is a lease agreed in advance". (In his article, a remark follows about IV *3ḫt* 16 as the ending date of the lease. There is some mistake, since the source does not mention any final date of the lease or the specific date IV *3ḫt* 16). Bennett's suggestion would increase the differences between observationally and cyclically determined LDs 2 on the one hand, and the presumed beginning of the service month on the other.

Is it possible that the service month began earlier than expected? The contract is dated I *3ḫt* 8 alex., corresponding to a LD 25. On that day, *i.e.* about 4 or 5 days before the end of the current lunar month, no one could know whether the month would have 29 or 30 days; the same is true for the following lunar month, and therefore the beginning of a lunar month in II *3ḫt* alex. was uncertain by about 2 days. Even if the lunar month that was running on I *3ḫt* 8 alex. and the following month also were presumed to be 30 day months when the contract was signed, the date II *3ḫt* 17 alex. would nevertheless have fallen on a LD 4.

The question whether the service month began earlier than the parties to the contract expected, is irrelevant for the issue at hand, viz. the relationship of II *3ḫt* 17 alex. to the next LD 2, be it observationally or cyclically determined. The difference of +2 days exceeds the ± 1 day difference that is admissible in lunar observation; in a properly used cycle there is no leeway at all.

Bennett's interpretation "that one portion of the 60 days starts on II *3ḫt* 17" is not supported by Kaplony-Heckel who describes the "*Wendung ... dir gehört dein Anteil (p3j=k wn) aus den 60 Tagen am 17 Paophi ...*" as "*mir unklar*". The text as it stands does not indicate that a portion of the 60 days starts on II *3ḫt* 17. In view of this uncertainty and the other difficulties cited, I conclude that the parties to the contract did not expect the beginning of a service month on II *3ḫt* 17 alex.

No. 23: Since uncertainties about the expected start of the phyle month are present in the text, I omit it from further consideration;

No. 24 (oTHebes D 31 = oDem Zauzich 31; Thompson, 1913: 51f; Kaplony-Heckel, 2004: 313-314) – The text refers to a lease of a month of temple service at Thebes that was agreed in 30 Commodus = AD 190. Thompson read the year as '12'; for the correct reading '30', Parker was obliged to Hughes (Parker, 1950: 67); for the date "30 Commodus", see also Kaplony-Heckel (2004: 313 n. 101). The contract was signed on IV *prt* 15 alex. [= II *šmw* 8 civ.], 13 days before the start of the service.

The phyle service was to last *n jbd 4 prt sw 28 r tpj šmw sw 27.* Parker realized that the dates are to be understood in terms of the Alexandrian Calendar, if the service month should be lunar. Thus the service was to last "from IV *prt* 28 alex. [= II *šmw* 21 civ. = April 23] to I *šmw* 27 alex. [= III *šmw* 20 = May 22]". Parker computed II *šmw* 19 civ. = April 21 in AD 190 as old crescent day and thus the first service day II *šmw* 21 civ. as LD 2. As table 39 indicates, it is not impossible, but improbable that old crescent was sighted on that day. Therefore, April 20 is to be preferred as old crescent day, resulting in a LD 2 on II *šmw* 20, one day earlier than the first service day that is cited in the text; Bennett (2008: Table 2) also prefers April 20 as old crescent day. As table 40 shows, the first service day coincided in the Carlsberg Cycle with a LD 1, not with a LD 2. Relying on his postulate that LD 1 was the first day of lunar temple service months, Parker accepted the cyclical coincidence.

Regardless of the results of astronomical computation, the fact remains that the contract was signed 13 days before the beginning of the service month. At that time it could not be known whether lunar observation would result in IV *prt* 28 alex. being a LD 2. There is a similar problem with the concluding date I *šmw* 27 alex. = III *šmw* 20 civ. of the contract. If the starting date is counted as a LD 2, then the concluding date would be a LD 1 or LD 2, depending on inclusive or exclusive counting of the last service day. In Carlsberg Cycle year 21, a LD 1 fell on III *šmw* 20 or 21 civ., and therefore the concluding date could have been a cyclically determined LD 1. By contrast, opening and concluding dates could have been observationally correct only by guessing.

There is an alternative. Thompson (1913: 51f.) noted that the service dates "*n jbd 4 prt sw* 28 *r tpj šmw sw* 27 " are written above the line in the original" (*cf.* fig. 27, second line from above). This suggests the possibility that the service dates were added after the contract was written and when the service had started, *i.e.* on *tpj šmw sw* 27 alex. at the earliest.

No. 24: If the service dates were added to the contract on *tpj šmw sw* 27 alex. at the earliest, then at least the first service day can be accepted as observationally correct, implying that in AD 190 on April 21 = IV *prt* 28 alex. = II *šmw* 19 civ. old crescent was observed under unfavorable circumstances. The opening date was not cyclically determined; the concluding date may have been.

No. 25 (oDem Zauzich 32; Kaplony-Heckel, 2004: 314-316) – According to Kaplony-Heckel, this contract from Thebes refers to "*Tempeldienst an verschiedenen Festen, und zwar am Tag 8 im ersten Überschwemmungsmonat, sodann am Tag 14 und am Tag 19 im vierten Überschwemmungsmonat*" in year 8 of [Severus and Caracalla] corresponding to AD 199. Bennett (2008: 540) interprets the text as a lease of a month of temple service which had been agreed in advance; he considers I *3ḫt* 8 the start of the service month. Kaplony-Heckel leaves it open whether No. 25 is dated in the Alexandrian or the civil calendar; Bennett decides in favor of the civil calendar, since otherwise there is no lunar match. Since month and day are omitted in the dating of the contract and only year 8 is given, it is possible that it was signed on one of the days I *3ḫt* 1–7 or perhaps as late as day 8 itself.

It appears that old crescent could have occurred either on I *3ḫt* 5 = July 9 in AD 199, or less probably, on I *3ḫt* 6 = July 10 (see table 41); thus I *3ḫt* 8 = July 12 in AD 199 would have coincided observationally with an LD 3, or less probably, with an LD 2. Bennett (2008: Table 2) prefers old crescent occurring on I *3ḫt* 6 = July 10 in AD 199 which results in I *3ḫt* 8 as an observationally determined LD 2. I concur, because I *3ḫt* 5 = July 9 was a LD 29 if the preceding LD 1 was correctly determined.

If the contract was signed before (*sic*) I *3ḫt* 6, then the parties could not have known what correspondence observation would yield between a lunar day and I *3ḫt* 8. The contract could have been signed on I *3ḫt* 6 on a LD 30 recognized as such by observation, provided the preceding LD 1 was correctly determined; under these circumstances it could have been clear that I *3ḫt* 8 would fall on LD 2. As table 42 shows, the Carlsberg Cycle allowed I *3ḫt* 8 or I *3ḫt* 9 as possibilities for interpolated LDs 2.

No. 25 summarizing: I *3ḫt* 8 as presumed LD 2 could have been determined either by observation or by the interpolated Carlsberg Cycle with the chances for LD 2 about 50:50.

Ptolemaic and Roman Lunar Dates Nos. 26-32: 'Inferred Direct Civil/Temple Service Synchronisms from Dime'

Studying grain receipts based on an offering of 1 artaba per day, Lippert and Schentuleit deduced about sixteen first and last service days between 6 [Augustus] and 10 Domitian (Lippert & Schentuleit, 2006: 181-83; Lippert, 2009: 183-194). Bennett accepts six cases of first service days (or LDs 2) and one case of a last service day (a LD 1). I follow Bennett's evaluation and numbering of the seven dates. There is nothing to be added to his assignment of the Egyptian dates to regnal years and their equivalents in the Julian calendar. I present the positions of the seven old crescents within the azimuth-altitude diagram (figure 28) and list their DAZ-h values in table 43. There is only one case close to the seasonal zone of uncertainty, namely the March crescent No. 27 which stood at sunrise of –23/3/7 just on the lower visibility border of the cool season. Thus the old crescent of No. 27 was probably observed on –23/3/6 far above the seasonal uncertainty zone resulting in an LD 2 as the reported first service day.

As table 43 shows, dates Nos. 26-32 have been determined observationally or cyclically: 3 cases by observation, 2 cases by observation or by the cycle, 1 or 2 cases by the cycle. Apparently expecting that all dates from Dime ought to be determined in the same way, Lippert (2009: 188) concluded that "observation was not the method used by the priests of Soknopaiou Nesos". But it can be expected that in one and the same temple and over a period of 110 years the service times were determined in different ways: by careful or sloppy observation of the moon; by correct or sloppy use of a cycle; in irregular fashion when occasion arose.

No. 32 presents a problem. The date was deduced as follows: a receipt for 30 artabas of wheat for offerings by phyle 4 documents an apparent 30 day lunar service month beginning on [I *šmw*] ⌜12⌝ and ending on II *šmw* 1[1] in 10 Domitian = AD 91, corresponding to Carlsberg Cycle year 22; the receipt is dated to II *šmw* 11. A later receipt for offerings documents the first five service days of phyle [5] from III *šmw* 12 to III *šmw* 16, immediately following the service of No. 32 (Lippert & Schentuleit, 2006: (no.) 49, B & C).

Observationally, the starting date [I *šmw*] ⌜12⌝ of No. 32 coincided with a LD 3. According either to Parker's or Depuydt's reconstruction, the starting date coincided cyclically with a LD 3, since both agree on I *šmw* 10 as cyclical LD 1. Regardless, since the recorded Carlsberg Cycle presents IV *prt* 11 and I *šmw* 10 as LDs 1 in cycle year 22, I *šmw* 10 or 11 might be interpolated as a LD 1. Thus there seems to be a 50:50 chance that [I *šmw*] ⌜12⌝ as the starting date of No. 32 was determined cyclically as a LD 2 or LD 3. On the other hand, the concluding date II Smw 11 fell on a LD 2, be it observationally or cyclically determined. These results do not conform to the expectation that lunar service months lasted from LD 2 to LD 1.

No. 32: The starting date corresponded cyclically to an interpolated LD 2 or LD 3 at 50:50

odds; observationally, the starting date corresponded to a LD 3. The concluding date of No. 32 was a 30th service day which, both cyclically and observationally, coincided with an LD 2, not with the expected LD 1.

Saite, Ptolemaic, and Roman Lunar Dates Nos. 33-40: 'Fixed Egyptian Lunar Synchronisms'

No. 33 (pLouvre 7848) – In 1956-57, Michel Malinine and Parker used photographs to read together through the Louvre collection of papyri in abnormal hieratic (Parker, 1957: 210; Depuydt, 1997: 267). In pLouvre 7848, they found the date "of an oath to be taken *m-bꜣḥ Ḫnsw-m-Wꜣst-nfr-ḥtp ḥꜣt-sp 12t ꜤII šmw13 n smdt* [hieroglyphic transcription of *smdt*] I *šmw13*, before Khonsu ... in year 12, II *šmw13*, being the 15th lunar day of (lunar) I *šmw*" (Parker, 1957: 210f.).

The document is dated to year 12 of Amasis. Parker used this double date to answer the question whether Amasis ruled for 43 or 44 years – or in terms of absolute chronology, whether 12 Amasis corresponded to 559 or 558 BC. II *šmw* 13, the civil part of the double date, coincided with October 19 both in 559 and in 558 BC. Since a lunar day 15 or calendaric full moon as the lunar part of the double date came close to October 19 or coincided with it only in 559 BC, Parker could conclude that 12 Amasis corresponded to 559, not to 558 BC.

Depuydt interprets the inclusion of (lunar) month I *šmw* in the double date as evidence for Parker's civil based lunar calendar (Depuydt, 1997: 162ff.); Juán A. Belmonte (2003: 14-16), who rejects in general the existence of the civil-based lunar calendar, argues against Depuydt's interpretation. He points out that IV prt of 12 Amasis would have been a 'blue month', possibly resulting in an uncertain correlation between lunar months and lunar days. According to this interpretation, the specification "*smdt* Ꜥ*I šmw*" is intended to dispel any doubt about the lunar month. The discussion cannot be considered to be concluded, since the lunar-civil double date contained in No. 42 below constitutes evidence for the civil-based lunar calendar.

Only recently did Koenraad Donker van Heel (1996: 93-99) publish the Amasis double date in its context; I thank him for information on the writing of the lunar day confirming Parker's transcription. The text pertains to the settlement of a disagreement about the ownership of a tomb. The two parties drew up a document on I *šmw* 21 in 12 Amasis stating that the conflict should be settled by an oath before Khonsu "in year 12, 2nd month of the *šmw* season, (day) 13, on the fifteenth day (festival) of the 1st month of the I *šhemu šmw* season" (Donker van Heel: 1996: 94). I wonder whether *n* 15*.t jbd*-1 *šmw* could be understood in apposition to the preceding civil date, with *n* indicating the apposition (Erichsen, 1954: 201); this seems to have been Parker's interpretation.

The document is dated to I *šmw* 21 = September 27 in 559 BC; the oath was to be taken 22 days later on II *šmw* 13 = October 19 in 559 BC, being a calendric full moon day. The oath is to be understood as a so-called temple oath, since it was to be taken before Khonsu (*m-bꜣḥ Ḫnsw-m-Wꜣst-nfr-ḥtp*), *i.e.* in the Khonsu Temple. Apparently there are no other cases of temple oaths dated to a lunar day (Kaplony-Heckel, 1963: passim).

The double date implies that a LD 1 possibly coincided with I *šmw* 29 = October 5 in 559 BC. As table 44 shows, visibility or invisibility of old crescent was possible on October 5 in −558, resulting in II *šmw* 12 or 13 as observational LD 15.

When the agreement was drawn up, it could not have been known whether the current lunar month would last 29 or 30 days; therefore the projected correspondence of II *šhemu* 13 civ. = lunar day 15 was uncertain by a day. The projection of a civil-lunar double date may indicate the use of some rule for predicting lunar days; at least until now, there is no evidence for a systematic prediction of lunar days in the Saite Period. The possibility of a cyclical fixing of the Amasis date was briefly discussed by Winfried Barta (1979: 4, 10) and Depuydt (1997: 162); they agree that the Carlsberg Cycle is not to be expected at such an early date.

No. 33: This is not taken into consideration below, since it is chronologically isolated from Nos. 1-32 and 34-42.

Nos. 34-37 are cited in the well-known building inscriptions in Edfu (Chassinat, 1929; 1932 [= Edfou IV & VII]). According to No. 34 (Edfou IV.14 & VII.5: Cauville & Devauchelle, 1984: 32; Depuydt, 1997: 122 ff. – Edfou VII.5: Kurth, 1994: 70; Kurth *et al.*, 2004: 6), the temple was founded in [month] *Jpt-ḥmt.s* [Epiphi = III *šmw*] 7 = *snwt* [LD 6] in 10 Ptolemy III, corresponding to August 23 in −236.

For the writing of *Jpt-ḥmt.s* = Epiphi, see Parker (1950: 21f.) and Spalinger (1990: 77). Edfou VII.5 mentions *snwt*; Edfou IV.14 cites only the civil date. The text indicates that LD 6 or *snwt* is an auspicious day for foundations; see also Gutbub (1973: 389f.).

As table 45 shows, old crescent might have been observable or unobservable on III *šmw* 1 = August 17 in –236, resulting in III *šmw* 7 as LD 6 or 7 for the foundation ceremony. Bennett (2008: Table 4) prefers August 17 as old crescent day with August 16 as his second choice. Parker, presumably using Schoch's age-of-the-moon method, computed August 17 = III *šmw* 1 as old crescent day (Parker, 1950: 102-104). Both dates are possible, insofar one day was a LD 29, and the other a LD 30, if the preceding LD 1 was correctly determined.

The foundation of Edfu Temple occurred in year 20 of the extrapolated Carlsberg Cycle; in that cycle year the days I *šmw* 2 or 3 can be interpolated as LDs 1. Thus the foundation date *Jpt-ḥmt.s* [III *šmw*] 7 was either a cyclical LD 6 or 5 (see table 46). In accordance with his astronomical computation and following the lead of the text, Parker reconstructed the missing LD 1 of I *šmw* in Carlsberg Cycle year 20 as III *šmw* 2; in Depuydt's reconstruction, LD 1 falls on III *šmw* 1.

No. 34: If the foundation date was determined by observation, then old crescent was seen on III *šmw* 1 = August 17. If the date was determined cyclically, then in that cycle, III *šmw* 2 was a LD 1. Since both the observational and the cyclical dates are ambiguous, the manner in which LD 6 was fixed for the foundation ceremony at Edfu remains open.

No. 35 (Edfou IV.7 and VII.6; Cauville & Devauchelle, 1984: 33 f.; Edfou VII.6: Kurth, 1994: 70; Kurth *et al.,* 2004: 7) refers to the end of the first great building period in Edfu in year 10 of Ptolemy IV on III *šmw* 7 = August 17 in –211. According to the text, III *šmw* 7 coincided with a LD 6. Parker (1950: 99-101) noted that the civil date of No. 35 "is exactly 25 years later than [No. 34]". He concluded that III *šmw* 7 = LD 6 was determined by using the Carlsberg Cycle. Knowledge of the complete cycle would not have been necessary in this case; knowing that a lunar double date tends to repeat after 25 Egyptian years would have sufficed to project III *šmw* 7 = LD 6 of No. 34 onto III *šmw* 7 = LD 6 of No. 35. It may be noted that in year 20 of the reported Carlsberg Cycle, interpolation yields III *šmw* 1 or 2 as LD 1, as implied in table 48. Parker also noticed that III *šmw* 7 in year 10 of Ptolemy IV was observationally a LD 7. By contrast to No. 34, there is in the case of No. 35 no doubt about the respective old crescent day. As table 45 indicates, the moon was invisible on August 11 in –211, and old crescent had occurred the day before. Bennett also computes August 10 as old crescent day. Thus observationally the date of No. 35 coincided with a LD 7, not with a LD 6 as the text states.

No. 35: The reported LD 6 is observationally incorrect; the reported LD 6 coincides with one of two dates which can be interpolated in the Carlsberg Cycle.

No. 36 (Edfou VII.7 + IV.2; Cauville & Devauchelle, 1984: 37f.; Edfou VII.7: Kurth, 1994: 71; Kurth *et al.*, 2004: 9) refers to the completion and dedication of Edfu Temple on IV *šmw* 18 in year 28 of Ptolemy VIII = September 10 in –141. Two additional inscriptions (Parker, 1950: 214-217) document that 95 Egyptian years had elapsed between the temple's foundation (No. 34) on *Jpt-ḥmt.s* [III *šmw*] 7 = LD 6 in 10 Ptolemy III [August 23, –236] and its dedication (No. 36) on IV *šmw* 18 = *dnjt snnw* [= LD 23] of III *šmw* in 28 Ptolemy VIII [September 10 in – 141].

As table 47 shows, old crescent might have been observed on LD 23 of the lunar month that began in III *šmw* or, less probably, on LD 24 of the same lunar month, but under no circumstances in –141 on August 18 = LD 25 of the respective lunar month. Therefore, observationally the dedication date would have coincided with a LD 25 or, less probably, LD 24, not with a LD 23 as stated in the text. Parker (1950: 86-88), presumably using Schoch's age-of-the-moon method, computed August 17 in –141 as old crescent day which results in a LD 24 for the dedication day. Bennett computes August 16 and 17 as possible old crescent days and decides for August 16 as old crescent day which results in a LD 25 for the dedication day. In other words, the dedication date was not determined by observation.

The dedication text implies that the reported LD 23 was counted from III *šmw* 26 as LD 1 in 142 BC. Parker (1950: 19) noticed that this LD 1 "fits nicely between II *šmw* 26 and IV *šmw* 25 of [Carlsberg] cycle year 15 in a cycle beginning 157 BC". Therefore he reconstructed III *šmw* 26 as LD 1 in Carlsberg Cycle year 15, whereas

Depuydt reconstructs III *šmw* 25. Parker could have been more precise by saying that III *šmw* 25 or 26 can be interpolated between the recorded LDs 1 in year 15 of the Carlsberg Cycle. Since the dedication date coincided cyclically with either a LD 23 or 24 (see table 50) and the text explicitly identifies the dedication date as a LD 23, Parker's approach was rational.

No. 36: The Edfu dedication date LD 23 was observationally not correct; the chance of coincidence with a cyclical date was 50:50.

No. 37 (Edfou VII.8 + III.86; foundation of the Edfu Pronaos on II Smw 9 = LD 6 in *ḥ3b Jnt* [Payni], year 30 of Ptolemy VIII = July 2 in –139.

Parker (1950: 19) interpreted *ḥ3b Jnt* as a rendering of the month name Payni; he is followed by Spalinger (1990: 78), Depuydt (1997: 183), Kurth (1994: 71), and Kurth *et al.* (2004: 10), whereas Cauville & Devauchelle (1984: 39) and Belmonte (2003: 15) understand "festival of the Valley". Parker's interpretation of *ḥ3b Jnt* is bolstered if not outright proven by parallel writings of *ḥ3b Jpt* for the month name of Paophi adduced by Wolfgang Waitkus (1993: 105ff.) and Spalinger (1990: 73ff.) for which see also below No. 41. Furthermore, there are apparently no indications that the Theban "festival of the Valley" was observed in Edfu (W. Waitkus, personal communication); neither is a *snwt* or sixth day of the Theban festival attested.

As table 51 implies, LD 6 fell observationally on the reported II *šmw* 9. Parker (1950: 83-85) acknowledged that observational LD 6 and cyclical LD 6 in Carlsberg Cycle year 17 coincided (see table 52).

No. 37: Whether the LD 6 of the foundation of the Pronaos was determined by observation or by use of the Carlsberg Cycle remains open; the chances were 50:50.

Nos 33 – 37: Table 53 shows that the observational LD would not have applied in two cases of the reported LDs; the cyclical LD would have applied in all four reported cases. If the same method was used throughout, then the reported lunar days were determined by the Carlsberg Cycle (or another equivalent cycle) in all four cases.

No. 38 (funerary stela of Ta-Imhotep from Memphis; BM EA 147): The text appears to provide a civil-lunar double date referring to the day the son of Ta-Imhotep was born, in year 6 of Cleopatra VII, corresponding to 46 BC.

Günther Vittmann (1984: 959f) recently translated the passage as follows: "*... Jahr 6, 3. Monat der šmw-Jahreszeit, Tag 15 (in der 8. Stunde des Tages), es war (pw) das jẖt-ḥr-ẖ3wj-Fest dieses herrlichen Gottes Imhotep, des Sohnes des Ptah.*" He comments: "*jẖt-ḥr-ẖ3wj ist der 5. Tag des Mondmonats, und m.W. ist es nicht sicher, dass es ein ausserhalb des Mondkalenders stehendes Fest dieses Namens – abgesehen von dem hier gewiss nicht in Frage kommenden letopolitanischen – gegeben hat.*" Note that Miriam Lichtheim (1980: 62) translated *jẖt ḥr ẖ3wt* as "Offering-feast", whereas Eve Reymond (1981: 176) translated: "Offerings were (placed) on the altar".

Heinrich Brugsch was the first to notice that *jẖt-ḥr-ẖ3wj* could mean LD 5 (Brugsch, 1891: 924); then Borchardt (1935: 40) interpreted the civil-lunar double date as "*deren* [= Ta-Imhotep's] *Geburtstag*" rather than that of her son. Brugsch read the civil date correctly as III *šmw* 15, whereas Borchardt cited the date as III *šmw* 13, following Gauthier's "*Livre des Rois*" (Gauthier, 1916: 411) without mentioning Brugsch's reading. Borchardt computed conjunction as occurring in 6 Cleopatra VII on 46 BC, July 8, 4h 53m local time Memphis. He concluded, "*der fragliche Geburtstag war also kalendarisch sicher ein 5. Mondmonatstag*".

Parker (1950: 80-82), Depuydt (1998: 1292), and Bennett (2008: 551f) more or less reproduced Borchardt's result on the basis of the wrong civil date, citing their respective predecessors as authorities.

Vittmann (1984: 960) noticed Parker's mistake, commenting, "*Die Differenz von zwei Tagen, die sich bei Verwendung des richtigen Datums ergibt, stellt (hoffentlich) kein grundsätzliches Hindernis für die Beobachtung dar, dass wir es mit einem Monddatum zu tun haben*". He presumes that the mistake in the civil date was inspired by the date which Petubastis III himself, the son of Ta-Imhotep, indicates as his birth day. The text of Petubastis's stela BM 188 refers to his birthday as III *šmw* 13, at least according to Gauthier (1916: 412). By contrast, Reymond (1981: 220) translated: "the 13th of Pamenhotep [= III *prt* 13], the festival day of Bastet"; Brugsch (1891: 929) also read III *prt* 13. Heinz Felber informs me that the date in the Demotic text of BM 188, as copied by Reymond and Brugsch, is to be read without a doubt III prt 13. Harimuthis, the brother of Ta-Imhotep and author of her stela (Reymond, 1981: 166), apparently cited the wrong date for the birth of his sister's son, whereas Gauthier introduced yet more mistakes.

III *šmw* 15, the date given in Ta-Imhotep's stela, corresponds in 46 BC to a cyclical LD 7 or an observational LD 8; III *prt* 13, the date given in Petubastis III's stela, corresponds in 46 BC to a cyclical LD 3 and to an observational LD 4. Thus no LD 5 is involved.

No. 38: Does not refer to a civil-lunar double date.

No. 39 (Bucheum stela 13; Mond & Myers, 1934: 32, Pl. XLIIIA, XLIII; Borchardt, 1935: 40) – The double date IV prt 21 = LD 16 (*mspr* 2) in year 1 of Augustus, corresponding to –28 April 22, refers to the day when a Buchis bull died. The double date implies that a LD 1 would have coincided with IV *prt* 6 = April 2 in –28. Parker (1950: 72-76) computed IV *prt* 6 as observational LD 1 which coincided with LD 1 in Carlsberg Cycle year 3; I concur (see table 54). Thus the Buchis bull would have died on a day when observational and cyclical LD 16 coincided (see table 55).

No. 39: The chances are even that the reported LD 16 was determined by observation or by using the Carlsberg Cycle.

No. 40 (Funerary papyrus pDem Rhind I; Möller, 1913: 14) – This papyrus from Thebes contains a double date which was the day when the person for whose benefit the papyrus was written (a certain *Mntw-<m>z3.f*) died. The double date consists of a LD 16 (*ḥbs-tp*) coinciding with III *šmw* 10 alex. (= III *šmw* 14 civ.) = July 4 in 21 Augustus. In his Calendars Parker (1950: 73-75) noted a slight uncertainty about *ḥbs-tp* as the name of LD 16; later he was able to resolve this problem (Parker, 1953: 50).

As table 56 implies, the observational LD 1 from which LD 16 = III *šmw* 14 civ. was counted, coincided with II *šmw* 29 civ. = June 19 in –8. As Parker realized, the date III *šmw* 14 civ. is also a LD 16 in Carlsberg Cycle year 23 (see table 57, and also Parker, 1950: 72-76; Borchardt, 1935: 40).

No. 40: The chances are even that the reported LD 16 was determined by observation or by using the Carlsberg Cycle.

No. 41 (Foundation of the Hathor Temple in Dendera; Amer & Morardet, 1983: 255-258) – The ceremony took place in year 27 on day 14 *n ḥ3b-jpt m rk ḥm nsw-bjt* (Ptolemy XIII Auletes) *snwt pw nt jbd pn*. Amer and Morardet understood *ḥ3b-jpt* as a writing of Epiphi, whereas Spalinger, citing Waitkus (1993, 105ff.), could show that Paophi (II Akhet) is meant and that the foundation date corresponds to October 19 in –54 (Spalinger, 1990: 73 ff.).

According to tables 58 & 59, observational LD 1 fell on October 12 in –54, resulting in LD 7 = Paophi 14 (= II *3ḫt* 14) civ.; whereas cyclical LD 6 fell on Paophi 14 (= II *3ḫt* 14) civ.

Furthermore, Spalinger realized that the *snwt* or LD 6 of the foundation ceremony coincided with LD 6 in year 3 of the reported Carlsberg Cycle. He cites Ronald A. Wells who computed October 12 as day of last visibility (Spalinger, 1990: 79); I concur in principle.

No. 41: The reported LD 6 was a cyclically correct date; the date was observationally not correct.

No. 42 (oAshmolean; Parker and Neugebauer, 1968: 231-234; Bohleke, 1996: 20f.) – The ostracon is written in Demotic; it contains the Greek word *σελήνης* in Demotic transcription. The text lists solar, lunar, and planetary positions pertaining to the time of birth of an unnamed individual. The implied birthday is dated in lines 1-2 to *ḥ3t-sp ... n t3 Pr-ʿ3t* IV *prt*, *sw* 22 (*nt n ?/ nty m?*) *hrw slns*: regnal year ... of the Queen, IIII Peret, day 22, day of the moon (*σελήνης*). In line 5, there is a second date: *ḥ3t-sp* 14 I <*šmw*> 4 ...: regnal year 14, I <Shemu> 4. As Neugebauer and Parker realized, the lunar and civil dates fit only in year 14 of Cleopatra VII. The dating is confirmed by the solar, lunar, and planetary positions listed in the horoscope. The two dates cited in the text can be expressed as a civil-lunar double date: regnal year 14, I <*šmw*> 4 = LD 22 of lunar month IV *prt*, corresponding to –37 May 3. It follows that the LD 1 from which LD 22 was counted fell on –37 April 12 = IV *prt* 13 civ. = lunar month IV *prt* day 1 in year 19 of the Carlsberg Cycle (Neugebauer & Parker, 1968: 233). The designation of the respective lunar month as IV prt can be understood as the monthly pairing of lunar and civil month, according to Depuydt's nomenclature (Depuydt, 1997: 190f.). Lunar month IV *prt* would have been the eighth lunar month since the preceding civil New Year's Day with no 'blue month' occurring in the course of the year (see above No. 33). Tables 60 & 61 show that the lunar date could have been determined by the Carlsberg Cycle or by observation from any location in Cleopatra's Egypt.

No. 42: The chances are even that the reported LD 22 was determined by observation or by using the Carlsberg Cycle.

Analysis of Ptolemaic and Roman Lunar Dates

The reported first and last lunar service dates and other lunar double dates considered above have been computed both as observational and as Carlsberg cyclical dates.

In four cases (Nos. 37, 39, 40 and 42) the Julian calendar day of a reported civil-lunar double date is the same by observational and cyclical determination; in three cases (Nos. 35, 36 and 41) observational and cyclical dates differ.

If in the cases of Nos. 37, 39, 40 and 42 the days are counted back from the respective LDs of the double dates, then it follows that the Egyptian lunar month began on the Julian calendar day d+1 that followed old crescent day d.

By contrast, if in the cases of Nos. 35, 36 and 41 the days are counted back, then it follows that the lunar month began cyclically on the Julian calendar day d+1 that followed old crescent day d, whereas observationally the lunar month would have begun on old crescent day d. It might be conjectured that the lunar month was begun by mistake a day early. But this solution is not viable, since Nos. 35, 36 and 41are August and October dates when no clouds are to be expected in Upper Egyptian Edfu (Nos. 35 and 36) and Dendera (No. 41). Note that these arguments presuppose that the dates are fixed in absolute chronology; otherwise the reasoning would be circular.

Under these circumstances, I conclude that Egyptian LD 1 corresponded to Julian calendar day d+1. This allows the identification of the first day of the temple service (*ꜣbd, wrš*) as Egyptian LD 2 or Julian calendar day d+2. Furthermore, the correspondence of LD 1 and Julian calendar day d+1 confirms that Egyptian LD 1 was not determined as the day of conjunction, at least not in the case of no. 5 when conjunction fell on d+2.

Following this line of argument, a correct first service day is understood below as an observationally or cyclically correct LD 2; in the case of a lunar double date, a correct date is understood as a date which coincided observationally or cyclically with a reported LD.

I have omitted Bennett's Nos. 14-15, 16-17, 18, 20, 21-23, 33 and 38 from consideration for the reasons cited; I have added two bis-numbers and Nos. 41-42. Of the 33 dates which I deem fit for chronological analysis, 30 are not ambiguous and can be assigned to the following five groups:

A(stronomical): 6 dates that are certainly or highly probably correct by observation, but not in the reported Carlsberg cycle: 5, 6 (?), 24 (?), 26, 27, 30;

C(yclical): 4 dates that are not correct or probably not correct by observation, but are correct in the reported Carlsberg cycle: 7, 9, 28, 41;

AC: 10 dates that are correct or probably correct by observation and also in the reported Carlsberg cycle: 1, 3, 4, 11, 13, 17bis, 37, 39, 40, 42;

C*: 6 dates that are not correct or probably not correct by observation and which coincide with one of two interpolated dates in the Carlsberg Cycle: 10, 13bis, 25, 35, 36;

AC*: 5 dates that are correct or probably correct by observation and which coincide with one of two interpolated dates in the Carlsberg Cycle: 2, 8, 12, 29, 31.

No. 34 is ambiguous and may be belong to either A, C* or AC*; the allocation of Nos. 19 and 32 remains open.

The five groups are assignable to the compartments of the Venn-diagram in table 62. The presence of dates in A and C implies that Ptolemaic-Roman lunar dates were determined by observation or by using the Carlsberg Cycle. The occurrence of dates in AC is explained by the fact that observed lunar dates coincide with dates in the Carlsberg Cycle in 70% of the cases, provided that cycle and observation are in general agreement (Parker, 1950: 121); in other words, dates are to be expected in AC under any circumstances. The existence of dates in C* and AC* suggests that an interpolated form of the Carlsberg cycle was in use.

The distribution of Nos. 7-13bis (consecutive dates from Gebelein) in C, AC, C*, AC*, with no examples in A, is compatible with all being cyclical dates. By contrast, the distribution of the grain receipt dates Nos. 26-31 from Dime indicates that they were not determined uniformly. The grain receipt dates in A (26, 27, & 30) were determined observationally; those in AC* (29 & 31) either observationally or by the interpolated cycle and the remainder in C (28) by the recorded cycle.

Furthermore, Table 62 shows that the lunar-civil double dates based on the recorded Carlsberg Cycle (C, AC) and those based on the interpolated Carlsberg Cycle (C*, AC*) are more or less evenly distributed; the relationship of the remaining cyclically to observationally determined dates is about 3:2. Thus, according to the available lunar data, the Egyptians of Ptolemaic and Roman times would have dated more than half the time by consulting the Carlsberg Cycle, rather than by astronomical observation.

In general, the lunar dates in our material concern specific temples and their personnel. There is some information about the population at large and their attitudes towards lunar days, *cf.* the funerary document No. 40 citing the lunar day of a death, and the horoscope No. 42 citing the lunar day of a birth. Discrepancies between observational and cyclical dates should not have impaired the use of the cycle, since anyone relying on a schematic cycle rather than on observation appears to be willing to accept discrepancies.

Parker presumed that the Carlsberg Cycle had been introduced in the 4th century BC, but this seems too early (Depuydt, 1998: 1295). The earliest example cited by Parker refers to 237 BC (No. 34 above); the supposed earlier use of the Carlsberg Cycle for the Macedonian calendar during the reign of Ptolemy II is not supported by the sources (Jones, 1997: 162-166).

The supposition that the lunar temple service of Ptolemaic and Roman times began regularly on a LD 2 does not result in contradictions. The dates which refer to lunar temple service are marked + in table 62. There are two groups of such dates. In Nos. 1–13bis, 17bis, and 26–31, the scribes used lunar dates which were already established, whereas in Nos. 19 and 24–25, the lunar dates were yet to be determined. The latter group consists of private lease contracts which are difficult to interpret. Since I do not read Demotic, I have had to rely on transcriptions, translations, commentary and personal advice of my colleagues who specialize in Demotic. Whenever the scribes used established civil-lunar double dates, the starting date of the lunar temple service either is a LD 2 or a possible LD 2, be it observationally or cyclically determined.

There remains No. 32 from Dime, and presumably also the dates before and after, as examples for a possible exceptional start of the lunar temple service on LD 3. No. 20 may be cited as a possible case in which the lunar temple service began in the last days of a lunar month. Regardless, the regular day for starting lunar phyle services seems to have been LD 2, and the concluding day LD 1.

The question arises whether the lunar temple service (*ꜣbd, wrš*) started on LD 2 not only in the Middle Kingdom and the Ptolemaic-Roman period, but also during the intervening centuries as well. Few lunar temple service dates seem to be preserved from that time span. An exception is the *wrš*-date in year 5 of [Sheshonq I]. I have shown that the procession on *ḥꜣb.f nfr n wrš* (his beautiful feast of the *wrš*-service) of the god Seth in Dakhla probably took place on a LD 1 in 939 BC as year 5 of Sheshonq I (Krauss, 2005b; for further chronological confirmation see Payraudeau 2008 who could fix the last year of Psusennes II in 944 and thus 5 Sheshonq I in 939 BC). A *wrš*-feast on LD 1 accords with a temple service that starts on a LD 2 and ends on a LD 1 (see the comments on No. 6, above).

Bennett (2008: 543-548) deduced a hit rate of ca. 55% correct old crescent observations from the Ptolemaic-Roman data. He based his calculation on 38 of 40 dates, having excluded Nos. 21-22 from his final analysis. Bennett should also have omitted No. 33, since it is not a Ptolemaic-Roman date. The hit rate of 55% results from his acceptance of uncertain dates and the exclusion of cyclical dates. He accepts the uncertain dates Nos. 14-15 and No. 20. In those cases, old crescent would have been determined too late or too early, if the first service day were a LD 2. Furthermore, he reckons the Gebelein dates Nos. 7, 9, and 10 and the Dime grain receipts Nos. 28 and 32 as observationally incorrect dates. He does not take into account that these dates (with the exception of No. 32) coincide with cyclical LDs 2. He also reckons Nos. 35-36 (the Edfu inscriptions) and No. 38 (Ta-Imhotep stela) as observationally incorrect dates without considering that Nos. 35-36 coincided with cyclical dates, whereas No. 38 has to be deleted.

The nature of the error which Bennett postulates that the Egyptians made remains unclear to me. Did they correctly observe old crescent, but count incorrectly from it to LD 2, or did they incorrectly report old crescent on a day after old crescent, and then counted from that day on correctly to LD 2?

The observationally incorrect dates of C and C* in table 62 were apparently determined cyclically. Since they were not determined by observation, they cannot be used for reckoning the hit rate of the observers. The dates in AC and AC* are observationally, as well as cyclically, correct; whether these dates were determined observationally or cyclically is moot. Thus none of the dates in AC and AC* are relevant for determining the hit rate of the observers. In other words, no data seem to exist for computing the proportion of correctly as opposed to incorrectly observed dates. It is the existence of cyclically determined lunar dates that makes it impossible to determine a hit rate of the observers. A mere 6 dates in A are available to determine the quality of Ptolemaic-Roman lunar observation. All that can be deduced from them is that in one possible case (No. 24?) out of 6 all told, old crescent would have been determined under exceptionable conditions.

Egyptologists can live without knowing how well the moon – old crescent in particular – was observed in Ptolemaic-Roman times. Babylonian new and old crescent observations, not to mention modern ones, offer a standard for comparison.

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Postscript

Allen's empirical formula for lunar brightness as function of phase as cited on pp. 8 and 26 is based on incomplete observational data with a significant gap around new moon; it is therefore not reliable near new moon (information provided by Robert van Gent).

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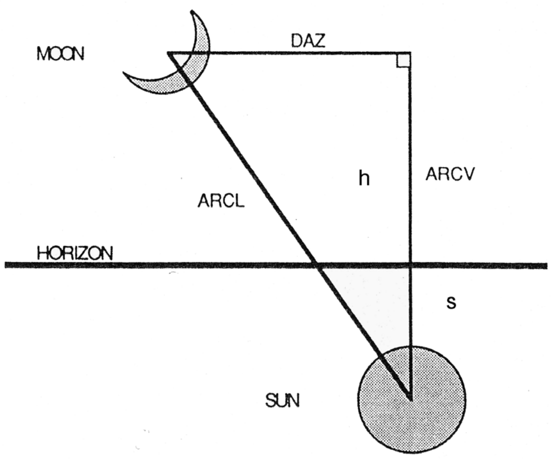

Figure 1. Positions of the sun and new/old crescent relative to the horizon; adapted from Schaefer (1998: Fig. 1).

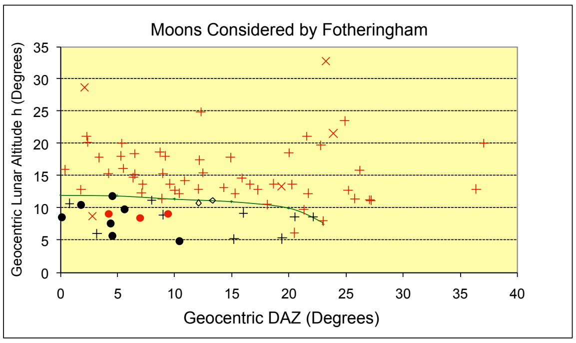

Figure 2. Crescents of Mommsen's list and analyzed by Fotheringham.

- \+ naked-eye (new crescent) sightings
- \+ naked-eye (new crescent) non-sightings
- ● sightings with telescope
- x naked-eye (old crescent) sightings
- ○ Mommsen's doubtful non-sightings
- ● non-sightings with telescope

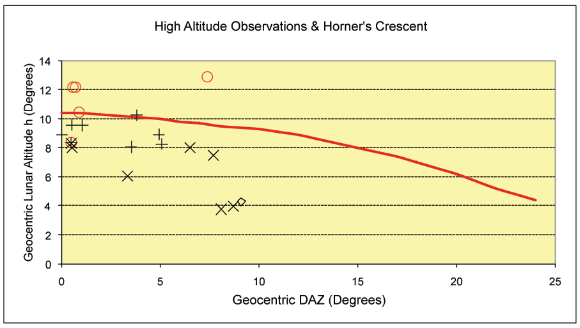

Figure 3. Horner's crescent referred to optically aided, high altitude observations.

- ○ naked-eye sightings after optical detection
- \+ optically aided sightings
- x optically aided non-sightings
- ◊ Horner's crescent
- ○ Schaefer nos. 94, 111, 128, 189, 278
- \+ Schaefer nos. 157, 199, 238, 250, 279, 289; SAAO list: 1999/9/10 (Al-Sharah);2000/1/7 ("nr Isfahan"; actually ca. 250 km SE of Isfahan)
- x Schaefer nos. 122, 194, 195, 242, 280, 288; SAAO list: 2000/1/7 (Sutherland, South Africa)

Red line: Schoch's visibility line of 1929/30.

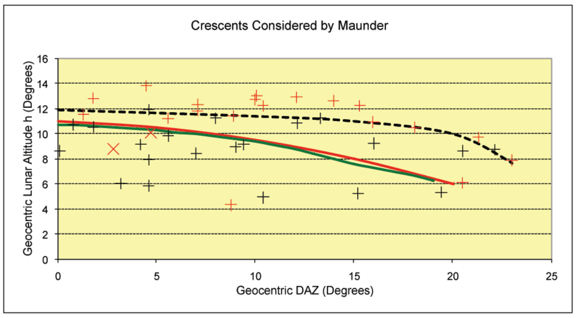

Figure 4. Crescents considered by Maunder.

- \+ new crescent sightings, Horner's crescent included
- x old crescent sightings
- \+ new crescent non-sightings

Solid red line: Maunder's line; broken black line: Fotheringham's line; green line: Schoch's line of 1927/28

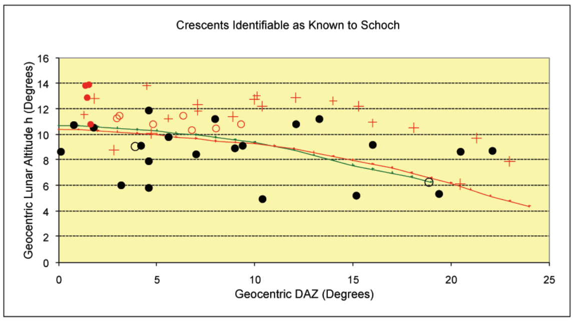

Figure 5. Crescents identifiable as known to Schoch.

- \+ new and old crescent sightings according to Fotheringham and Maunder
- ● crescent non-sightings according to Fotheringham
- ● new crescent sightings by Schoch
- ○ Babylonian new crescent sightings according to Schoch
- ○ Babylonian new crescent non-sightings according to Schoch

Green line: Schoch's visibility line of 1927/28; red line: Schoch's line of 1929/30

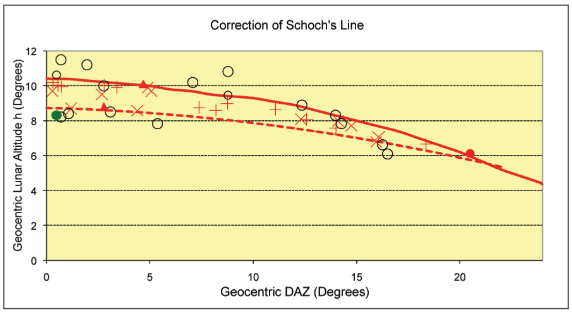

Figure 6. Babylonian and modern crescent observations referred to Schoch's visibility line of 1929/30.

Crescents known to Schoch:

- ● J. Schmidt's new crescent of 1859/10/27
- ▲ F. Schmidt's and Decroupet's old crescents

Crescents not known to Schoch:

- \+ modern new crescent sightings
- x Babylonian new and old crescent sightings
- ○ reported Babylonian new and old crescent non-sightings
- ● Pierce's new crescent sighting

Solid red line: Schoch's visibility line of 1929/30; broken red line: suggested correction of Schoch's line

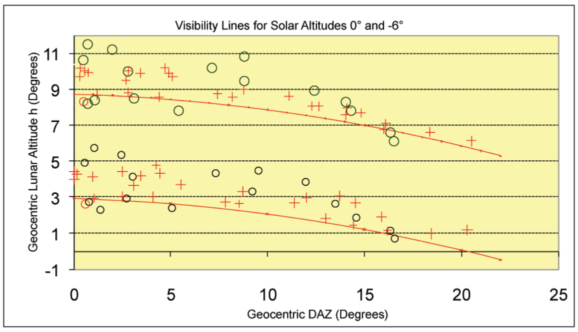

Figure 7. Visibility lines (red) for solar altitude 0° and –6°.

- + sighted crescents of Table 4
- ○ Pierce's crescent
- ○ non-sighted Babylonian crescents of table 4

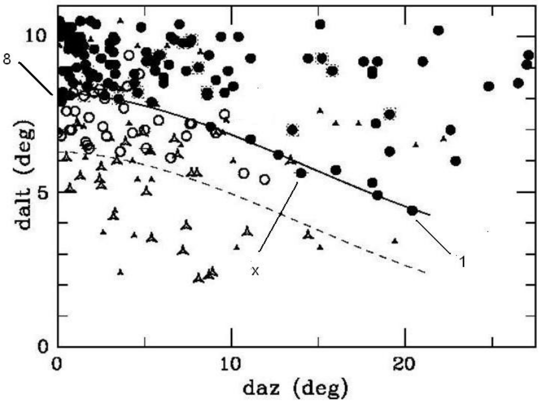

Figure 8. Crescents of the SAAO list; adapted from Caldwell & Laney (2005: Fig. 1).

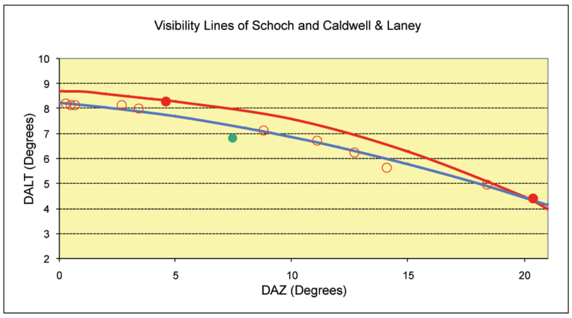

Figure 9. Minimum visibility lines of Schoch and Caldwell & Laney.

- ● Crescents defining Schoch's line
- ○ Crescents referring to Caldwell's and Laney's line
- ● Ramlah 1997/5/7

Red line: Schoch's line; blue line: Caldwell & Laneys's line

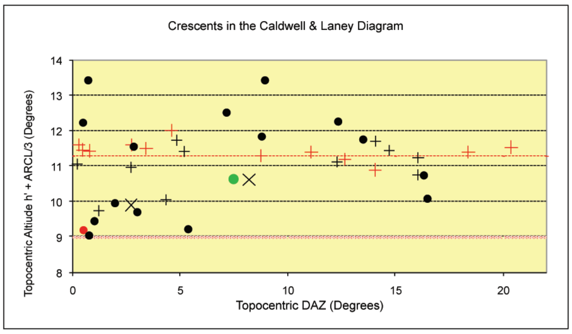

Figure 10. Sighted and not sighted moons of table 4 transferred to the Caldwell-Laney diagram.

\+ modern sighted crescents used by Caldwell & Laney
Babylonian and modern sighted and non-sighted crescents not used by Caldwell & Laney:
\+ Babylonian sighted crescents
x Athens 1864/4/23 & 1871/9/14
● Babylonian non-sighted crescents
● Ramlah 1997/5/7
● Pierce's new crescent
Broken red line: h' + ARCL/3 = 11.3°; shaded red line: border line for impossible sightings

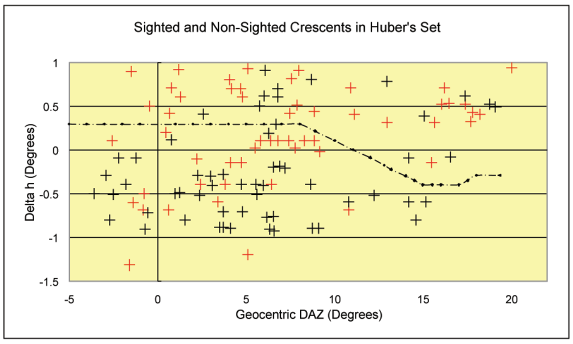

Figure 11. Huber's set of 602 new crescents; distribution of sighted and non-sighted moons close to Schoch's visibility lines.

\+ sighted crescents
\+ non-sighted crescents
x –axis : Schoch's line of 1929/30
broken line: difference between Schoch's lines of 1929/30 and 1927/28
Adapted from Huber (1982: Figure 5.2)

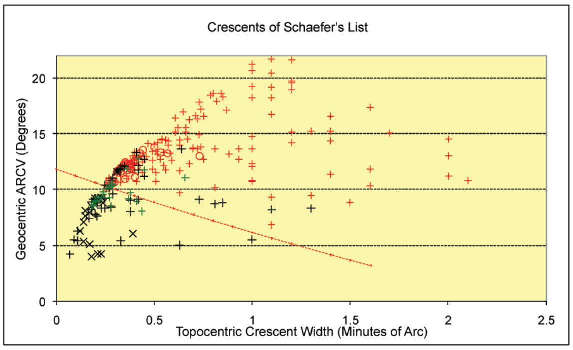

Figure 12. Crescents of the Schaefer-Doggett list within a diagram fitted to Yallop's terms.

\+ V: naked-eye sightings
○ V F = V(V): naked-eye sightings after optical detection
\+ I: naked-eye non-sightings
\+ V B = I(V): sightings with binoculars
x I B = I(I) : not visible with binoculars (Schaefer's codes)
--- Schoch's line expressed as function of crescent width and ARCV

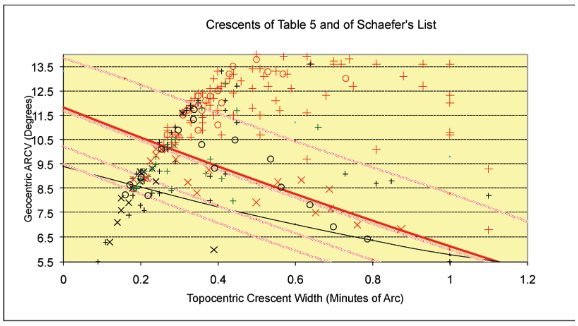

Figure 13. Detail of figure 12 with crescents of table 5 added.

- \+ V: naked-eye sightings
- ○ V F = V(V) naked-eye sightings after optical detection
- \+ I: naked-eye non-sightings
- \+ V B = I(V) sightings with binoculars or telescope
- x I B = I(I): not visible with binoculars
- x Babylonian sightings
- ○ Babylonian non-sightings

solid red line: reference line after Schoch;
green line: Schoch's line corrected;
shaded red lines: border lines of visibility types A, B, C, D and E.

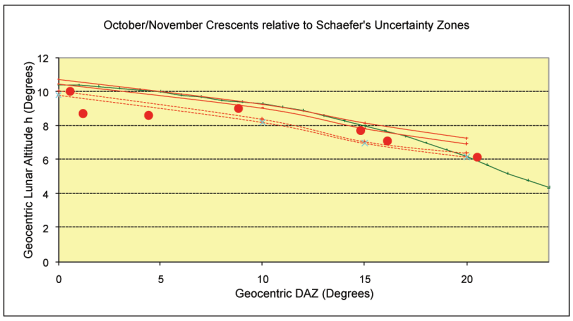

Figure 14. October and November crescents relative to Schaefer's seasonal uncertainty zones.

● October and November crescents (see table 8)
Solid red lines: October (upper) and November (lower) average lines
broken red lines: October (upper) and November (lower) lower border of uncertainty zones
green line: Schoch's visibility line of 1929/30

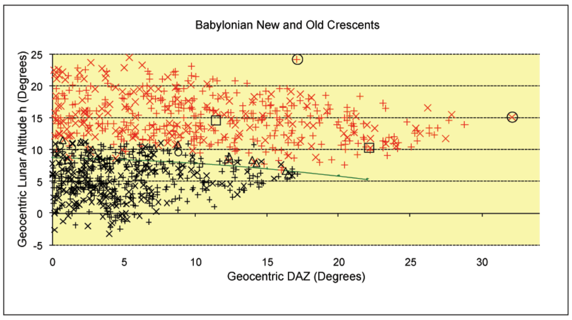

Figure 15. Babylonian new and old crescents.

+, x sighted new and old crescents
+, x implied non-sighted new and old crescents
○, Δ reported non-sightings of new and old crescents
□: implied non-sightings of a new and an old crescent
+, x combined with ○: the corresponding moons sighted one day later or earlier, respectively

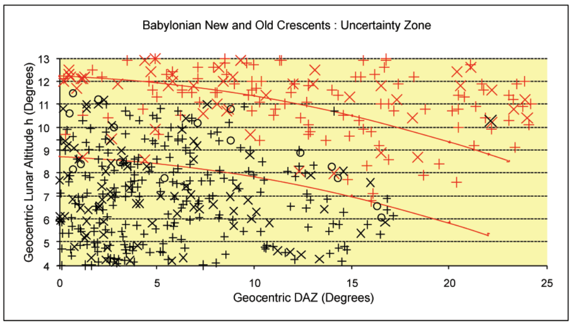

Figure 16. Babylonian crescents: uncertainty zone.

+, x sighted new and old crescents
+, x implied non-sighted new and old crescents
○, Δ reported non-sightings of new and old crescents
□: implied non-sightings of a new and an old crescent
+, x combined with ○: the corresponding moons sighted one day later or earlier, respectively
Red lines: upper and lower borders of the uncertainty zone

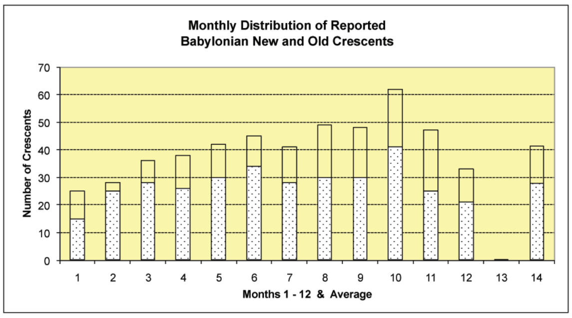

Figure 17. Distribution of reported Babylonian new and old crescents by month. New crescents dotted; old crescents blank.

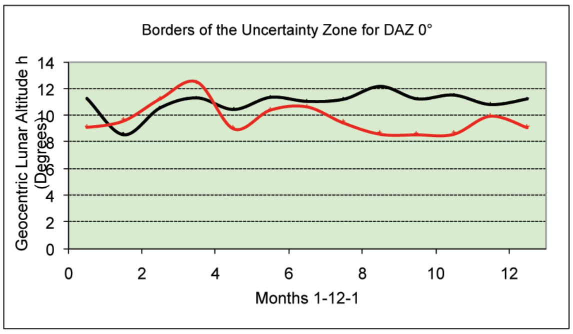

Figure 18. Upper and lower borders of the Babylonian crescent uncertainty zone.

Red line: lower border of sighted crescents
Black line: upper border of non-sighted crescents

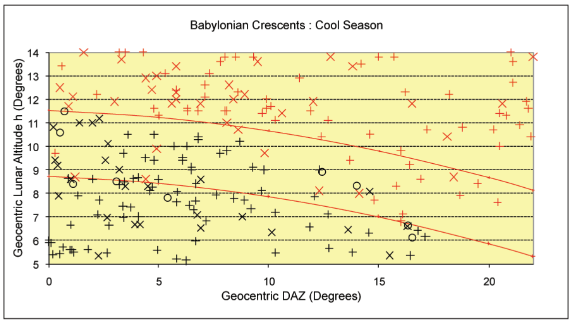

Figure 19. Babylonian crescent reports of the cool season.

+, x sighted new and old crescents
+, x implied non-sighted new and old crescents
○, Δ reported non-sightings of new and old crescents
□: implied non-sightings of a new and an old crescent
+, x combined with ○: the corresponding moons sighted one day later or earlier, respectively

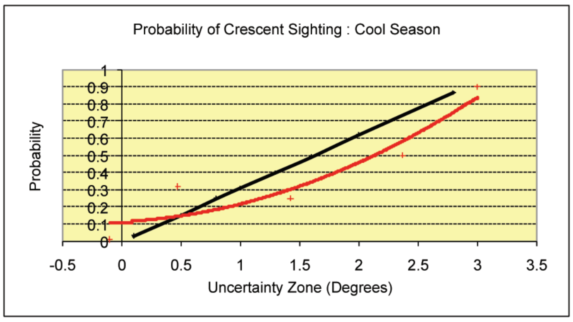

Figure 20. Development of crescent sighting probability in the uncertainty zone (cool season).

+ single probability points of table 9
Red line: polynomial fitted to single points
Black line: development of probability after Huber's model

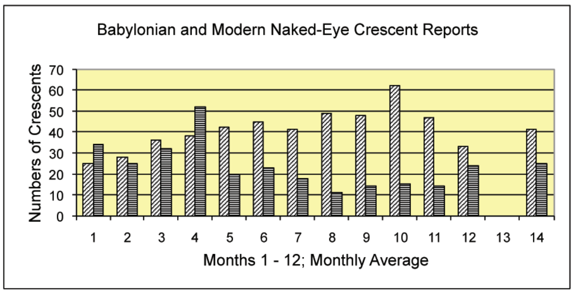

Figure 21. Month-wise distribution of Babylonian and modern naked-eye crescent reports.

Row 1 (Reihe 1): Babylonian reports
Row 2 (Reihe 2): modern reports

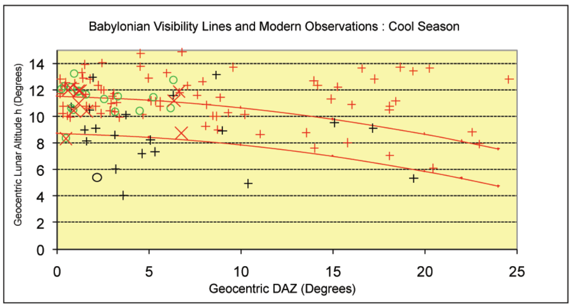

Figure 22. Modern observations referred to Babylonian crescent visibility lines (cool season).

+ naked-eye sightings
○ naked-eye sightings after optical detection
X sightings in high altitudes (naked-eye and after optical detection)
+ naked-eye non-sightings
○ validity doubted by Caldwell & Laney

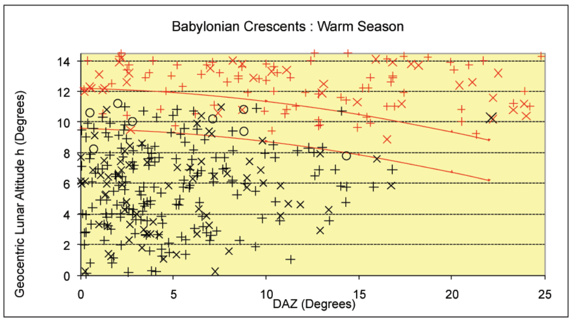

Figure 23. Babylonian crescent reports March to September.

+, x sighted new and old crescents
+, x implied non-sighted new and old crescents
○, Δ reported non-sightings of new and old crescents
□: implied non-sightings of a new and an old crescent
+, x combined with ○: the corresponding moons sighted one day later or earlier, respectively

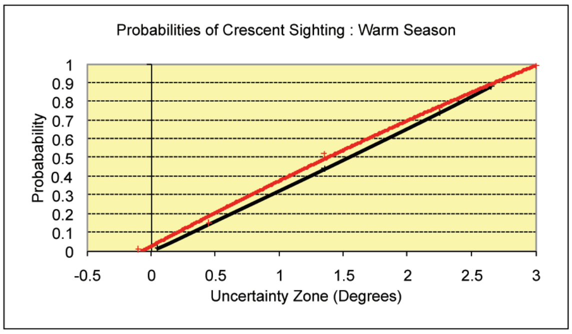

Figure 24. Development of crescent sighting probability in the uncertainty zone of the warm season.

+ single probability points of table 12
Red line: polynomial fitted to single points
Black line: development of probability after Huber's model

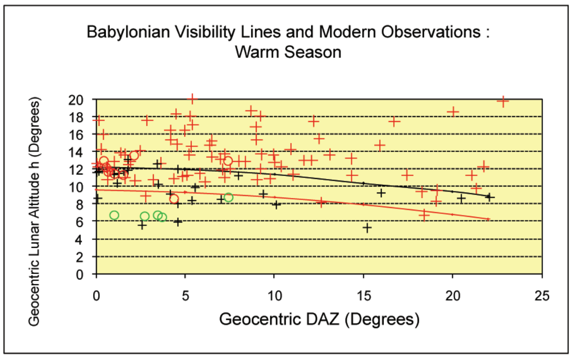

Figure 25. Modern observations referred to Babylonian crescent visibility lines (warm season).

+ naked-eye sightings
○ naked-eye sightings after optical detection, low and high altitudes
+ naked-eye non-sightings
○ validity doubted by Caldwell & Laney
red lines: borders of Babylonian uncertainty zones

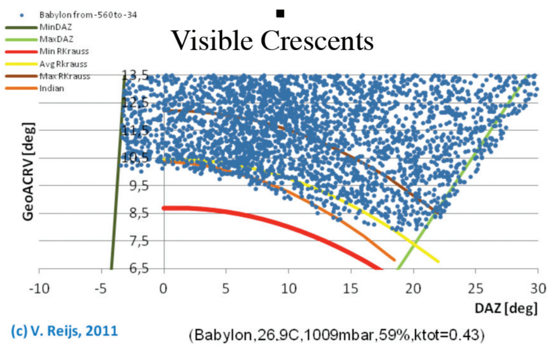

Figure 26. Calculated Babylonian new crescents for the period 570 BCE to 34 BCE.

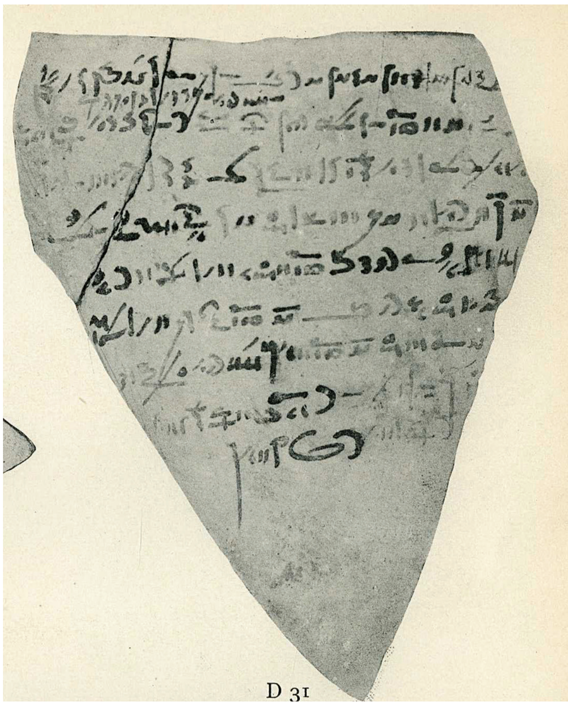

Figure 27. Ostracon D 31; after Thompson (1913).

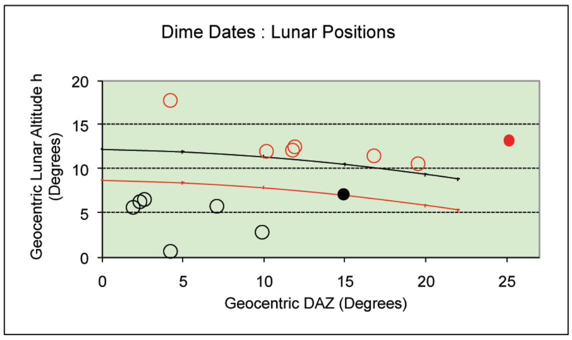

Figure 28. Moons relating to Nos. 26-32 within the azimuth-altitude diagram.

○: old crescents in observable positions
○ : the same crescents being invisible one day later
●: No. 27 on –23/3/6
●: No. 27 on –23/3/7 .
red line: lower border of uncertainty zone in the cool season
black line: upper border of uncertainty zone in the warm season

Schaefer code 1	Schaefer code 2	Schaefer's codes interpreted	SAAO code verbatim
V	V	naked-eye sighting	A: visually, regardless of optical aid; B: visually, but remarked/ inferred as being very near the limit of feasibility
VF	V(V)	naked-eye sighting after optically aided detection	
I	I	not visible to the naked eye	E: not visually, no optical aid mentioned
VB	I(V)	visible with binoculars or telescope	C: not visually, but with optical aid other than telescope; D: not visually, but with telescope only
VT		changed to I(V); no. 106	
IB	I(I)	not visible with binoculars or telescope	F: not even with optical aid

Table 1. Crescent visibility codes of the Schaefer-Doggett list and the SAAO list.
code 1: Schaefer (1988: 513); code 2: Schaefer-Doggett (1994: 394); Schaefer (1996: 761).

Stern, Table 2, predicted new crescents	DAZ	Schoch 1929/1930 h	delta h	early new crescents after Stern
–366/5/19	8.8°	9.4°	0°	possibly early
–326/1/29	5.1°	8.0°	–2.0°	early
–322/6/11	0.6°	9.7°	–0.7°	possibly early
–302/5/31	0.6°	10.1°	–0.3°	possibly early
–273/11/4	3.9°	9.0°	–1.1°	possibly early
–272/3/2	3.9°	9.7°	–0.4°	possibly early
–251/6/6	6.2°	9.5°	–0.3°	possibly early
–251/12/1	8.6°	9.1°	–0.3°	possibly early
–232/11/1	8.6°	9.5°	+0.1°	possibly early
–196/1/3	5.5°	10.0°	–0.3°	possibly early
–161/12/27	3.0°	9.9°	–0.2°	possibly early
–135/3/17	2.5°	9.9°	–0.4°	possibly early
–134/4/5	0.7°	9.9°	–0.5°	possibly early
–97/3/18	1.1°	10.2°	–0.3°	possibly early
–82/4/1	5.6°	9.9°	+0.1°	possibly early
–72/11/2	10.8°	9.1°	0°	possibly early
Appendix 1 predicted old crescents				
–250/11/19	1.1°	8.4°	–2.0°	early
–232/3/8	16.5°	6.1°	–1.4°	early
–194/11/29	3.1°	8.5°	–1.7°	early
–189/6/10	0.7°	8.2°	–2.2°	early
–154/4/14	14.3°	7.8°	–0.3°	early
–140/12/31	5.4°	7.8°	–2.1°	early
–121/3/11	16.3°	6.6°	–1.0°	early

Table 2. Predicted Babylonian crescents compared to Schoch's visibility line of 1929/30
h: geocentric lunar altitude at solar altitude 0°
delta h: difference between h and lunar minimum altitude according to Schoch's visibility line of 1929/30

no.	new crescent	Schoch	Babylonian source	observed / predicted	visibility according to Schoch
a	–544 July 5	1927, XXXVIII	not identified		visible
b	–522 Sep. 29	1928, 100	not identified		not visible
c	–386 Oct. 25	1927, XXXVIII	not identified		visible
d	–385 Nov. 13	1927, XXXVIII	not identified		visible
e	–384 Oct. 3	1927, XXXVIII	not identified		visible
f	–384 Dec. 1	1927, XXXVIII	not identified		visible
g	–328 Oct. **13	1927, XXXVIII	Stern's list 1; SH 1	observed	visible
h	–273 Nov 4	1928, 98	Stern's list 2; SH 1	predicted	not visible
i	–74 March 3	1928, 100	not identified		Visible

Table 3. Babylonian new crescents cited by Schoch (1927; 1928).

Table 4 (next page).

DAZ	0°	5°	10°	15°	20°	24°
ARCV	10.4°	10.0°	9.3°	8.0°	6.2°	4.4°
W (minarc)	0.246	0.284	0.422	0.652	0.987	1.33

Table 5. Values of DAZ, ARCV and crescent width w for Schoch's line of 1929/30.

Table 6 (next page).

DAZ	March / September	June	September / March	December
	critical geocentric lunar altitude			
0°	11° ± 0.8°	11.6° ± 0.7°	11° ± 0.8°	10.2° ± 0.6°
10°	9.5° ± 0.9°	10.1° ± 1.1°	9.5° ± 0.9°	8.8° ± 0.8°
15°	8.5° ± 1.2°	9.0° ± 0.8°	8.5° ± 1.2°	7.5° ± 0.7°
20°	7.6° ± 0.9°	7.9° ± 0.8°	7.6° ± 0.9°	6.6° ± 0.7°

Table 7. Set of DAZ/altitude critical visibility lines according to Schaefer.

source : Appendix 2	Babylonian new crescents	delta h Schoch	DAZ	h	h' + ARCL/3
	–381/8/31	–0.3°	5.1°	9.7°	11.4°
	–378/11/25	–0.3°	14.8°	7.7°	11.4°
	–284/11/6	–0.6	16.1°	7.1°	11.2°
	–264/9/26	–0.9°	16.0°	6.8°	10.73°
	–199/2/3	–0.7°	0.3°	9.7°	11.1°
	modern new crescents				
Schmidt 1868, 207ff					
Athens	1868/4/23	–0.9°	8.2°	8.58°	10.62°
source : Schaefer's list					
no. 2 Athens	1859/10/27	+0.2°	20.5°	6.1°	11.5°
no. 147 Nova Scotia	1978/3/9	–0.3°	3.43°	9.9°	11.5°
no. 86 Cape Town	1913/11/28	–0.4°	0.57°	10.0°	11.5°
no. 158 Florida	1979/1/28	–0.2°	0.33°	10.2°	11.6°
no. 162 Iowa	1979/1/28	–0.2°	2.8°	10.0°	11.6°
no. 278 Mt. Collins	1990/2/25	–2.1°	0.50°	8.3°	9.2°
source: SAAO list					
Ashdod	1990/9/20	–0.2°	18.4°	6.6°	11.4°
Signal Hill	1997/2/8	–0.2°	14.0°	7.6°	10.9°
Arad	1997/8/4	–0.8°	12.6°	8.1°	11.2°
Signal Hill	1998/2/27	–0.5°	11.1°	8.6°	11.4°
Ramlah	1996/10/3	–0.4°	8.8°	9.0°	11.3°
Ramlah	1997/5/7	–0.9°	7.4°	8.75°	10.6°
Islamabad	1991/2/15	–0.5°	0.8°	9.9°	11.4°
source: Appendix 1	Babylonian old crescents				
	–284/11/4	–1.5°	4.4°	8.6°	10.1°
	–248/10/27	–1.7°	1.2°	8.7°	9.8°
	–225/8/16	–0.7°	2.7°	9.5°	11.0°
	–206/1/21	–0.7°	12.3°	8.1°	11.1°
	–203/12/8	–0.1°	4.9°	9.9°	11.7°
	–192/1/16	–0.3	14.1°	8.0°	11.7°
source : Schaefer's list	modern old crescents				
no. 44 Athens	1871/9/14	–1.4°	2.8°	8.8°	9.9°
no. 78 [Soumagne]	1889/11/22	+0.1°	4.7°	10.1°	12.0°
source : Appendix 2	non-sighted Babylonian new crescents				
	–461/3/22		0.5°	10.6°	
	–366/5/19		8.78°	9.45°	
source : Appendix 1	non-sighted Babylonian old crescents				
	–250/11/19		1.1°	8.4°	
	–248/1/6		12.4°	8.9°	

Table 4. New and old crescents below Schoch's visibility line of 1929/30 and reportedly non-sighted new and old Babylonian crescents.

h = geocentric lunar altitude at solar altitude 0°

delta h Schoch: difference between h and lunar minimum altitude according to Schoch's visibility line of 1929/30

location	date	topocentric crescent width w'	geocentric ARCV	q	visibility type
sighted crescents					
Babylon E(vening)	–378/11/25	0.689 minarc	8.46°	+0.063	B
Babylon M(orning)	–192/1/16	0.617	8.83°	+0.06	B
Babylon E	–381/8/31	0.315	10.19°	+0.027	B
Babylon M	–206/1/21	0.556	8.76°	+0.02	B
Babylon M	–203/12/8	0.288	10.28°	+0.02	B
Ashdod E, SAAO	1990/9/20	0.875	6.84°	–0.002	B
Babylon E	–284/11/6	0.693	7.67°	–0.013	B
Signal Hill E, SAAO	1998/2/27	0.508	8.68°	–0.013	B
Islamabad E, SAAO	1991/2/15	0.241	10.12°	–0.023	C
Ramlah E, SAAO	1996/10/13	0.378	9.26°	–0.029	C
Babylon E	–264/9/26	0.762	7.01°	–0.043	C
Babylon E	–199/2/3	0.241	9.85°	–0.050	C
Signal Hill E, SAAO	1997/2/8	0.65	7.51°	–0.047	C
Babylon M	–225/8/16	0.227	9.62°	–0.08	C
Arad E, SAAO	1997/8/4	0.500	7.92°	–0.09	C
Ramlah E, SAAO	1997/5/7	0.323	8.74°	–0.11	C
Athens E	1868/4/23	0.346	8.32°	–0.14	C
Babylon M	–284/11/4	0.217	8.92°	–0.15	C
Athens M, Schaefer no. 44	1871/9/14	0.21	8.9°	–0.163	D
Babylon M	–248/10/27	0.181	8.86°	–0.18	D
non-sighted crescents					
Babylon M	–122/8/17	0.338	11.34°	+0.156	B
Babylon M	–119/6/15	0.446	10.5°	0.135	B
Babylon M	–234/11/22	0.340	11.75°	+0.19	B
Babylon M	–248/1/6	0.537	9.71°	+0.107	B
Babylon E	–461/3/22	0.299	10.88°	0.087	B
Babylon M	–110/9/3	0.359	10.30°	0.064	B
Babylon M	–86/3/14	0.565	8.55°	0.007	B
Babylon E	–366/5/19	0.393	9.35°	–0.011	B
Babylon M	–143/9/7	0.256	10.12°	–0.014	B
Babylon M	–154/4/14	0.640	7.86°	–0.020	C
Babylon M	–121/3/11	+0.701	6.94°	–0.078	C
Babylon M	–232/3/8	0.786	6.44°	–0.083	C
Babylon M	–194/11/29	0.202	8.96°	–0.162	D
Babylon M	–250/11/19	0.172	8.67°	–0.21	D
Babylon M	–140/12/31	0.220	8.21°	–0.227	D
Babylon M	–189/6/10	0.160	8.26°	–0.258	E

Table 6. New and old crescents of table 3 expressed in terms of Yallop's diagram.

source : Appendix 2	Babylonian new crescents	DAZ	h	Δ(h – h')
	–381/8/31	5.1°	9.7°	+0.2°
	–378/11/25	*14.8°*	*7.7°*	*+0.7°*
	–284/11/6	*16.1°*	*7.1°*	*+0.3°*
	–264/9/26	16.0°	6.8°	–0.4°
	–199/2/3	0.3°	9.7°	+0.1°
	modern new crescents			
Athens, Schmidt 1868	**1868/4/23**	**8.2°**	**8.58°**	**–0.5°**
source: Schaefer's list				
no. 2 Athens	1859/10/27	20.5°	6.1°	–0.2°
no. 147 Nova Scotia	1978/3/9	3.43°	9.9°	+0.2°
no. 86 Cape Town	*1913/11/28*	*0.57°*	*10.0°*	*+0.4°*
no. 158 Florida	1979/1/28	0.33°	10.2°	+0.3°
no. 162 Iowa	1979/1/28	2.8°	10.0°	+0.5°
no. 278 Mt. Collins	1990//25	0.50°	8.3°	–1.3°
source: SAAO list				
Ashdod	1990/9/20	18.4°	6.6°	–0.2°
Cape Town	1997/2/8	14.0°	7.6°	+0.1°
Arad	1997/8/4	12.6°	8.1°	–0.1°
Cape Town	1998/2/27	11.1°	8.6°	+0.6°
Ramlah	*1996/10/13*	*8.8°*	*9.0°*	*+0.4°*
Ramlah	1997/5/7	7.4°	8.75°	–0.6°
Islamabad	1991/2/15	0.8°	9.9°	+0.3°
source : Appendix 1	Bavbylonian old crescents			
	–284/11/4	**4.4°**	**8.6°**	**–0.7°**
	–248/10/27	**1.2°**	**8.7°**	**–1.1°**
	–225/8/16	2.7°	9.5°	–0.4°
	–206/1/21	12.3°	8.1°	+0.4°
	–203/12/8	4.9°	9.9°	+1.1°
	–192/1/16	14.1°	8.0°	+0.7°
source: Schaefer's list	modern old crescents			
no. 44 Athens	1871/9/14	2.8°	8.8°	–1.0°

Table 8. Adapted from table 4.

	Δh sighted	Δh non-sighted
1	0°	0.06°
2	0.01°	0.11°
3	0.1°	0.15°
4	0.31°	0.15°
5	0.58°	0.18°
6		0.31°
7	0.8°	0.33°
8	0.88°	0.41°
9	0.96°	0.46°
10	1.44°	0.66°
11	1.44°	0.69°
12	1.69°	0.77°
13	1.79°	0.79°
14	1.82°	0.93°
15	1.98°	1.0°
16	2.13°	1.0°
17	2.39°	1.05°
18	2.42°	1.09°
19	2.63°	1.09°
20		1.12°
21		1.13°
22		1.15°
23		1.4°
24		1.47°
25		1.48°
26		1.53°
27		1.61°
28		1.67°
29		1.67°
30		1.87°
31		1.93°
32		2.02°
33		2.03°
34		2.06°
35		2.12°
36		2.15°
37		2.30°
38		2.33°
39		2.55°
40		2.77°

Table 9. Altitudes of sighted and non-sighted crescents within the uncertainty zone of the cool season.

crescent position relative to uncertainty zone	Δh	approximate sighting probability
below	–0.1°	0.01
average lower third	0.475°	0.318
average middle third	1.425°	0.25
average upper third	2.375°	0.5
above	3.0°	0.99

Table 10. Babylonian crescent positions and corresponding sighting probabilities in the cool season.

	Δh sighted	Δh non-sighted
1	0°	0.07°
2	0.38°	0.28°
3	0.69°	0.28°
4	0.88°	0.29°
5	0.92°	0.32°
6	1.89°	0.34°
7	1.28°	0.37°
8	1.31°	0.39°
9	1.39°	0.39°
10	1.41°	0.4°
11	1.51°	0.5°
12	1.51°	0.53°
13	1.68°	0.54°
14	1.79°	0.57°
15	1.87°	0.61°
16	1.93°	0.61°
17	1.99°	0.61°
18	2.12°	0.72°
19	2.21°	0.83°
20	2.38°	0.84°
21	2.48°	0.87°
22	2.48°	0.99°
23	2.49°	1.07°
24	2.52°	1.24°
25	2.59°	1.47°
26	2.62°	1.48°
27		1.49°
28		1.65°
29		1.88°
30		2.07°
31		2.6°

Table 11. Altitudes of sighted and non-sighted crescents within the uncertainty zone of the warm season.

crescent position relative to uncertainty zone	Δh	approximate sighting probability
below	–0.1°	0.01
average lower third	0.45°	0.15
average middle third	1.35°	0.52
average upper third	2.25°	0.76
above	3.0°	0.99

Table 12. Babylonian crescent positions and corresponding sighting probabilities in the warm season.

	September to March	March to September	
	Babylonian	**Babylonian**	**Athenian**
DAZ	h*	h*	h*
0°	10.1° ± 1.5°	10.8° ± 1.4°	10.6° ± 1.8°
5°	10.0°	10.7°	10.5°
10°	9.4°	10.1°	9.95°
15°	8.4°	9.2°	9.0°
20°	7.1°	7.8°	7.6°
22°	6.4°	7.1°	7.0°

Table 13. Empirical seasonal crescent visibility criteria in the azimuth-altitude diagram according to Babylonian and modern observations.

year	Akhet				Peret				Shemu			
	I	II	III	IV	I	II	III	IV	I	II	III	IV
1		1		30		29		28		27		26
2		20		19		18		17		16		15
3		9		8		7		6		5		4
4		28		27		26		25		24		23
5		18		17		16		15		14		13
6		7		6		5		4		3		2
7		26		25		24		23		22		21
8		15		14		13		12		11		10
9		4		3		2		1		30		29
10		24		23		22		21		20		19
11		13		12		11		10		9		8
12		2		1		30		29		28		27
13		21		20		19		18		17		16
14		10		9		8		7		6		5
15		30		29		28		27		26		25
16		19		18		17		16		15		14
17		8		7		6		5		4		3
18		27		26		25		24		23		22
19		16		15		14		13		12		11
20		6		5		4		3		2		1
21		25		24		23		22		21		20
22		14		13		12		11		10		9
23		3		2		1		30		29		28
24		22		21		20		19		18		17
25		12		11		10		9		8		7

Table 14. The cycle of pCarlsberg 9; after Parker (1950: 15).

year BC	Julian date of I Akhet 1	local time of conjunction 30° East	difference
257	October 26	9 h 12 m	
232	October 20	12 h 2 m	9125 d + 2 h 50 m
207	October 14	20 h 51 m	9125 d + 8 h 49 m

Table 15. Coincidences of conjunctions and I *ꜣḫt* 1 as beginnings of Carlsberg cycles.

date No.	last visibility	DAZ	h	h*	probability
1	– 55 Dec 22 = IV A 18	6.4°	12.2°	9.7° ± 0.9°	~1

Table 16. Visibility circumstances of date No.1 in the azimuth-altitude diagram. - ‚Probability' = probability of observing last visibility.

date No.	service day 1	observational LD of service day 1	LD of service day 1 in Carlsberg Cycle year 2
1	–55 Dec 24 = IV A 20	LD 2	LD 2

Table 17. Observational and cyclical correspondances to Service Day 1 of No. 1.

date No.	last visibility	DAZ	h	h*	probability
2	–54 Jan 20 = I P 17	12.9°	18.4°	8.7° ± 0.9°	~1
2	–54 Jan 21 = I P 18	4.1°	9.7°	9.8° ± 0.9°	medium

Table 18. Visibility circumstances of date No. 2 in the azimuth-altitude diagram.

date No.	service day 1	observational LD of service day 1	LD of service day 1 in CC year 2
	–54 Jan 22 = I P 19 LD 2 or less	LD 2 or less	
2	probable LD 1 LD 1 or 2	probable LD 1	LD 1 or 2

Table 19. Observational and cyclical correspondences to Service Day 1 of No. 2.

date No.	last visibility	DAZ	h	h*	probability
3	–47 Aug 26 = IV S 27	0.14°	21.75°	10.6° ± 1.1°	~1
3	–47 Aug 27 = IV S 28	1.9°	11.0°	10.5° ± 1.1°	medium to sizeable

Table 20. Visibility circumstances of date No. 3 in the azimuth-altitude diagram.

date No.	service day 1	observational LD of service day 1	LD of service day 1 in CC year 9
3	–47 Aug 29 = IV S 30	LD 2 or 3	LD 2

Table 21. Observational and cyclical correspondences to Service Day 1 of No. 3.

date No.	last visibility	DAZ	h	h*	probability
4	–36 Feb 1 = II P 3 7.	7.8°	12.2°	9.5° ± 0.9°	~1

Table 22. Visibility circumstances of date No. 4 in the azimuth-altitude diagram.

date No.	service day 1	observational LD of service day 1	LD of service day 1 in CC year 20
4	–36 Feb 3 = II P 5	LD 2	LD 2

Table 23. Observational and cyclical correspondences to Service Day 1 of No. 4.

date No.	last visibility	DAZ	h	h*	probability
5	AD 66 April 11 = I S 8	22.28°	12.0°	6.4° ± 1.2°	~1
5	AD 66 April 12 = I S 9	13.2°	4.9°	8.9° ± 1.2°	~0

Table 24. Visibility circumstances of date No. 5 in the azimuth-altitude diagram.

date No.	service day 1	observational LD of service day 1	LD of service day 1 in CC year 22
5	AD 66 April 13 = I S 10	LD 2	LD 30 or 1

Table 25. Observational and cyclical correspondences to Service Day 1 of No. 5.

date No.	last visibility	DAZ	h	h*	probability
6	–130 Nov 11 = II A 18	10.3°	20.9°	9.4° ± 1.45°	~1
6	–130 Nov 12 = II A 19	5.7°	7.4°	9.9° ± 1.45°	~0

Table 26. Visibility circumstances of date No. 6 in the azimuth-altitude diagram.

date No.	service day 1	observational LD of service day 1	LD of service day 1 in CC year 1
6	–130 Nov 13 = II A 20	LD 2	LD 1

Table 27. Observational and cyclical correspondences to Service Day 1 of No. 6.

No. 8	I Peret 19 to II Peret 18
No. 9	II Peret 19 to III Peret 18
No. 7	IV Akhet 20 to I Peret [18]
No. 10	III Peret 19 to IV Peret 17
No. 11	IV Peret 18 to I Shemu 16
No. 12	[I Shemu 1]7 to II Shemu 16
No. 13	[II Shemu 17 to] III Shemu 16
No. 13bis	[III Shemu 17 to ...]

Table 28. The *wrš*-dates of pDemot Cairo 30901.

date No.	last visibility	DAZ	h	h*
7	–129/1/9 = IV A 17	12.2°	17.1°	9.1° ± 0.9°
7	–129/1/10	3.3°	7.2°	9.9° ± 0.9°
8	–129/2/8 = I P 17	7.8°	12.6°	9.5° ± 0.9°
9	–129/3/9 = II P 16	13.0°	17.1°	9.0° ± 1.2°
9	–129/3/10	3.7°	9.7°	10.1° ± 1.2°
10	–129/4/8 = III P 16	8.9°	14.3°	9.6 ± 1.2°
10	–129/4/9	0.6°	6.7°	10.2° ± 1.2°
11	–129/5/8 = IV P 16	5.8°	11.1°	10.0° ± 1.2°
12	–129/6/6 = I S 15	9.8°	16.6°	9.9° ± 1.1°
12	–129/6/7	3.9°	7.2°	10.4° ± 1.1°
13	–129/7/6 = II S 15	7.1°	13.5°	10.5° ± 1.1°
13bis	–129/8/4	7.8°	21.1°	10.1° ± 1.1°
13bis	–129/8/5 = III S 15	5.7°	9.6°	10.4° ± 1.1°

Table 29. Visibility circumstances of dates Nos. 7-13bis in the azimuth-altitude diagram.

date No.	service day 1	observational LD of service day 1	LD of service day 1 in CC year 2
7	IV A 20	LD 3	LD 2
8	I P 19	LD 2	LD 1 or 2
9	II P 19	LD 3 or less probably LD 2	LD 2
10	III P 19	LD 3	LD 2 or 3
11	IV P 18	LD 3 or LD 2	LD 2
12	I S 17	LD 2	LD 1 or 2
13	[II S 17]	LD 2	LD 2
13bis	[III S 17]	LD 3 or far less probably LD 2	LD 2 or 3

Table 30. Observational and cyclical correspondences to Service Day 1 of Nos. 7-13bis.

date No.	last visibility	DAZ	h	h*	probability
16	–40 April 13 = IV P 14	11.8°	13.2°	9.2° ± 1.2°	~1
17	–40 May 13 = I S 14	5.6°	12.4°	10.0° ± 1.2°	~1
17bis	–40 Aug 10 = IV S 13	0.3°	11.8°	10.6° ± 1.1°	~1

Table 31. Visibility circumstances of dates Nos. 16-17bis in the azimuth-altitude diagram.

date No.	service day 1	observational LD of service day 1	cyclical LD of service day 1
16	IV peret 17 (Chauveau)	LD 3	LD 2
17	I shemu 15 (Chauveau)	LD 1	LD 29/30 or 1
16	IV peret 15 ? (Bennett)	LD 1	29/30
17	I shemu 15 ? (Bennett)	LD 1	LD 29/30 or 1
	last service day		
17bis	IV shemu 14	LD 1	LD 1

Table 32. Observational and cyclical correspondences to Service Day 1 of Nos. 16-17 and last Service Day of No. 17bis.

date No.	last visibility	DAZ	h	h*	probability
18a	AD 5 Nov 19 = III A 30	0.3°	16.5°	10.1° ± 1.45°	~1
18b	AD 6 Jan 17 = I P 29	6.5°	15.7°	9.6° ± 0.9°	~1

Table 33. Visibility circumstances of start and end date of No. 18 in the azimuth-altitude diagram.

date No.	last visibility	DAZ	h	h*	probability
19	AD 10 April 1 = IV P 14 civ. = IV P 6 alex.	22.5°	12.3°	6.4° ± 1.4°	~1
19	AD April 2	13.5°	4.9°	9.1° ± 1.4°	~0

Table 34. Visibility circumstances of date No. 19 in the azimuth-altitude diagram.

date No.	possible service day 1	observational LD of service day 1	LD of service day 1 in CC year 16
19	IV P 8 alex. =IV P 16 civ. = AD 10 April 3	LD 2	LD 1
19	IV P 9 alex. =IV P 17 civ. = AD 10 April 4	LD 3	LD 2

Table 35. Observational and cyclical correspondences to Service Day 1 of No. 19.

date No.	last visibility	DAZ	h	h*	probability
20	AD 69 Dec 30 = I P 27 civ.	17.4°	18.6°	7.9° ± 1.1°	~1
20	AD 69 Dec 31	11.75°	8.8°	9.2° ± 1.1°	medium

Table 36. Visibility circumstances of date No. 20 in the azimuth-altitude diagram.

date No.	last visibility	DAZ	h	h*	probability
23	AD 147 Oct 11 = III A 26 civ. = II A 13 alex.	4.9°	15.5°	10.0° ± 1.45°	~1
23	AD 147 Oct 12	4.7°	4.0°	10.0° ± 1.45°	~0

Table 37. Visibility circumstances of date No. 23 in the azimuth-altitude diagram.

date No.	service day 1	observational LD of service day 1	LD of service day 1 in CC year 4
23	AD 147 Oct 15 = II A 17alex. = III A 30 civ.	LD 4	LD 3 or 4

Table 38. Observational and cyclical correspondences to Service Day 1 of date No. 23.

date No.	last visibility	DAZ	h	h*	probability
24	AD 190 April 20 = II S 18 civ. = IV P 25 alex.	16.5°	17.4°	8.4° ± 1.2°	~1
24	AD 190 April 21	8.5°	9.1°	9.9° ± 1.2°	slight

Table 39. Visibility circumstances of date No. 24 in the azimuth-altitude diagram.

date No.	service day 1	observational LD of service day 1	LD of service day 1 in CC year 21
24	IV P 28 alex. =II S 21 civ. = AD 190 April 23	LD 3 or less probably LD 2	LD 1

Table 40. Observational and cyclical correspondences to Service Day 1 of date No. 24.

date No.	last visibility	DAZ	h	h*	probability
25	AD 199 July 9 = I A 5	3.7°	20.2°	10.4° ± 1.1°	~1
25	AD 199 July 10 = I A 6	1.0°	10.3°	10.6° ± 1.1°	medium

Table 41. Visibility circumstances of date No. 25 in the azimuth-altitude diagram.

date No.	service day 1	observational LD of service day 1	LD of service day 1 in CC year 6
25	AD 199 I A 8	LD 3 or less probably LD 2	LD 1 or LD 2

Table 42. Observational and cyclical correspondences to Service Day 1 of date No. 25.

date No.	service day 1	observational	cyclical	determination of service day 1	old crescent DAZ / h
26	-23 Feb 7 = II Peret 13	LD 2	LD 1	observational	-23/2/5 19.5°/10.5°
27	-23 March 8 = III Peret 12	LD 2	last LD or LD 1	observational	-23/3/6 25.1°/13.2°
28	AD 68 Nov 15 = IV Akhet *12	LD 3	LD 2	cyclical	68/11/12 4.2°/ 17.8°
29	AD 68 Dec 14 =I *Peret [11]	LD 2	LD 1 or 2	observational or cyclical	68/121/12 10.7°/12.0°
30	AD 90 March 20 = IV Peret 22	LD 2	LD 1	observational	90/3/18 16.8°/11.5°
31	AD 90 April 19 = I Shemu 22	LD 2	LD 1 or 2	observational or cyclical	90/4/17 11.8°/12.1°
32	AD 91 April 9 = [I Shemu] *12	LD 3	LD 2 or 3	cyclical ?	91/4/6 11.9°/12.5°

Table 43. Observational and/or cyclical determination of Nos. 26-32.

date No.	last visibility	DAZ	h	h*	probability
33	-558 Oct 4 = I S 28	6.0°	21.4°	9.9° ± 1.45°	~ 1
33	-558 Oct 5 = I S 29	5.4°	9.5°	9.9° ± 1.45°	medium

Table 44. Visibility circumstances of No. 33 in the azimuth-altitude diagram.

date No.	last visibility	DAZ	h	h*	probability
34	–236 Aug 16	7.3°	22.9°	10.2° ± 1.1°	~1
34	–236 Aug 17 = III S 1	4.1°	10.3°	10.4° ± 1.1°	medium

Table 45. Visibility circumstances of date No. 34 in the azimuth-altitude diagram.

date No.	reported LD	observational LD	LD in CC year 20
34	III S 7 = LD 6	LD 6 or 7	LD 5 or 6

Table 46. Observational and cyclical correspondences to date No. 34.

date No.	last visibility	DAZ	h	h*	probability
35	–211 Aug 10	1.5°	21.45°	10.6° ± 1.1°	~1
35	–211 Aug 11 = III S 1	3.9°	7.4°	10.4° ± 1.1°	~0

Table 47. Visibility circumstances of date No. 35 in the azimuth-altitude diagram.

date No.	reported LD	observational LD	LD in CC year 20
35	III Shemu 7 = LD 6	LD 7	LD 5 or 6

Table 48. Observational and cyclical correspondences to date No. 35.

date No.	last visibility	DAZ	h	h*	probability
36	-141 Aug 16 = III S 23	4.4°	23.9°	10.4° ± 1.1°	~1
36	–141 Aug 17	0.8°	10.2°	10.6° ± 1.1°	medium
36	–141 Aug 18	2.3°	–3.5°	–	0

Table 49. Visibility circumstances of date No. 36 in the azimuth-altitude diagram.

date No.	reported LD	observational LD	LD in CC year 15
36	LD 23	LD 25 or less probably LD 24	LD 23 or 24

Table 50. Observational and cyclical correspondences to date No. 36.

date No.	last visibility	DAZ	h	h*	probability
37	–139 June 26 = II S 3	11.1°	17.1°	9.7° ± 1.1°	~1
37	–139 June 27	4.1°	7.1°	10.4° ± 1.1°	~0

Table 51. Visibility circumstances of date No. 37 in the azimuth-altitude diagram.

date No.	reported LD	observational LD	LD in CC year 17
37	LD 6	LD 6	LD 6

Table 52. Observational and cyclical correspondences to date No. 37.

reported LD	observational	cyclical
No. 34: LD 6	LD 6 or 7	LD 5 or 6
No. 35: LD 6	LD 7	LD 5 or 6
No. 36: LD 23	LD 25 or less probably LD 24	LD 23 or 24
No. 37: LD 6 in Payni	LD 6	LD 6

Table 53. Observational and cyclical correspondences to dates Nos. 34 – 37.

date No.	last visibility	DAZ	h	h*	probability
39	–28 April 1 = IV P 5	18.0°	9.2°	7.8° ± 1.2°	~1

Table 54. Visibility circumstances of date No. 39 in the azimuth-altitude diagram.

date No.	reported LD	observational LD	LD in CC year 3
39	LD 16	probably LD 16	LD 16

Table 55. Observational and cyclical correspondences to date No. 39.

date No.	last visibility	DAZ	h	h*	probability
40	– 8 June 18 = II S 28 civ.	4.7°	12.0°	10.4° ± 1.1°	~1

Table 56. Visibility circumstances of date No. 40 in the azimuth-altitude diagram.

date No.	reported LD	observational LD	LD in CC year 23
40	LD 16	LD 16	LD 16

Table 57. Observational and cyclical correspondences to date No. 40.

date No.	last visibility	DAZ	h	h*	probability
41	–54 Oct 12	6.1°	18.4°	9.9° ± 1.5°	~ 1
41	–54 Oct 13	4.2°	7.2°	10.0° ± 1.5°	~ 0

Table 58. Visibility circumstances of date No. 41 in the azimuth-altitude diagram.

date No.	reported LD	observational LD	LD in CC year 3
41	LD 6	LD 7	LD 6

Table 59. Observational and cyclical correspondences to date No. 41.

date No. 42	last visibility	DAZ	h	h*	probability
Alexandria	–37/4/11	13.1°	15.4°	9.5° ± 1.4°	~ 1
	–37/4/12 = IV P 13	1.3°	7.1°	10.8 ± 1.4°	~0
Syene	–37/4/11	11.2°	16.9°	8.6° ± 1.4°	~ 1
	–37/4/12	0.4°	7.2°	10.8 ± 1.4°	~0

Table 60. Visibility circumstances of date No. 42 in the azimuth-altitude diagram.

date No.	reported LD	observational LD	LD in CC year 19
42	LD 22	LD 22	LD 22

Table 61. Observational and cyclical correspondences to date No. 42.

C	AC	A
	1+	
	3+	
	4+	5+
7+	11+	6?
9+	13+	
	17bis +	
28+	37	24?+
	39	
41+	40	26+
	42	27+
C*	**AC***	
10+	2+	
13bis+	8+	30+
25	12+	
35	29+	
36	31+	

Table 62. Distribution of lunar dates classifiable as observed and/or cyclical dates.

Appendix 1. Observed Old Crescents in Astronomical Diaries 1 - 3 and 5 - 6.

The comments below refer to the individual entries in the Astronomical Diaries and should be read in tandem with these entries. For the use of [....] in citations from SH, see Hunger SH 1, 37.

Since interest here centers on the critical conditions for sighting new and old crescent, reports in the Diaries about rain, clouds, or dense mist on potential new or old crescent days are not relevant but rather those reports of actual sightings, as well as expected sightings which turned out to be negative under unexceptional circumstances. Note that virtually all reports in the Diaries are from the city of Babylon itself.

The Babylonian observer recorded six lunar time intervals which have been termed "Lunar Six" by Sachs: the lag between sunset and moonset (called *na*) on the first day of the month or new crescent day; four intervals relating to setting and rising of sun and moon around full moon; the lag between moonrise and sunrise (called *KUR*) on last crescent day (Hunger SH 1, 20-22; Fatoohi *et al.*, 1999: 53). Lags were measured in *UŠ*. For *UŠ* as "time degree", translated as 1° and corresponding to 4 time minutes, also for 1 NINDA as 1/60th of UŠ, see Hunger SH 1, 16 and Steele (2009: 45-46).

For the evaluation of the reported lags around full moon, see Brack-Bernsen (1999: 15, 37). Steele has analysed reports about the different kinds of lags (Steele, 2009: 47-51; with older literature). Accordingly, the Babylonians defined rising and setting of sun and moon as the moment when the upper limb of the luminary crossed the horizon. On the basis of ca. 100 *KUR* entries Steele found a mean error of 2.1° (time degrees) or 8.4 minutes between computed and observed rising of moon and sun on old crescent day with maximum differences of ca. ± 8° (time degrees) or ± 32 minutes.

Steele accepts as possibly measured those lags without the comment "measured" or "not seen": "Unless there is some mention of bad weather in the record, I have assumed that the timings in the third category [that have no comment attached] were measured. This may have caused some predicted material to be included in the analysis, but as there is no significant change in the result if this group is ignored, it would appear that, on the whole, these do indeed represent measured lunar six values" (Steele, 2009: 47 n. 79).

To be on the safe side, I follow Stern's lead and exclude any of the lunar six lacking the comment "measured". A reported lag value with an asterisk [*] in the list below indicates that the *KUR* lag of an observed old crescent is not qualified by "measured". Note that different observers reported different lags for one and the same occasion (Nos. 51, 52 and 72); the differences do not exceed 3° (time degrees).

There are about 112 measured *KUR* lags which are textually certain (designated by # in the list below). Rising and setting times I controlled with Uraniastar 1.1 which takes refraction into consideration and parallax (see Pietschnig & Vollmann, 1995, Handbuch I-6). The errors in astronomically computed minus measured lags are distributed between −3.0° and +8.75° (time degrees); the mean error and standard deviation amount to 2.51° ± 2.27° (time degrees) or about 10 minutes. Fig.1a shows the 112 values divided into classes of 1° (time degree) and the number of occurrences in a class. It appears that the distribution of the textually certain *KUR* lag errors is a normal distribution.

Besides the general remark "I have watched," the reports describe old or new crescent as "faint", "bright", and so forth; a report of the position of new or old crescent relative to a fixed star or a planet also implies actual observation of the moon (Stern, 2008: 20). Most useful for the identification of old crescents reported on a 26th, 27th or 28th day of a Babyonian lunar month are those cases in which the 1st day of the month is also reported and, furthermore, computed by Fatoohi *et al.* and/or by Stern. Whenever information about the preceding new crescent day was lacking, I used the lags and lunar, stellar, or planetary positions which are reported for certain days of the month in

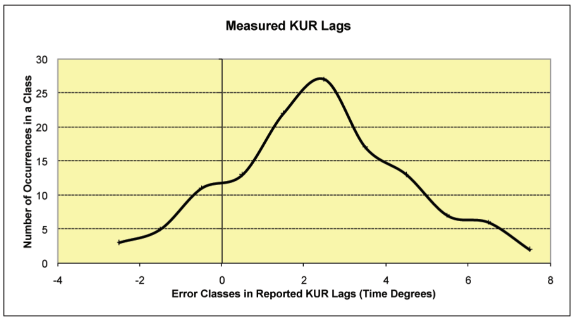

Figure 1a. Errors in measured *KUR* lags.

question to identify old crescent day within the Julian calendar. For the cubit and its subdivision into 24 'fingers' as units of measure for the distances between the heavenly bodies, see Hunger SH 1, 22; for the correspondence of 1 cubit to 2.2°, see Fatoohi *et al.* (1999: 55).

In some reports only the position of the moon relative to a star is preserved; such information is tantamount to an implicit observation of old crescent. I computed several of these cases and added them at the end of the list below but did not include them in the final analysis.

The table below lists the dates of old crescent and the geocentric values of DAZ and h which are relevant for the azimuth-altitude diagram. Dates which were not reported but which I calculated for elucidation of a reported date, are marked with +; a line is left vacant following each pair or triplet of such dates.

source	**date**	**observation / measured lag**	**computed lag**	**lag in min**	***DAZ***	***h***	**reference to number in figure text**
SH 1	–373/10/29	*23°	21.75°	87	*9.5°*	*17.6°*	1
SH 1	–372/9/18	*16°	15.5°	62	*7.0°*	*13.5°*	2
SH 1	–371/4/13	*22°	20.75°	83	*18.1°*	*18.1°*	3
	–371/4/14+				*7.6°*	*11.0°*	
SH 1	–345/1/28	# 11°	12.25°	49	*9.8°*	*9.7°*	4
SH 1	–324/8/29	*29°	26.75°	107	*0.3°*	*21.0°*	5
	–324/8/30+				*3.3°*	*8.6°*	
SH 1	–284/11/4	# 10° 30'	10°	40	*4.4°*	*8.6°*	6
SH 1	–278/12/27	# 21°	26.25°	105	*14.1°*	*18.30°*	7
	–278/12/28+				*4.5°*	*8.3°*	
SH 1	–273/11/2	# 24°	28.75°	115	*2.7°*	*23.6°*	8
	–273/11/3+		12.75°		*1.4°*	*11.0°*	
SH 1	–266/5/22	[meas.d]		57	*13.1°*	*12.5°*	9
SH 2	–255/10/14	[meas.d]		109	*3.5°*	*22.7°*	10
	–255/10/15+				*0.4°*	*9.2°*	
SH 2	–253/11/22	observed		60	*0.5°*	*12.5°*	11
SH 5 no. 38	–251/5/5	# 12°	12.5°	50	*23.9°*	*11.2°*	12
SH 5 no. 38	–251/10/30	# 20°	27°	108	*0.1°*	*22.4°*	13
	–251/10/31+				*2.0°*	*11.0°*	
SH 5 no. 38	–250/1/27	# 18°	16.5°	66	*16.5°*	*11.8°*	14
	–250/11/18+				*3.9°*	*19.3°*	
SH 6 no. 2	–250/11/19	not seen *9°	10°		*1.1°*	*8.4°*	15
SH 2	–248/1/6	not seen 11°	12.5°		*12.4°*	*8.9°*	16
SH 6 no. 5	–248/10/27	# 10°	9.75°	39	*1.2°*	*8.7°*	17
SH 6 no. 5	–247/1/23	# 16°	17.5°	70	*20.6°*	*11.8°*	18
SH 2	–245/4/30	# 16°	17°	68	*16.5°*	*15.1°*	19
	–245/5/1+				*6.92°*	*9.41°*	

source	date	observation / measured lag	computed lag	lag in min	*DAZ*	*h*	reference to number in figure text
SH 2	–234/11/22	not seen *9°	13.75°		*0.7°*	*11.5°*	20
SH 2	–233/4/17	# 12° 30'	11.75°	47	*24°*	*10.4°*	21
SH 6 no. 10	–233/6/15	# 14°	16.25°	65	*17.8°*	*13.8°*	22
SH 6 no. 10	–233/10/12	# 18°	20.75°	83	*2.1°*	*17.7°*	23
SH 6 no.10	–233/12/10	# 22°	28.25°	113	*8.1°*	*21.3°*	24
	–233/12/11+				*2.7°*	*10.1°*	
SH 6 no. 10	–232/3/7+		18.75°		*28.1°*	*14.2°*	25
	–232/3/8	not seen 21°10'	6.75°		*16.5°*	*6.1°*	
SH 2	–232/11/29	observed *11°	15°	60	*3.0°*	*11.9°*	26
SH 2	–225/8/16	10+[x]°	11.25°	45	*2.7°*	*9.5°*	27
SH 6 no. 20	–211/4/14	# 11° 30'	11.25°	45	*22.2°*	*10.2°*	28
SH 6 no. 20	–211/6/12	# 19°	23°	92	*16.7°*	*19°*	29
	–211/6/13+				*6.5°*	*10.4°*	
SH 6 no. 20	–211/7/12	# 18° 30'	21.75°	87	*5.1°*	*16.6°*	30
SH 6 no. 20	–211/8/10	# 21°	29.75°	119	*1.9°*	*22°*	31
	–211/8/11+		10.25°		*2.65°*	*8.44°*	
SH 6 no. 20	–211/9/9	# 8° 10'	15°	60	*2.5°*	*12.6°*	32
SH 6 no. 20	–211/10/8	# 19°	20.5°	82	*0.6°*	*16.7°*	33
SH 6 no. 20	–211/11/6	# 21°	27.5°	110	*5.0°*	*22.8°*	34
	–211/11/7+		11.0°		*2.6°*	*9.4°*	
SH 6 no. 20	–211/12/6	10 + x°	18.5°	74	*9.5°*	*13.6°*	35
SH 6 no. 20	–210/4/2	# 17°	18.5°	74	*32.1°*	*15.0°*	36
	–210/4/3+				*22.1°*	*10.3°*	
SH 2	–210/8/29+		27.75°		*0.2°*	*21.8°*	37
	–210/8/30	# 23° 10'	9.5°	38	*2.4°*	*8.1°*	
SH 2	–209/5/22	# 14°	15.75°	63	*11.1°*	*13.4°*	38
SH 2	–209/8/19	# 16°	18.5°	74	*0.25°*	*14.7°*	39
SH 2	–209/9/17	# 22° 30'	25.25°	101	*0.9°*	*20.9°*	40
	–209/9/18+				*0.0°*	*7.7°*	
SH 2	–208/9/6	26° ? /*16° ?	14.25°	57	*1.3°*	*12.2°*	41
SH 6 no. 22	–207/7/27	# 17° 30'	21.25°	85	*3.5°*	*16.3°*	42
SH 6 no. 22	–206/1/21	# 9°	9.75°	39	*12.3°*	*8.1°*	43
SH 2	–203/12/8	# 12°	12.75°	51	*4.9°*	*9.9°*	44
SH 2	–202/8/31	# 10° 20'	12.25°	49	*7.4°*	*10.6°*	45

source	date	observation / measured lag	computed lag	lag in min	*DAZ*	*h*	reference to number in figure text
SH 6 no. 27	−200/4/12	# 12° 30+[x]'	12.5°	50	*14.9°*	*11.3°*	46
SH 6 no. 27	−200/5/10	15° ?	25.75°	103	*30.1°*	*22.10°*	47
	−200/5/11	15° ?	17.5°	70	*21.2°*	*15.2°*	
	−200/5/12+		8.75°		*12.9°*	*7.9°*	
SH 2	−197/11/3	# 12°	16.5°	66	*1.6°*	*14.0°*	48
SH 2	−196/2/29	# 12°	13°	52	*18.1°*	*10.4°*	49
SH 6 no. 29	−195/5/16	# 15°	17.5°	70	*27.9°*	*15.4°*	50
	−195/5/17+		7.75°		*16.0°*	*7.6°*	
SH 2	−195/10/11	[x]+7°	17.25°	69	*2.6°*	*15.1°*	51
& SH 6 no. 29		& # 14°					
SH 2	−194/1/8	# 17°	16.5°	66	*12.0°*	*11.9°*	52
& SH 6 no. 29		& # 19°					
SH 6 no. 29	−194/2/6	# 17°	19.5°	78	*22.0°*	*13.8°*	53
SH 6 no. 31	−194/7/4	# 17°	22.75°	91	*11.8°*	*18.2°*	54
SH 6 no. 31/32	−194/8/3	# 13°	15.5°	62	*0.6°*	*12.2°*	55
SH 6 no. 31	−194/10/30	# 15°	17°	68	*0.3°*	*14.4°*	56
SH 6 no 31/32	−194/11/28+		25.5°		*7.0°*	*19.7°*	57
	−194/11/29	not seen *9°50'	10.25°		*3.1°*	*8.5°*	
SH 2	−193/5/25	# 12° 50'	15.5°	62	*18.2°*	*13.7°*	58
SH 2	−193/10/19	# 22°	27°	108	*0.4°*	*22.5°*	59
	−193/10/20+		9.75°		*1.0°*	*8.6°*	
SH 2	−192/1/16	# 10°	11.25°	45	*14.1°*	*8.0°*	60
SH 2	−191/7/31	# 13° 30'	16°	64	*0.5°*	*12.3°*	61
SH 2	−190/7/20	20° [md.?]	17.25°	69	*1.0°*	*13.1°*	62
SH 6 no. 35	−189/6/9+		21°		*9.0°*	*16.7°*	63
	−189/6/10	not seen *9°	10°		*0.7°*	*8.2°*	
SH 6 no. 35	−189/7/9	# 13°	17.5°	70	*2.6°*	*13.2°*	64
SH 6 no. 35	−189/9/6	# 16°	19.25°	77	*2.3°*	*16.1°*	65
SH 6 no. 35	−189/10/6	# 13°	14.75°	59	*4.4°*	*12.9°*	66
SH 6 no. 35	−189/11/4	# 22°	25°	100	*9.7°*	*20.1°*	67
	−189/11/5+		10.25°		*6.9°*	*8.6°*	
SH 6 no. 35	−189/12/4	# 13° 30'	19.25°	77	*12.8°*	*13.8°*	68
SH 6 no. 37	−187/8/14	# 20° 30'	24.5°	98	*6.1°*	*19.5°*	69
	−187/8/15+				*3.6°*	*7.4°*	

source	date	observation / measured lag	computed lag	lag in min	*DAZ*	*h*	reference to number in figure text
SH 2	−186/3/10	[measured]		63	*13.8°*	*13.4°*	70
SH 6 no. 39/40	−185/4/27	# 20° 30'	20.25°	81	*16.6°*	*17.6°*	71
	−185/4/28+		11.25°		*6.3°*	*9.9°*	
SH 6 no. 39/40	−185/5/27	# 12° / # 15°	15.75°	63	*10.3°*	*13.3°*	72
SH 6 no. 39	−185/6/25	# 18° 30'	22°	88	*12.4°*	*17.6°*	73
SH 6 no. 39	−185/9/21	# 21° ?	23.25°	93	*8.6°*	*19.7°*	74
SH 6 no. 39	−185/10/21	# 12° ?	14°	56	*6.9°*	*11.8°*	75
SH 2	−184/5/15	[measured]		71	*14.7°*	*15.3°*	76
SH 2	−183/7/31+				*16.6°*	*27.1°*	
	−183/8/1	measured ?		75	*11.4°*	*15.8°*	77
	−183/8/2+				*6.8°*	*3.9°*	
SH 2	−181/2/13	[measured]		60	*8.9°*	*12.2°*	78
SH 2	−179/3/21	# 18°	20.75°	83	*26.2°*	*16.5°*	79
	−179/3/22+				*14.6°*	*8.1°*	
SH 2	−172/4/2	# 13° 40'	13.25°	53	*22.2°*	*11.6°*	80
SH 6 no. 45	−171/8/18	# 12° ?	14.75°	59	*0.2°*	*12.0°*	81
SH 2	−170/11/4	# 13°	15.25°	61	*8.2°*	*12.6°*	82
SH 6 no. 46	−169/6/28	# 15°	19.5°	78	*5.8°*	*15.0°*	83
SH 6 no. 46	−169/7/27	# 20° 30'	24.75°	99	*5.7°*	*19.0°*	84
	−169/7/28+				*1.5°*	*6.4°*	
SH 2	−168/10/12	*21°	19.5°	78	*8.1°*	*16.3°*	85
SH 2	−168/12/10	# 16°	22°	88	*13.5°*	*15.6°*	86
SH 3	−162/9/7	14+x° 30+x'	16.25°	65	*5.4°*	*16.3°*	87
SH 6 no. 48/50	−161/8/27	# 18° 30'	21°	84	*5.8°*	*17.4°*	88
SH 6 no. 48/50	−161/9/26	# 16°	16.75°	67	*0.8°*	*14.3°*	89
SH 6 no. 48/50	−161/12/24	# 19° 30'	23°	92	*6.8°*	*17.5°*	90
	−161/12/25+				*0.4°*	*7.9°*	
SH 3	−158/8/24	# 22°	27°	108	*2.2°*	*21.1°*	91
SH 3	−155/3/25	# 13°	14.5°	58	*27.5°*	*11.6°*	92
	−155/3/26+				*16.7°*	*5.9°*	
SH 6 no.54	−154/4/14	not seen # 9° 10'	8.25°		*14.3°*	*7.8°*	93
SH 6 no. 54	−154/5/13	# 14°	15.75°	63	*16.8°*	*13.9°*	94
SH 6 no. 54	−154/6/12	# 15°	17.5°	70	*8.2°*	*14.0°*	95
SH 6 no. 54	−154/7/12	# 14°	16°	64	*1.2°*	*12.1°*	96
SH 6 no. 54	−154/10/9	# 9°	13.25°	53	*0.9°*	*11.7°*	97
SH 6 no. 54	−154/11/7	# 22°?	23.25°	93	*6.6°*	*18.8°*	98

source	date	observation / measured lag	computed lag	lag in min	*DAZ*	*h*	reference to number in figure text
	–154/11/8+				*4.1°*	*6.7°*	
SH 6 no. 54	–154/12/7	# 14°	15.75°	63	*10.6°*	*11.4°*	99
SH 3	–144/11/16	# 15°	19.25°	77	*2.2°*	*15.8°*	100
SH 3	–143/8/8	# [13°]	16.25°	65	*8.7°*	*13.8°*	101
SH 3	–143/9/7	not seen 10°20'	11.25°		*2.8°*	*10.0°*	102
SH 3	–141/10/14	15? °	25.75°	103	*0.5°*	*21.6°*	103
	–141/10/15+		10°		*3.2°*	*9.0°*	
SH 3	–141/11/13	# 16 °	19°	76	*0.2°*	*15.7°*	104
SH 3	–140/8/5	# 21°	25°	100	*6.9°*	*19.6°*	105
	–140/12/30+				*12.4°*	*17.9°*	106
SH 3	–140/12/31	not seen *9°30'	10°		*5.4°*	*7.8°*	
SH 3	–137/11/29	# [2]1°30'	23.5°	94	*8.7°*	*17.8°*	107
SH 3	–136/10/19	measured		73	*1.6°*	*15.4°*	108
SH 6 no. 69	–135/6/12	dense mist*9°50'	14.25°		*4.7°*	*11.4°*	109
SH 6 no. 69	–135/7/11	# 20° 30'	22.75°	91	*4.0°*	*17°*	110
SH 6 no. 69	–135/9/8	# 20°	19.75°	79	*0.27°*	*18.4°*	111
SH 6 no. 69	–135/11/7	# 15° 30'	14.75°	59	*5.8°.*	*12.0°*	112
SH 6 no. 69	–135/12/6	# 22°	24.5°	98	*14.0°*	*17.4°*	113
	–135/12/7+		9°		*8.8°*	*7.0°*	
SH 6 no. 69	–134/3/5	# 11°	10.5°	42	*18.4°*	*8.7°*	114
SH 6 no. 69	–134/4/3	# 16°	15°	60	*22.5°*	*13.2°*	115
SH 3	–134/9/27	# 16°	18.25°	73	*2.3°*	*15.6°*	116
SH 3	–134/10/27	# 11° 30'	14°	56	*5.6°*	*11.8°*	117
SH 3	–132/11/3	# 13° 30'	14.75°	59	*8.4°*	*12.0°*	118
SH 3	–129/6/6	# 16°	19°	76	*12.0°*	*15.4°*	119
SH 3	–129/8/4	# 24°	25.25°	101	*10.8°*	*20.3°*	120
	–129/8/5+		10.75°		*7.1°*	*9.2°*	
SH 6 no. 74	–127/6/13	# 8°	14.25°	57	*13.0°*	*11.9°*	121
SH 6 no. 74	–127/7/12	20 + [x]°	22.75°	91	*15.3°*	*18.6°*	122
	–127/7/13+		10.25°		*9.8°*	*8.8°*	
SH 6 no. 74	–127/8/11	# 14°?	20°	80	*10.8°*	*16.7°*	123
	–127/8/12+				*7.2°*	*6.5°*	
SH 6 no. 74	–127/10/10	14 + x°	14.25°	57	*4.7°*	*12.4°*	124
SH 3	–124/9/6	# 20°	23°	92	*4.9°*	*19.0°*	125
SH 3	–123/1/3	# 15°	17°	68	*5.8°*	*13.4°*	126

source	date	observation / measured lag	computed lag	lag in min	*DAZ*	*h*	reference to number in figure text
SH 3	−123/6/29	# 14° [?]	16.75°	67	*15.9°*	*14.1°*	127
SH 6 no. 77	−122/8/17	not seen *10°20'	13.25°		*2.0°*	*11.2°*	128
SH 6 no. 77	−122/9/15	# 16°	20°	80	*0.4°*	*16.7°*	129
SH 6 no. 77/ SH 3	−122/10/14	# 21°	26.5°	106	*0.7°*	*22.1°*	130
	−122/10/15+				*3.5°*	*8.3°*	
SH 6 no. 77	−122/11/13	# 13°	17°	68	*0.6°*	*14.2°*	131
SH 6 no. 77	−121/1/11	# 11°	14.25°	57	*8.6°*	*10.7°*	132
SH 6 no. 77	−121/3/10+		16.75°		*25.4°*	*13.2°*	133
	−121/3/11	not seen *18°	7.5°		*16.3°*	*6.6°*	
SH 3	−119/6/15	not seen *11°30'	13°		*8.8°*	*10.8°*	134
SH 6 no. 79	−118/5/5	# 15°	14°	56	*21.1°*	*12.6°*	135
SH 6 no. 79	−118/8/31+		31.75°		*0.2°*	*24.9°*	136
	−118/9/[1]	# 13°?	17.5°	70	*2.2°*	*14.2°*	
SH 6 no. 79	−118/12/28	# 20°	25.25°	101	*17.8°*	*16.8°*	137
SH 3	−111/5/18	# 13°	15.25°	61	*9.3°*	*12.9°*	138
SH 3	−110/9/3	not seen *9°	11.5°		*7.1°*	*10.2°*	139
SH 3	−109/11/20	# 13°	16.25°	65	*4.9°*	*13.0°*	140
SH 3	−108/5/14	# 16°	18.25°	73	*19.2°*	*15.9°*	141
SH 3	−108/8/11	# 11°	12.5°	50	*8.7°*	*10.8°*	142
SH 3	−108/11/8	# 14°	16.5°	66	*3.3°*	*13.7°*	143
SH 3	−107/11/27	measured		59	*1.1°*	*12.1°*	144
SH 3	−105/6/11	# 11°	12.75°	51	*17.3°*	*11.2°*	145
SH 3	−105/7/10	# 17°	22.75°	91	*17.3°*	*18.7°*	146
	−105/7/11+		8.5°		*9.12°*	*7.42°*	
SH 3	−105/9/7	# 17°	23.75°	95	*4.4°*	*19.7°*	147
SH 3	−99/10/29	# 22°	26.5°	106	*3.8°*	*21.9°*	148
	−99/10/30+				*2.3°*	*11.2°*	
SH 3	−89/1/18	# 18° 40' ?	15.5°	62	*5.8°*	*12.3°*	149
SH 5 no. 23	−88/9/28	# 16° 30'	17.5°	70	*3.3°*	*15.1°*	150
SH 5 no. 23	−88/10/27	# 23° ?	23.75°	95	*3.7°*	*21.2°*	151
	−88/10/28+				*0.3°*	*9.4°*	
SH 5 no. 23	−87/4/23	# 15° ?	13.5°	54	*20.4°*	*12.4°*	152
SH 5 no. 23	−87/5/23	[measu]red		39	*16.5°*	*8.9°*	153
SH 3	−87/9/17	# 21°	29.5°	118	*5.4°*	*24.5°*	154
	−87/9/18+				*1.4°*	*10.8°*	

source	date	observation / measured lag	computed lag	lag in min	*DAZ*	*h*	reference to number in figure text
SH 3	–86/3/13+		18.5°		*22.3°*	*15.0°*	
	–86/3/14	not seen *16°10'	9.25°		*13.5°*	*8.3°*	155
SH 3	–86/12/4	? x + 5°	30°	120	*6.3°*	*23.2°*	156
	–86/12/5	? x + 5°	13.25°	53	*0.2°*	*10.8°*	
SH 3	–84/3/20	14°	15.5°	62	*25.7°*	*12.5°*	157
	–84/3/21+				*15.5°*	*5.4°*	
SH 3	–81/4/17	15 + [x]°	12°	48	*23.3°*	*10.8°*	158
SH 3	–78/10/8	# 15° ?	18.25°	73	*3.9°*	*15.7°*	159
SH 3	–77/7/1	# 22°	25.5°	102	*7.4°*	*19.4°*	160
SH 3	–77/7/31	# 15°	18°	72	*2.1°*	*13.9°*	161
SH 3	–77/8/29	# 25°	24°	96	*3.0°*	*19.4°*	162
SH 3	–60/11/17	10 + [x]°	14°	56	*8.1°*	*11.0°*	163
SH 5 no. 31	–9/6/29	# 17° 30'	18°	72	*12.5°*	*14.7°*	164
	–9/6/30+		4°		*1.7°*	*3.0°*	
+	+	+	+		+	+	+
SH 1	–381/9/28	observed			*2.6°*	*16.1°*	165
SH 1	–374/2/16	observed			*23.4°*	*14.1°*	166
	–374/2/17+				*10.9°*	*7.1°*	
SH 1	–373/11/28	observed			*11.1°*	*13.8°*	167
	–373/11/29+	–			*6.0°*	*4.2°*	
SH 1	–332/9/26	?			*7.7°*	*19.2°*	168
	–332/9/27+	–			*4.7°*	*7.1°*	
SH 1	–288/10/19	observed			*0.8°*	*23.5°*	169
	–288/10/20+	–			*2.8°*	*10.4°*	
SH 1	–266/8/18	observed			*0.8°*	*17.1°*	170
	–266/8/19+				*3.5°*	*4.4°*	
SH 1	–266/11/15	observed			*2.2°*	*11.9°*	171
SH 2	–182/10/18	observed			*4.3°*	*17.2°*	172
	–182/10/19+				*0.3°*	*4.2°*	
SH 2	–170/6/9	observed			*7.2°*	*15°*	173
	–170/6/10+				*1.8°*	*5.2°*	

1) Checked: position of moon on 24th relative to α Virginis. The observer seems to have seen old crescent on the 26th, since he wrote in A, Obv. 15f: "The 26th, moonrise to sunrise: 23°; the moon ...". The remark, "in front of α Librae", which follows after a lacuna should refer to the moon, since Saturn and Venus which are mentioned on the 25th were behind α Librae; 2) The observer identified the 27th as old crescent day, since he noted the *KUR* lag, though without qualification "measured". Nevertheless, his remark, the "moon was 2/3 cubit behind Mercury, the moon being 3 cubits [low to the south ...]" shows that he has seen old crescent; 3) The observer identified the 27th as old crescent day, since he noted the *KUR* lag, though without the qualification "measured". His remark, "the

moon was 3 cubits above Venus", shows that he has seen the moon after the rise of Venus. There was a very sizeable chance on the succeeding morning of sighting the crescent;4) Checked: position of moon on 20th relative to β Scorpii and position of Mercury relative to β Capricorni on 23rd; 5) Checked: 1st day and positions of moon on 5th relative to β Librae, on the 25th relative to ε Leonis and on the 27th relative to Mercury. On the 27th = –324/8/29 the observer noted "moonrise to sunrise: 29°; I did not watch. The moon stood 3 cubits in front of Mercury to the west [...]". The remark "I did not watch" presumably refers to moonrise and/or sunrise; the observer has seen Mercury and the moon at some moment and thus he has seen the moon as old crescent. – On the succeeding morning the moon stood below the lower border of the seasonal uncertainty zone; 6) Checked: position of moon on 11th (occultation of Mars) and *KUR* lag on 27th. Old crescent was sighted on the 27th = –284/11/4, although the moon stood just on the lower border of the seasonal uncertainty zone; 7) Checked: position of moon on 26th relative to θ Ophiuchi and *KUR* lag on 27th. Old crescent was sighted on the 27th = –278/12/27 under a lag of 105 m; at sunrise of the 28th the moon stood below the seasonal visibility zone; 8) Checked: positions of moon on 22nd relative to α Leonis, on 26th relative to α Virginis and on 27th (old crescent day) relative to α Librae. There was a sizeable chance of sighting the crescent on the 28th; 9) Checked: position of moon on 14th: "2 2/3 cubits [behind] α Scorpii."; 10) Checked: 1st day and positions of moon on 3rd relative to   Scorpii and on 27th relative to α Virginis and Mercury; on the 27th = –255/10/14 the moon stood "in front of" Mercury as reported, but rather "behind" and not "in front of"   Virginis. Presumably "[... moonrise to sunrise: nn°] measured" refers to the 27th, although there was a slight chance of sighting the moon on the 28th; 11) Checked: positions of moon on 11th relative to β Arietis and on [28th] relative to Saturn. At sunrise of the 29th = –253/11/23 the moon stood below the horizon; 12) Checked: lag of 6° on 15th between sunrise and moonset, corresponding to astronomically computed 4.15°; 13) Checked: 1st day and lag on 14th. – The observer reported old crescent on the 27th = –251/10/30; the difference between astronomically computed lag of 27° and the reported *KUR* lag of 20° amounts to an acceptable 7°. There was a sizeable chance of sighting the crescent on the succeeding morning; 14) Checked: reported *KUR* lag of 18° on 27th as reported old crescent day.; 15) Checked: measured lags on 13th and 14th. The observer predicted a *KUR* lag of 9° for the 28th = –250/11/19 which was more or less identical with astronomically computed 10°; nevertheless the moon stood at sunrise below the seasonal uncertainty zone and will thus have been unobservable; 16) Checked: position of moon on 24th relative to θ Ophiuchi. – The observer reported that he did not see the moon on the 27th = –248/1/6, although he "watched"; there had been a medium chance of sighting the crescent. Note, that the observer saw the first appearance of Saturn on the same morning, Saturn being "2/3 cubits behind Mars to the east". The moon rose before the onset of civil dawn, about half an hour after Saturn, and about 20 minutes after Mars; 17) Checked: measured lag of 8° between sunrise and moonset on the 15th, corresponding to astronomically computed 7.5°. The measured *KUR* lag of 10° on the 28th corresponds to astronomically computed 9.75°. The measured lags confirm that old crescent was indeed sighted on the 28th = –248/10/27, though on the lower border of the seasonal uncertainty zone; 18) Checked: measured lag of 7°, sunrise to moonset on 13th, corresponding to astronomically computed 4.25°. – Note that the measured lags on the 14th and 13th (moonrise to sunset) do not correspond well with astronomical computation; 19) Checked: position of moon on 25th relative to Venus, the latter being below the moon, not vice versa as reported. Old crescent was observed on the 27th = –245/4/30 as the reported *KUR* lag shows; there was a slight chance of sighting the crescent on the succeeding morning' 20) Checked: position of moon on 24th relative to   Virginis. – The observer expected old crescent on the 28th = –234/11/22; in spite of a sizeable chance he missed it; 21) Checked: 1st day and lag between moonrise and sunrise on 15th; 22) Checked: measured lag of 9° 30' on 15th, corresponding to astronomically computed 7.75°. – Note that in SH 6 no. 10 the numbering of the months does not refer to the year of observation, but to the Goal Year instead, see Brack-Bernsen 1999, 29-37; 23) See also n. 22. - Checked: 1st day and measured lag on 14th; 24) See also n. 22. – Checked: 1st day and measured lags on 13th and 14th. The 27th = –233 9/10 was reported as old crescent day; there was a medium chance of sighting the crescent on the succeeding morning; 25) See also n. 22. – Checked: 1st day and measured lags on 13th and 14th. The observer expected old crescent on the morning of the 27th = –232/3/8; he had predicted a *KUR* lag of 21° 10', but he did not see the moon which stood below the seasonal uncertainty zone. The predicted lag of 21° 10' evidently refers to the 26th = –232/3/7, whereas the astronomically computed lag on the 27th = –323/3/8 would have amounted to 6.75° or 27 minutes; 26) Checked: positions of moon on 23rd relative to α Virginis and on 27th relative to α Scorpii. – The *KUR* lag seems to have been predicted; regardless the crescent was sighted on the 28th = –222/11/29 as shown by the remark "it was low"; 27) Checked: positions of moon on 26th relative to ε Leonis and on 27th relative to α Leonis and Mars. – The observer sighted old crescent on the 28th = –225/8/16 in spite of only a very slight chance and measured the *KUR* lag despite "mist"; 28) Checked: 1st day; 29) Checked: 1st day. – Old crescent was reported for the 27th = –211/6/12; there was a medium chance of sighting the crescent on the succeeding morning; 30) Checked: 1st day; 31) Checked: 1st day and measured lags on 13th and 14th (evening and morning). – Note the difference between 21° as reported *KUR* lag and 29.75° as astronomically computed lag on the 27th; 32) Checked: lag sunrise to moonset on 15th. – On the 28th = –211/9/9 the observer did not see the crescent although it stood above the seasonal uncertainty zone, because of "mist"; 33) Checked: 1st day; 34) Checked: 1st day. – Old crescent was reported on 27th = –211/11/6; there was a slight chance of sighting the moon on the succeeding morning; 35) Checked: 1st day; 36) The observer identified old crescent day as 26th = –210/4/2; the measured lag of 17° agrees with astronomically computed 18.75°. Actually old crescent ought to have been visible on –210/4/3 under astronomically unexceptionable circumstances. There are only five other reports for the month in question; in each case the observer reported "clouds, I did not watch". Thus it is probable that the moon was obscured by clouds on –210/4/3; 37) Checked: positions of the moon on –210/8/20 = 18th relative to Saturn and on –210/8/27 = 25th relative to δ Cancri, and of Venus on 210/8/29 = 27th relative to α Leonis. Under these premises old crescent occurred on the 27th instead of the reported 28th. On the 27th = –210/8/29 the astronomically computed lag amounted to 111 minutes = 27.75°, corresponding to the measured *KUR* lag of 23° 10' which is mistakenly reported for the 28th = – 210/8/30. On the latter day the moon stood below the seasonal uncertainty zone; lag would have amounted to 9.5° (time degrees) or 38 minutes; 38) Checked: position of moon on 24th relative to β Arietis; 39) Checked: position of moon on 16th relative to η Piscium and in the "night of the 18th (error for 27th), last part of the night", relative to α Leonis; 40) Checked: position of moon on 24th relative to ε Leonis; 41) Checked: position of moon on 25th relative to α Leonis. – Hunger transcribes "⌜2⌝ 6?" for the *KUR* lag, but the astronomically computed lag amounted only to 14.25°; a difference of –11.75° between astronomically computed and measured *KUR* lag is otherwise not attested. Therefore one could restore "⌜1⌝ 6?" instead of "⌜2⌝ 6?". It is also possible that the observer measured a *KUR* lag of *16° and the scribe wrote *26 by mistake; 42) Checked: 1st day; 43) Checked: 1st day; 44) Checked: position of moon on 25th relative to α Scorpii; 45) Checked: position of moon on 26th relative to ϱ Leonis; 46) Checked: 1st day; 47) Only two partially preserved entries can be used to determine the details of [month I] in SE [111]. The measured lag of 15° between sunset and moonrise on the [1]6th ought to refer to the evening of –200/4/29. A measured *KUR* lag of 15° is reported for the ⌜26th?⌝ = –200/5/10. There are two problems with the ⌜26th?⌝ : old crescent would have been observable quite conveniently on the 27th, if the weather was unexceptional; furthermore, on the 26th the astronomically computed lag amounted to 25.75° resulting in a otherwiese unattested difference of 10.75° to the reported 15°. The problems can be solved by reading ⌜27th?⌝ .instead of ⌜26th?⌝ which would imply a difference in lag of 2.5° to the reported 15°; 48) Checked: 1st day and position of moon on 26th relative to α Librae, Mercury and Saturn; 49) Checked: 1st day and position of moon on 25th relative to γ Capricorni; 50) Checked: lag on 14th. The observer reported old crescent on the 26th = –195/5/16; there was a slight chance to see the moon on the succeeding morning; 51) Checked: measured lags sunset to moonrise on the evening of the 15th (SH 2) and sunrise to moonset on the morning of the 15th (SH 6); 52) Checked: positions of moon on 24th relative to θ Ophiuchi and on 27th relative to Mercury (SH 2); 53) Checked: 1st day; 54) Checked: measured lags on 14th and 15th; 55) Checked: 1st day and lags on 14th and 15th (evenings and mornings); 56) The text preserves only the concluding lines for month VII. The latter is identifiable since month VIII follows without a break. Furthermore, the measured lag of 15° corresponds closely to astronomically computed 17° on –194/10/30 as old crescent day of month VII; 57) Checked: 1st day. – The observer expected old crescent on the 28th = –194/11/29 when the moon stood just below the seasonal uncertainty zone; old crescent would have occurred on the 27th; 58) Checked: 1st day and position of moon on 2nd relative to Jupiter and Venus; 59) Checked: positions of moon on 23rd relative to α Leonis and on 27th relative to α Virginis; 60) Checked: position of moon on 16th relative to α Leonis; 61) Checked: position of moon on [25th] relative to γ Geminorum; 62) On –190/7/16 = 24th the moon was below β Tauri and on –190/7/19 = 27th behind β Geminorum. The reported *KUR* lag of "20° [....]", be it measured or not, corresponds to astronomically computed 17.25° on the 28th = -190/7/20. Since on the 27th the *KUR* lag would have amounted to ˜30°, it is preferable to identify the 28th rather than the 27th as old crescent day; 63) Checked: 1st day. – The observer expected old crescent on the 28th = –189/6/10. He explained his failure to see old crescent by "mist"; actually the moon stood below the seasonal uncertainty zone and old crescent had occurred on the 27th; 64) Checked: 1st day; 65) Checked: lag moonrise to sunset on 14th; 66) Checked: 1st day; 67) Checked: 1st day. – Old crescent occurred on the 26th = –189/11/4 as reported; there was a slight chance of sighting the moon on the succeeding morning; 68) Checked: lags on 12th, 13th (both precise) and 27th (less precise, according to astronomical computation); 69) Checked: 1st day. – Old crescent occurred on the 27th = –187/8/14; the moon stood below the sesonal uncertainty zone on the succeeding morning; 70) Checked: lag moonrise to sunset on 13th; position of moon on 23rd relative to Jupiter; 71) Checked: 1st day. – The observer reported the 27th = –185/4/27 as old crescent day; there was a medium chance of sighting the moon on the succeeding morning; 72) Checked: 1st day. – *KUR* lag is reported as 12° in SH 6 no. 39, in SH 6 no. 40 as 15° which corresponds quite well to astronomically computed 15.75°; 73) Checked: 1st day; 74) Checked: measured lag on 16th; 75) Checked: 1st day; 76) Checked: positions of moon on 26th relative to α Arietis and on 27th relative to Venus; 77) Checked: lag moonrise to sunset on 13th and lag sunset to moonrise on the 14th. Probably muš/measured in C, Rev. 8 refers to the *KUR* lag; since a note about the northwind on the 28th follows, old crescent day seems to have been on the 27th = –183/8/1. At sunrise of the 28th the moon stood far below the seasonal uncertainty zone; on the 26th any observer ought to have judged that the moon was too high for being old crescent. Thus it is probable that old crescent was reported on the 27th; 78) Checked: position of moon on 27th relative to Mercury and <rising> of Mercury <to sunrise> on 28th; 79) Checked: position of moon on 24th relative to δ Capricorni. – Old crescent was reported on the 26th = –179/3/21. There was a medium chance of sighting the crescent on the 27th, but it rained in "the last part of the night"; 80) Checked: positions of moon on 18th relative to α Scorpii and of Venus on 30th relative to β Tauri; 81) Checked: 1st day and lag sunrise to moonset on 15th; 82) Checked: position of moon on 27th relative to β Librae and Mercury; 83) Checked: 1st day; lags on 15th (moonset to sunrise) and 16th (sunrise to moonset); 84) Checked: 1st day. – At sunrise of –169/7/28 the moon stood far below the seasonal uncertainty zone; 85) Checked: positions of moon on 25th relative to β Virginis and on 27th as old crescent day relative to α Virginis.; 86) Checked: positions of moon on 25th relative to α Scorpii and of Venus on 27th relative to γ Capricorni; 87) Checked: 1st day and positions of moon on 19th relative to α Tauri, on 23rd relative to δ Cancri and on 26th relative to ϱ [Leo]nis. Last visibility was reported for day "⌜26⌝ + [x]" with a *KUR* lag of 14+ [x]° 30 + [x]'. Since the astronomically computed lag amounted to 16.25° on the 27th = –162/9/7, last visibility occurred on that day; 88) Checked: 1st day. Old crescent occurred on the 27th = –161/8/27 as reported by the observer of SH 6 no. 48; the 27th is confirmed as old crescent day by the measured lags on the 15th and 16th in SH 6 no. 48. The observer or more likely the scribe of SH 6 no. 50 referred old crescent in error to the "28th" and he reported for that day the *KUR* lag of the day before; 89) Checked: 1st day; 90) Checked: 1st day. – Old crescent occurred as reported on the 27th = –161/12/24; at sunrise of the succeeding morning the moon stood below the seasonal uncertainty zone; 91) Checked: positions of moon relative to ε Leonis on 26th and to α Leonis on 27th; 92) Checked: position of moon on 8th relative to α Geminorum and on 11th relative to α Leonis and Jupiter, resulting in 26th = –155 March 24/25. Old crescent was reported for the morning of the 26th = –155/3/25; 93) Checked: lags on 15th and 16th. The observer expected old crescent on the 27th and predicted a lag of 9° 10' which compares quite well with astronomically computed 8.25°. There was a slight chance of sighting the crescent, but the observer did not sight it; 94) Checked: 1st day; 95) Checked: 1st day.; 96) Checked: 1st day; 97) Checked: 1st day; 98) Checked: 1st day. – Old crescent occurred on the 27th = –154/11/7 as reported; at sunrise of the succeeding morning the moon stood below the seasonal uncertainty zone; 99) Checked: 1st day; 100) Checked: 1st day. – The observer reported correctly that on the 26th the moon was 3 cubits below α Librae; he reported incorrectly that the moon was 1 cubit above Mercury which was actually the case on the 27th when he observed old crescent; 101) Checked: positions of moon on 21st relative to η Tauri and on 27th relative to ε Leonis; 102) Checked: 1st day. - On the 27th the observer reported: "dense mist, when I watched I did not see it." He had predicted a *KUR* lag of 10° 20' which came very close to astronomically computed 11.25°. He had seen Venus rising more than 2 hours before moonrise and measured her distance to α Leonis; the mist will have developed some time later; 103) Checked: positions of moon on 26th relative to γ Virginis and on 27th relative to α Virginis. Old crescent was reported for the 27th = –141/10/14; there was a slight chance of sighting the moon on the succeeding morning. Hunger transcribes *KUR* lag on old crescent day as ⌜15?⌝; the astronomically computed lag amounted to 25.75°. An error of 12.75° would be a remarkable outlier; presumably ⌜25?⌝ is to be restored which would reduce the difference to 2.75°; 104) Checked: 1st day and positions of moon on 2nd relative to θ Ophiuchi, on 5th relative to β Capricorni and on 24th relative to α Virginis; 105) Checked: 1st day and position of the moon on 24th relative ffi Geminorum and relative to Venus on 27th; 106) Checked: 1st day and positions of moon relative to Mercury, Jupiter and Mars on 27th. The observer expected to see old crescent on the 28th = –140/12/31 and predicted a *KUR* lag of 9° 30', but "when I watched I did not see it". Correspondingly at sunrise of the 28th the moon stood below the seasonal uncertainty zone and ought to have been invisible; thus the observer has seen old crescent on the 27th = –140/12/30; 107) Checked: positions of moon on 19th relative to α Leonis and on the 25th relative to α Librae; 108) Checked: 1st day and positions of moon on 23rd relative to θ Leonis and on 26th relative to α Virginis. On the 23rd the moon was indeed 1 cubit behind θ Leonis as reported, but far less than "4 cubits low to the south". The day of old crescent is in a lacuna; only *KUR* itself is partially preserved and "it was bright, measured". Old crescent day can be restored with certainty as 27th = –136/10/19. The 28th is not possible, since old crescent day is referred to between reports about the position of the moon on the 26th and on the rise of Mercury on the 27th; 109) The observer was unable to see the moon because of "dense mist". The latter is comparable to rain or clouds, a category of obstacles which remains unconsidered in the present study as far as conditions of observation are concerned. Since the observer had predicted a *KUR* lag, the date could be used for analysis of lag values; 110) Checked: 1st day; 111) Checked: lags on 15th: sunset to moonrise and sunrise to moonset; 112) Checked: 1st day; 113) Checked: 1st day; 114) Checked: 1st day; 115) Checked: lags on 13th and 14th (moonrise to sunset); 116) For the date in general see SH 3, 193. - Old crescent on the 27th = –134/9/27 can be identified via the reported *KUR* lag of 16°, corresponding to astronomically computed 18.25°; 117) Checked: position of moon on 24th relative to γ Virginis; 118) Checked: 1st day and position of moon on 19th relative to β Geminorum; 119) Checked: positions of moon on 23rd relative to β Arietis and on 25th relative to η Tauris, also of Venus on 25th (first part of the night), relative to ε Leonis; 120) Checked: 1st day and position of moon on 18th relative to α Arietis. The observer reported old crescent for the 26th = –129/8/4; at sunrise of the succeeding morning there was a very slight chance of sighting the moon; 121) Checked: 1st day; 122) Checked: 1st day. – The observer reported old crescent on the 26th = –127/7/12; at sunrise of the succeeding morning the moon stood just on the lower border of the seasonal uncertainty zone. 123) The observer reported the 26th = –127/8/11 as old crescent day. Although there are no other dates of the month which can be checked, the reported old crescent day should be accepted, since the moon stood far below the seasonal uncertainty zone at sunrise of the succeeding morning; 124) The entries for month VI are mostly lost in lacunae. The observer reported the 28th = –127/10/10 as old crescent day; the report is to be accepted, since the measured *KUR* lag of 14+x° corresponds to the astronomically computed lag of 14.25°; 125) Checked: position of moon on 20th relative to α Tauri; 126) Checked: 1st day and position of moon relative to Venus on 25th; 127) Checked: 1st day and position of moon relative to Mercury on 27th = –123/6/29 as old crescent day. After the figure for the *KUR* lag, there is a lacuna in which the remark "measured" may or may not have disappeared; 128) Checked: lag on 15th. – The observer expected old crescent on the 28th = –122/8/17, but did not see it in spite of a sizeable chance; 129) Checked: 1st day; 130) Checked: 1st day and positions of Jupiter on 26th and of moon relative to α Virginis on 27th = –122/10/14 as old crescent day; 131) Checked: 1st day; 132) Checked: 1st day; 133) Checked: 1st day. – The observer expected old crescent on the 27th = –121/3/11; he did not see the moon which stood below the seasonal uncertainty zone. For the 27th he had predicted a *KUR* lag of 18°; the difference between astronomically computed lag of 7.5° and the *KUR* lag in the text amounts to –10.5°. Since an error of –10.5° is otherwise not attested within the textually certain data, I presume that there is a mistake. Perhaps the scribe wrote (10+8)° instead of 8° for the predicted *KUR* lag. Another possibility is that the *KUR* lag of 18° actually refers to the 26th, when the astronomically computed lag amounted to 16.75°; there might have been a similar situation in the case of no. 155 below; 134) Checked: positions of moon on 22nd relative to β Arietis and on 26th relative to Saturn. The observer expected old crescent on the 27th, though "when I watched I did not see it"; thus he missed the crescent in spite of a sizeable chance; 135) Checked: 1st day; 136) Checked: 1st day. – The number of old crescent day is only partially preserved. The reported *KUR* lag of "13° ?" corresponds to astronomically computed 17.5° on the 27th = –118/9/1; 137) Checked: lag sunset to moonrise on 14th; 138) Checked: position of moon relative to Venus on [26th]; 139) Checked: position of moon relative to η Geminorum on 20th. The observer expected old crescent on the 27th and predicted a *KUR* lag of 9°, though "when I watched I did not see it"; he missed the crescent in spite of a medium chance; 140) Checked: position of moon relative to α Virginis on 23rd; 141) Checked: position of moon relative to δ Capricorni on 21st; 142) According to the observer the moon was 6 cubits below ε Leonis on the [2]6th and old crescent occurred on the 27th. By contrast, according to the positions of the moon on the 8th relative to β Scorpii and on the 21st relative to α Tauri, the moon's position of 6 cubits below ε Leonis ought to refer to the 25th and not to the 26th. The reported *KUR* lag of 11° corresponds to astronomically computed 12.5° on the 26th = –108/8/11; thus old crescent was observed on the 26th and not as reported on the 27th; 143) Checked: position of moon relative to γ Virginis on [23rd]; 144) Checked: position of moon on 23rd relative to [α] Virginis. – The day number for old crescent is in a lacuna and also the figure for the *KUR* lag; the remark "measured" is preserved. Since an entry on the 28th follows, old crescent ought to have been reported either for the 27th or 28th. Old crescent occurred on the 27th = –108/11/8, since on the 28th the moon stood barely above the horizon at sunrise and a measuring of the *KUR* lag would have been impossible; 145) Checked: 1st day and position of moon relative to η Tauri and Mars on 25th. Old crescent occurred as reported on the 27th = –105/6/11. The report is preserved in two tablets; the *KUR* lag is once reported as 12° without remark "measured" and once as "11°, measured"; 146) Checked: 1st day and position of moon relative to Venus and α Leonis on the 3rd, relative to Mars and β Tauri on the 25th. Old crescent occurred as reported on the 27th = –105/7/10 for which day one of the observers reported a *KUR* lag of 17°, corresponding somehow to astronomically computed 22.75° on the same day, but not to 8.5° on the succeeding day; 147) Checked: positions of moon on 22nd relative to γ Geminorum, on 26th relative to ϱ Leonis and on 27th relative to θ Leonis; 148) The tablet preserves barely more than the lunar entries for the last days of the month. Day 26 as reported old crescent day can be identified with –99/10/29, because the measured *KUR* lag of 22° corresponds to astronomically computed 26.5°, whereas on the succeeding day the astronomically computed lag amounted to 13° only. On the 27th = –99/10/30 there was a sizeable chance of sighting the crescent, but it was not seen. – According to SH 3, 406 it is possible that the tablet is from Uruk; if so, the difference to observation in Babylon is negligible; 149) Checked: positions of moon on 23rd relative to α Scorpii and on 27th relative to Mercury; 150) Checked: 1st day. Old crescent occurred as reported on the 27th = –88/9/28; 151) Checked: 1st day. Old crescent occurred as reported on the 27th = –88/10/27; there was a slight to medium chance of sighting the crescent on the succeeding morning; 152) Checked: 1st day. Old crescent occurred as reported on the 27th = –88/4/23; 153) Checked: 1st day. Old crescent occurred as reported on the 27th = – 87/5/23; the *KUR* lag was measured, the figure is in a lacuna; 154) Checked: position of Venus relative to α Leonis on [14th]; positions of moon on 17th relative to α Arietis, on 25th relative to α Leonis and on 27th relative to β Virginis. Old crescent was reported for the 27th; there was a medium to sizeable chance of sighting the crescent on the 28th. There is a remarkable difference of 8.5° between astronomically computed lag and reported *KUR* lag on the 27th; 155) Checked: positions of moon on 6th relative to α Tauris and on 18th relative to α Scorpii. – The observer expected old crescent on the 27th = –86/3/14 and predicted a *KUR* lag of 16° 10', but "when I watched I did not see it". The difference between astronomically computed lag and predicted *KUR* lag amounted to –6.9°. Otherwise the observers made no comparable mistakes when they measured or predicted *KUR* lag. Perhaps 16°10' actually refers to the *KUR* lag on the 26th when the astronomically computed lag amounted to 18.5°. The relevant point is that the observer did not see the moon on the 27th in spite of a medium chance and thus old crescent has been observed on the 26th; 156) Checked: 1st day and positions of moon on 19th relative to ε Leonis and on 22nd relative to Mars. Old crescent was reported either for the 27th or 28th; the day is in a lacuna, a report on the first appearance of Venus on the 28th follows. The crescent stood high above the seasonal uncertainty zone on the 27th; on the 28th the moon stood just below the upper border of the seasonal uncertainty zone. Since old crescent is described as being "bright", the report may refer to the 27th, rather than to the 28th. The astronomically computed lag amounted to 30° on the 27th and to 13.25° on the 28th. It seems admissible to restore the partially preserved *KUR* lag of "[x] + 5°" to *25°, if it refers to the 27th or to *15°, if it refers to the 28th; 157) Checked: position of moon relative to ζ Tauri on 6th; 158) Checked: position of moon on 10th relative to θ Leonis; 159) Checked: positions of moon on 20th relative to α Geminorum and on 27th relative to α Virginis. – It is possible that the remark "measured" stood in a lacuna, after the figure for the *KUR* lag; in any case, the observer saw the moon on the 27th as old crescent day "1 cubit behind α Virginis"; 160) Checked: 1st day and position of moon on 27th relative to Venus and Mercury; 161) Checked: position of moon relative to Jupiter on 19th and on 28th relative to Venus; 162) Checked: 1st day and positions of moon relative to γ Cancri on 25th and on 26th relative to α Leonis; 163) Checked: position of moon relative to α Geminorum on 17th; 164) For the dating of the solar eclipse which the tablet reports on –9/6/30, see Steele 2001, 208-211 and Steele 2009, 37-39. The eclipse reportedly took place on the 28th of a lunar month, since the preceding new crescent ought to have been observed on –9/6/2. On the evening of –9/6/1 the moon was not observable, since it stood far below the seasonal uncertainty zone. On the other hand, the text reports a *KUR* lag of "17° 30' measured" on the 28th which does not correspond to astronomically computed 16 minutes = 4° on the morning of –9/6/30. Furthermore, the moon would not have reached the sun to produce an eclipse on the 28th, if it would have had to travel a distance corresponding to 17° 30' (time degrees) after sunrise. It appears that the entry about the *KUR* lag was meant to refer to the morning of the 27th (= –9/6/29) when the astronomically computed lag amounted indeed to about 72 minutes = 18°; 165) The observer wrote in No. -381, A Rev. 17: "Night of the 28th, last part of the night, the moon was ...[...γ Virginis ....]". Thus the moon seems to have been sighted on the 28th = –381/9/28; this day has to be identified as old crescent day since on the succeeding morning the moon stood far below the seasonal uncertainty zone. The 1st day of this month figures in Stern's list among early new crescents; see also Appendix 2; 166) Checked: 1st day and position of moon relative to Mars and Venus on 27th. Since at sunrise of the 28th the moon stood below the seasonal uncertainty zone the 27th = –374/2/16 is to be identified as old crescent day; 167) Checked: position of moon on 24th. According to -373 A Rev. 12 the moon was observed on the 27th = –373/11/28 as implied by the observer's remark "the moon was behind ...". Since the moon stood far below the seasonal uncertainty zone at sunrise of the 28th, the 27th was old crescent day; 168) According to astronomical computation the 26th = –332/9/26 coincided with old crescent day; for that day the position of the moon relative to α Virginis is reported, but there is no remark indicating old crescent either on that day or on the succeeding days. At sunrise of –332/9/27 the moon stood far below the seasonal uncertainty zone; 169) Checked: positions of moon relative to Saturn on 25th and to *Mercury on 27th. The text reports for the 27th "the moon stood 2 cubits in front of Jupiter to the west". The scribe has written Jupiter instead of Mercury; actually the moon stood about 50 cubits east of Jupiter. In the succeeding line the scribe recorded correctly that during the month Jupiter was in Cancer and Mercury in Libra. – For the 27th = –288/10/19 as old crescent day a predicted *KUR* lag of 24° is reported, corresponding to an astronomically computed lag of 28°; 170) Checked: lunar positions relative to β Tauri on 23rd and relative to α Leonis on –266/8/18 as 28th day and reported old crescent day. Since the moon stood far below the seasonal uncertainty zone on the 29th, old crescent was seen on the 28th; 171) Checked: 1st day and lunar position relative to β/δ Scorpii and Venus on the 27th = –266/11/15. Thus old crescent was sighted, since according to astronomical computation old crescent day fell on –266/11/15; on the succeeding morning at sunrise the moon stood at the horizon; 172) Checked: positions of moon on [26th] relative to α Virginis and on 27th = –182/10/18 relative to Mercury. The 27th is to be identified as old crescent day; the crescent was observed together with Mercury; on the 28th = –182/10/19 the moon stood far below the seasonal uncertainty zone; 173) Checked: lag moonset to sunrise on 15th and position of moon relative to Tauri and Jupiter on –170/6/9 = 28th as old crescent day; on the succeeding morning at sunrise the moon stood far below the seasonal uncertainty zone.

Appendix 2. Comments on observed new crescents in Astronomical Diaries 1-3 and 5-6

The Babylonian calendar day lasted from sunset to sunset. The first day of the Babylonian lunar month began at sunset just after or before sighting of new crescent (Stern, 2008: 19). Thus the Babylonian observer had a marked interest in new crescent. Two-thirds of the new crescents listed below were identified by Fatoohi *et al.* (1999: 59) on the basis of SH 1-3. Stern controlled the latter list and added the new crescents in SH 5-6 (Stern, 2008). Victor Reijs controlled both lists and noted as errors in Stern's list: –206/1/13, -194/6/6 and -145/2/8. http://www.iol.ie/˜geniet/eng/compareBabylonian.htm; bullet 3).

Here I follow Stern and Fatoohi *el al.* here, citing only the dates of the new crescents without numbering them. Although Stern's identifications are trustworthy, I nevertheless looked at each report in the Diaries. The table below lists the reported date of a new crescent and the figures which are relevant for the azimuth-altitude diagram, viz. DAZ and lunar altitude h. Dates which are not reported and which I calculated to elucidate a reported date are marked with +; a line is skipped after such pairs of dates.

Stern interpreted certain cases as early or late. He does not specify his visibility criteria, simply stating that he "used a number of computer programs and visibility criteria for the calculation of astronomical data and new moon visibility" (Stern, 2008: 39). My descriptions of sighting probabilities as "slight", "medium" or "sizeable" refer to the seasonal visibility values in table 7 of the main article. I use "predicted" below for dates calculated by the Babylonian observer, and "astronomically computed" for those calculated with astronomical software.

source	date of new crescent	DAZ	h	Stern	lag in minutes	comments
SH 1	–567/4/22	6.8°	13.7°		64	1
SH 1	–567/5/21+	5.6°	9.6°			
	–567/5/22	9.7°	23.0°		117	2
SH 1	–567/6/20	9.9°	17.9°		91	3
SH 1	–566/2/12	9.3°	13.8°		69	
SH 1	–566/3/13+	6.5°	9.1°			4
	–566/3/14	8.6°	21.7°		104	
SH 1	–463/9/8	23.3°	11.2°		50	5
SH 5 no. 56	–461/3/22	0.5°	10.6°			6 not seen
	–461/3/23+	4.2°	24.3°			
SH 1	–418/10/19	8.1°	9.8°			
	–418/10/20+	18.0°	15.2°	late	76	7
SH 1	–-381/5/6	7.3°	15.7°		76	
SH 1	–381/7/4	12.1°	18.9°		95	

source	date of new crescent	DAZ	h	Stern	lag in minutes	comments
SH 1	−381/8/31	5.1°	9.7°	early	43	8
SH 1	−378/10/27	20.8°	11.3°		59	
SH 1	−378/11/25	14.8°	7.7°		42	
SH 1	−374/1/20	6.3°	11.5°		59	
SH 1	−374/3/20	1.6°	17.1°		80	
SH 1	−372/2/27	2.2°	12.2°		57	
SH 1	−372/3/28	1.8°	21.0°		101	
SH 1	−372/7/23	14.9°	9.6°		103	
SH 1	−370/8/1	20.5°	10.8°		48	
SH 1	−370/10/28	14.6°	10.5°		52	
SH 1	−368/7/10	16.5°	11.0°		50	
SH 1	−366/5/19	8.78°	9.45°			9 not seen
	−366/5/20+	13.4°	20.8°			
SH 1	−366/6/18	12.8°	11.8°		56	
SH 1	−366/8/17	18.8°	15.0°		69	
SH 1	−346/12/2	11.7°	15.4°		83	
SH 1	−346/12/31+	4.8°	9.4°			
	−345/1/1	12.2°	18.2°	late	101	10
SH 1	−345/3/1	8.7°	19.4°		94	
SH 1	−342/12/17	16.2°	12.3°		72	
SH 1	−333/6/14	10.4°	12.0°		58	
SH 1	−333/8/12+	16.8°	6.9°			
	−333/8/13	27.7°	14.6°		67	
SH 1	−332/9/29	21.1°	12.7°		60	
SH 5 no. 12	−331/2/23	1.8°	21.3°		102	
SH 1	−328/10/13+	6.8°	10.3°			
	−328/10/14	16.1°	16.7°	late	81	11
SH 1	−328/12/12	7.8°	13.6°		73	
SH 5 no. 13	−326/2/28	6.8°	11.5°		53	
SH 1	−324/4/6	6.4°	18.4°		87	
SH 1	−324/7/3	4.3°	10.8°		54	
SH 1	−324/8/2	10.4°	15.5°		74	
SH 1	−324/9/30	15.7°	13.8°		64	
SH 5 no. 36	−322/7/11	4.9°	13.0°		66	
SH 5 no. 36	−322/8/10	10.4°	13.6°		64	
SH 1	−322/12/7	21.0°	14.0°		87	
SH 1	−321/1/5	14.2°	13.5°		77	
SH 1	−321/2/3	8.1°	11.5°		58	
SH 1	−321/4/3	1.4°	17.0°		81	
SH 5 no. 36	−321/6/30	3.5°	12.9°		68	

source	date of new crescent	DAZ	h	Stern	lag in minutes	comments
SH 1, SH 5 no. 36	–321/7/30	10.0°	13.8°		66	12
SH 1, SH 5 no. 36	–321/8/29	15.5°	12.2°		54	
SH 1	–307/6/26	10.0°	15.2°		76	
SH 1	–307/8/24	15.2°	16.9°		78	
SH 1	–302/7/29	6.2°	10.5°		49	
SH 1	–302/8/28	12.8°	10.1°		45	
SH 1	–302/10/27	19.5°	8.4°		42	
SH 1	–302/11/26	20.5°	11.4°		68	
SH 1	–302/12/25+	12.3°	9.0°			
	–302/12/26	19.1°	18.5°	late	112	13
SH 1	–301/1/24	9.7°	16.6°		88	
SH 1	–301/6/19	1.2°	10.5°		55	
SH 1	–294/5/4	8.7°	20.1°		98	
SH 1	–293/1/25	1.1°	14.2°		70	
SH 1	–291/5/1	9.2°	15.0°		71	
SH 1	–291/6/29	16.7°	16.9°		82	
SH 1	–291/8/26	10.3°	9.4°		42	
SH 1	–289/6/8	10.8°	16.6°		83	
SH 1	–286/6/3+	2.06°	9.14°			
	–286/6/4	6.6°	20.0°		105	14
SH 1	–286/9/30+	09 mrt	9.1°		68	
	–286/10/1	20.0°	14.3°	late/OK		15
SH 1	–284/11/6	16.1°	7.1°		36	
SH 1	–283/10/26+	16 aug	6.4°			
	–283/10/27	26.7°	12.3°	late ?	69	16
SH 1	–281/11/4	25.5°	12.9°		74	
SH 1	–277/3/28	0.5°	22.1°		106	
SH 1	–277/4/26	2.6°	16.1°		78	
SH 1	–277/5/25+	04 sep	9.3°			
	–277/5/26	11.0°	19.8°		98	
SH 1	–273/12/4	5.8°	12.4°		74	
SH 1	–266/10/19	20.2°	10.7°		53	
SH 1	–266/11/18	21.7°	11.9°		70	
SH 1	–264/9/26	16.0°	6.8°	early	29	17
SH 2	–255/3/25	5.4°	18.8°		89	
SH 2	–255/9/17	14.0°	11.6°		52	
SH 6 no. 1	–253/5/1	8.5°	12.8°		60	
SH 6 no. 1	–253/9/26	15.2°	15.6°		73	
SH 5 no. 38	–251/5/8	7.9°	16.2°		78	

source	date of new crescent	DAZ	h	Stern	lag in minutes	comments
SH 5 no. 38	−251/7/6	8.5°	13.0°		64	
SH 2	−251/10/3	9.9°	12.0°		55	
SH 2, SH 5 no. 38	−250/2/27+	6.69°	7.68°			
	−250/2/28	10.0°	21.5°		105	
SH 6 no. 2	−250/5/27	7.8°	19.1°		98	
SH 6 no. 2	−250/6/25	6.5°	13.1°		68	
SH 6 no. 2	−250/7/25	10.8°	16.3°		79	
SH 6 no. 2	−250/8/23+	6.1°	10.7°			
	−250/8/24	15.3°	17.0°	late	79	18
SH 2	−249/8/12+	5.0°	10.4°			
	−249/8/13	14.7°	17.5°	late	81	19
SH 2	−246/1/15	10.3°	11.1°		60	
SH 2	−246/4/14	0.2°	15.0°		73	
SH 2	−246/5/13+	1.2°	10.3°			
	−246/5/14	2.3°	22.5°	late	119	20
SH 2	−246/10/8	23.8°	10.9°		54	
SH 2	−245/5/3	0.4°	18.0°		92	
SH 2	−245/7/1	9.0°	17.0°		86	
SH 2	−237/7/3	21.5°	18.4°		87	
SH 2	−237/8/1	20.1°	13.4°		61	
SH 6 no. 9	−234/4/1	7.9°	16.7°		77	
SH 2	−234/9/25+	5.4°	9.2°			
	−234/9/26	15.7°	16.1°		76	21
SH 2, SH 6 no. 9	−234/11/24	14.9°	16.9°		93	
SH 2	−233/2/20	8.6°	17.1°		84	
SH 2	−233/3/21	7.1°	12.1°		55	
SH 6 no. 10	−233/4/20	8.8°	19.1°		91	
SH 6 no. 10	−233/6/18	11.0°	16.2°		81	
SH 6 no. 10	−233/8/16	10.0°	12.7°		58	
SH 6 no. 10	−233/9/15	12.9°	14.6°		66	
SH 6 no. 10	−233/10/14+	6.3°	10.0°			
	−233/10/15	16.4°	16.1°	late	78	22
SH 6 no. 10	−233/11/13	9.9°	11.6°		58	
SH 6 no. 10	−232/2/9+	6.4°	7.5°			
SH 6 no. 10	−232/2/10	11.5°	20.8°		107	
SH 2	−232/10/2+	6.1°	9.5°			
	−232/10/3	15.7°	15.0°	late ?	70	23

source	date of new crescent	DAZ	h	Stern	lag in minutes	comments
SH 2	–232/12/31	14.0°	14.8°		85	
SH 2	–231/2/27+	6.6°	7.3°			
	–231/2/28	9.5°	20.8°		102	
SH 2	–225/1/23	7.6°	18.1°		94	
SH 2	–225/6/19+	3.4°	10.7°			
	–225/6/20	10.5°	19.2°	late	98	24
SH 6 no. 14	–224/6/7+	2.4°	9.8°			
	–224/6/8	9.2°	19.5°	late ?	99	25
SH 6 no. 14	–224/7/7	9.9°	10.7°		51	
SH 6, no. 16	–222/4/18	0.4°	14.0°		68	
SH 2	–218/10/28	14.0°	15.6°		78	
SH 2	–217/2/23	4.9°	18.1°		87	
SH 6 no. 20	–211/3/18	5.0°	11.3°		52	
SH 6 no. 20	–211/4/17	3.7°	17.8°		86	
SH 6 no. 20	–211/5/16	2.5°	13.4°		68	
SH 6 no. 20	–211/6/15	8.5°	22.1°		117	
SH 6 no. 20	–211/7/14	9.8°	17.6°		89	
SH 6 no. 20	–211/9/11	18.6°	14.9°		68	
SH 6 no. 20	–211/10/10	13.7°	9.4°		43	
SH 6 no. 20	–211/11/9	18.8°	10.8°		59	
SH 6 no. 20	–211/12/9	20.6°	14.6°		90	
SH 6 no. 20	–210/2/6	9.6°	16.5°		83	
SH 6 no. 20	–210/4/6	2.1°	14.3°		67	
SH 2	–210/7/3+	2.4°	11.0°			
	–210/7/4	11.2°	21.1°	late	107	26
SH 2	–209/5/24	0.2°	12.1°		63	
SH 6 no. 21	–208/3/14	0.6°	13.4°		52	
SH 2	–207/4/2	1.9°	16.1°		76	
SH 2	–207/5/1	2.5°	10.4°		51	
SH 6 no. 22	–207/6/30	9.8°	16.3°		82	
SH 6 no. 22	–207/7/30	17.4°	14.3°		65	
SH 6 no. 22	–207/12/24+	8.9°	7.7°			
SH 6 no. 22	–207/12/25	16.4°	19.5°		115	
SH 6 no. 22	–206/1/23	6.9°	18.4°		96	27
SH 2	–203/12/10	08 mei	13.8°		74	
SH 2	–201/12/18	6.0°	15.4°		81	
SH 2, SH 6 no. 27	–200/3/16	2.4°	16.1°		75	
SH 2	–200/4/15	6.6°	18.8°		89	

source	date of new crescent	DAZ	h	Stern	lag in minutes	comments
SH 2	−199/2/3	0.3°	9.7°	early	46	28
SH 2	−198/6/21	14.8°	15.3°		74	
SH 2	−197/3/13+	5.0°	8.6°			
	−197/3/14	8.6°	22.2°		106	
SH 2	−197/10/6+	6.8°	10.4°			
	−197/10/7	16.1°	16.6°	late	79	29
SH 2	−197/11/5	8.6°	12.3°		60	
SH 2	−196/2/2	8.8°	16.6°		85	
SH 6 no. 29	−195/5/19	8.4°	17.5°		87	
SH 6 no. 29	−195/10/13	9.7°	11.4°		52	
SH 2, SH 6 no. 29	−195/11/12	11.9°	11.5°		59	
SH 2, SH 6 no. 29	−194/1/11	13.3°	16.1°		90	
SH 2	−194/6/6+	1.98°	4.59°	early		30
	−194/6/7	7.6°	17.6°		91	
SH 6 no. 31	−194/7/6	7.1°	13.9°		70	31
SH 2	−194/10/2+	7.8°	9.7°			
	−194/10/3	19.2°	15.7°	late	75	32
SH 6 no. 31	−194/11/1	12.4°	10.4°		51	
SH 2	−193/4/27+	2.4°	9.5°			
SH 2	−193/4/28	3.9°	14.0°		68	
SH 2	−193/5/28	6.8°	21.8°		114	
SH 2	−193/10/22	21.1°	13.6°		70	
SH 2	−192/1/18+	9.8°	9.0°			
	−192/1/19	14.9°	19.1°	late	94	33
SH 2	−192/2/17	8.0°	13.8°		68	
SH 2	−192/3/18	5.0°	17.9°		83	
SH 2	−192/9/11	18.7°	14.2°		65	
SH 2	−191/10/30	26.9°	13.4°		77	
SH 2	−190/3/25+	0.4°	9.9°			
	−190/3/26	1.1°	21.6°		104	
SH 6 no. 34	−190/4/24	0.6°	15.0°		7	
SH 2	−190/5/24	2.4°	19.3°		102	
SH 6 no. 34	−190/8/21	16.7°	12.6°		54	
SH 6 no. 34	−190/9/20	23.9°	12.0°		56	
SH 6 no. 35	−189/3/15	0.3°	18.0°		85	
SH 6 no. 35	−189/5/13	0.7°	18.7°		99	
SH 6 no. 35	−189/6/11	1.7°	12.5°		66	

source	date of new crescent	DAZ	h	Stern	lag in minutes	comments
SH 6 no. 35	−189/7/11	8.7°	13.6°		67	
SH 6 no. 35	−189/9/9	20.5°	10.0°		45	
SH 2, SH 6 no. 35	−189/10/9	25.1°	10.4°		52	
SH 2	−189/11/7	18.7°	7.9°		42	
SH 6 no. 35	−188/2/3	4.3°	14.0°		69	
SH 2	−188/4/2	1.9°	19.1°		92	
SH 6 no. 37	−188/6/29	7.5°	13.2°		66	
SH 6 no. 37	−188/7/29	15.2°	12.2°		56	
SH 6 no. 37	−188/11/25	17.2°	10.7°		61	
SH 2	−187/10/16	25.5°	11.4°		60	
SH 2	−187/11/14	16.4°	9.2°		50	
SH 2	−185/3/2	1.4°	20.1°		96	
SH 6 no. 39	−185/3/31	2.2°	14.5°		68	
SH 6 nos. 39, 40	−185/4/29+	0.7°	9.6°			
	−185/4/30	3.8°	22.2°		112	34
SH 6 nos. 39, 40	−185/5/29	8.2°	16.9°		85	
SH 6 no. 39	−185/6/27	11.0°	10.5°		50	
SH 6 no. 39	−185/9/24	24.2°	11.0°		52	
SH 6 no. 39	−184/2/19	0.9°	18.0°		85	
SH 2	−183/5/7	6.9°	16.5°		80	
SH 2	−183/8/4	24.8°	14.3°		65	
SH 2	−183/10/31	19.3°	17.0°		90	
SH 2	−181/2/15	2.5°	18.0°		86	
SH 2	−179/3/24	7.2°	17.9°		84	
SH 6 no. 42	−179/4/22	7.9°	12.6°		59	
SH 6 no. 42	−179/5/22	12.9°	18.9°		92	
SH 6 no. 42	−179/6/20	12.1°	11.9°		57	
SH 2	−179/7/20	15.9°	14.7°		70	
SH 2	−178/8/7+	8.2°	9.4°			
	−178/8/8	17.3°	17.6°	late ?	82	35
SH 2	−178/9/6	11.3°	12.9°		59	
SH 2	−178/10/6	13.4°	14.8°		69	
SH 2	−176/9/13	9.0°	11.9°		53	
SH 2	−176/10/13	15.0°	13.8°		66	
SH 2	−175/5/8+	4.1°	10.3°			
	−175/5/9	6.7°	22.2°	late	112	36
SH 2	−175/12/1	20.4°	15.0°		90	
SH 2	−173/11/10	21.8°	11.6°		66	
SH 2	−173/12/9	16.0°	9.8°		57	
SH 2	−172/2/6	9.0°	17.3°		88	

source	date of new crescent	DAZ	h	Stern	lag in minutes	comments
SH 6 no. 45	−171/7/21	7.2°	11.1°		54	
SH 6 no. 45	−171/8/20	12.8°	9.7°		43	
SH 2	−170/8/9	12.9°	9.8°		44	
SH 2	−170/10/8	20.4°	7.6°		36	
SH 2	−170/11/7	21.9°	10.4°		59	
SH 2	−169/2/3	3.5°	14.0°		68	
SH 2	−169/5/2	1.2°	15.3°		78	
SH 6 no. 46	−169/6/30	11.3°	16.1°		80	
SH 6 no. 46	−169/10/27	22.2°	9.9°		52	
SH 6 no. 46	−168/3/22+	3.4°	9.7°			
	−168/3/23	1.8°	22.8°		110	37
SH 2	−168/8/17	25.3°	12.6°		57	
SH 2	−168/12/13	11.4°	12.9°		71	
SH 2	−164/6/5	12.4°	14.0°		67	
SH 2	−164/10/31	18.4°	17.0°		89	
SH 3	−163/4/25	5.5°	12.3°		58	
SH 3	−163/5/25+	11.4°	14.6°			
	−163/5/26	17.1°	24.2°		119	38
SH 3	−163/11/18+	4.4°	9.5°			
	−163/11/19	14.5°	18.6°		100	
SH 3	−162/3/16	3.2°	14.0°		65	
SH 6 no. 47	−162/7/12+	13.05°	8.34°			
	−162/7/13	20.6°	16.6°	late ?	78	39
SH 3	−162/8/11	15.4°	10.3°		45	
SH 3	−162/9/10	16.8°	13.0°		60	
SH 6 no. 50	−161/6/2	12.6°	14.5°		71	
SH 6 nos. 48, 50	−161/7/31	14.3°	11.4°		52	
SH 6 nos. 48, 50	−161/8/30	16.5°	14.1°		64	
SH 3, SH 6 no. 50	−161/9/28+	8.7°	10.2°			
	−161/9/29	17.7°	16.6°	late	79	40
SH 6 nos. 48, 50	−161/11/27	9.9°	15.3°		80	
SH 6 no. 48	−160/1/25	5.5°	14.2°		71	
SH 3	−158/6/29	13.6°	20.4°		104	
SH 3	−158/8/26	9.6°	12.8°		59	
SH 3	−156/12/1	23.3°	16.5°		103	
SH 3	−154/1/18	13.3°	16.8°		93	
SH 6 no. 54	−154/4/16	1.4°	14.8°		71	

source	date of new crescent	DAZ	h	Stern	lag in minutes	comments
SH 6 no. 54	−154/5/15+	0.3°	9.0°			
	−154/5/16	2.7°	20.0°		104	
SH 6 no. 54	−154/6/14	2.6°	13.7°		73	
SH 6 no. 54	−154/7/14	8.3°	15.7°		79	
SH 6 no. 54	−154/9/11+	10.4°	8.5°			
	−154/9/12	20.9°	13.7°		63	
SH 6 no. 54	−154/10/11	15.8°	8.3°		39	
SH 6 no. 54	−154/11/10	21.4°	10.9°		62	
SH 3	−151/3/15	0.8°	18.5°		87	
SH 3	−149/11/14	18.2°	12.2°		67	
SH 6 no. 57	−147/3/30	1.3°	14.9°		70	
SH 3	−145/1/9	6.8°	21.1°		51	
SH 3	−145/2/7	0.3°	16.0°		77	
	−145/2/8+	5.1°	28.8°	late		41
SH 6 no. 63	−145/4/7	3.1°	16.0°		75	
SH 3	−144/9/20+	14.1°	10.7°			
	−144/9/21	23.1°	17.3°	late	83	42
SH 3	−144/10/20	14.7°	14.7° !		72	
SH 3	−144/11/18	6.3°	11.6°		58	
SH 3	−143/8/10+	12.3°	8.3°			
	−143/8/11	21.1°	16.0°		74	
SH 3	−143/9/9	14.0°	11.6°		52	
SH 3	−143/10/9	14.6°	14.6° !		70	
SH 3	−142/11/26	7.1°	12.6°		64	
SH 3	−141/5/23	12.8°	20.7°		103	
SH 3	−141/10/16+	5.7°	10.0°			
	−141/10/17	16.1°	16.7°	late	82	43
SH 3	−140/4/12	8.6°	18.2°		86	
SH 3	−140/7/9	12.3°	15.8°		78	
SH 3	−140/12/3	10.1°	11.7°		52	
SH 3	−139/1/31+	6.7°	8.4°			
	−139/2/1	11.2°	18.5°		96	
SH 6 no. 68	−137/6/7	5.2°	16.7°		88	
SH 3, SH 6 no. 68	−137/12/31	16.4°	15.3°		91	
SH 3	−136/3/28	3.7°	17.2°		81	
SH 3	−136/9/22	17.4°	11.9°		54	
SH 6 no. 69	−135/6/14	6.3°	19.9°		106	

source	date of new crescent	DAZ	h	Stern	lag in minutes	comments
SH 6 no. 69	−135/9/11	16.9°	11.0°		50	
SH 6 no. 69	−135/10/11	20.5°	10.0°		49	
SH 6 no. 69	−135/11/10	23.5°	11.9°		69	
SH 6 no. 69	−134/1/7+	9.7°	8.1°			
	−134/1/8	15.9°	19.6°		112	
SH 6 no. 69	−134/2/6	7.7°	17.8°		90	
SH 3	−134/10/30	23.1°	10.3°		57	
SH 3	−133/2/25	2.5°	18.5°		88	
SH 3	−133/8/20	15.6°	9.9°		43	
SH 3	−133/9/19	21.3°	9.2°		41	
SH 3	−133/10/19	24.1°	10.1°		52	
SH 3	−132/3/15	1.2°	19.3°		91	
SH 3	−132/10/7	23.4°	9.1°		45	
SH 3	−131/10/26	23.1°	11.4°		61	
SH 3	−129/7/8+	11.7°	8.1°			
	−129/7/9	20.6°	15.9°		74	
SH 6 no. 74	−127/5/17	9.2°	16.4°		80	
SH 6 no. 74	−127/6/16	16.9°	16.8°		81	
SH 3	−124/12/6+	3.6°	10.5°			
	−124/12/7	11.6°	18.0°	late	98	44
SH 3	−123/2/4	5.0°	14.3°		71	
SH 3	−123/6/2	12.5°	15.8°		77	
SH 6 no. 77	−122/8/19	17.9°	18.0°		83	
SH 6 no. 77	−122/9/17	13.6°	14.8°		68	
SH 6 no. 77	−122/10/16	8.0°	11.5°		53	
SH 6 no. 77	−122/12/14+	6.1°	9.4°			
	−122/12/15	15.1°	17.9°		102	
SH 6 no. 77	−121/2/12	8.8°	16.0°		79	
SH 6 no. 77	−121/3/13+	6.5°	9.0°			
	−121/3/14	8.7°	19.6°		93	
SH 3	−119/4/19	5.3°	16.3°		78	45
SH 3	−119/6/17	5.2°	13.7°		70	
SH 6 no. 79	−118/4/8	3.7°	13.2°		61	
SH 3, SH 6 no. 79	−118/5/8	4.1°	19.8°		100	
SH 6 no. 79	−118/8/4+	5.2°	10.8°			
	−118/8/5	14.0°	17.4°	late	82	46
SH 6 no. 79	−118/9/3+	9.5°	10.9°			
	−118/9/4	19.4°	16.3°	late	72	47

source	date of new crescent	DAZ	h	Stern	lag in minutes	comments
SH 6 no. 79	–118/12/1+	12.7°	7.1°			
	–118/12/2	20.8°	15.3°		95	
SH 3	–117/10/22	16.9°	8.6°		42	
SH 3	–111/3/22	2.5°	16.1°		76	
SH 3	–111/6/19	10.6°	13.2°		65	
SH 3	–111/8/18	26.3°	12.5°		57	
SH 3	–107/4/7	5.3°	18.6°		88	
SH 3	–105/4/15+	6.3°	9.5°			
	–105/4/16	9.7°	21.6°		104	
SH 3	–105/5/15	11.2°	16.2°		79	
SH 3	–105/6/13	11.6°	11.0°		52	
SH 3	–105/9/9	11.1°	10.8°		48	
SH 3	–105/10/9	15.6°	16.0°		76	
SH 3	–104/8/29	14.4°	14.5°		66	
SH 3	–96/5/5	0.3°	19.6°		102	
SH 3	–95/5/24	3.9°	20.4°		107	
SH 5 no. 23	–88/9/1	19.9°	13.9°		63	
SH 5 no. 23	–88/9/30	12.7°	11.1°		50	
SH 5 no. 23	–88/10/30	13.9°	15.6°		79	48
SH 5 no. 23	–88/11/28	4.8°	11.6°		58	
SH 5 no. 23	–87/3/27	4.6°	12.0°		54	
SH 5 no. 23	–87/4/26	9.3°	17.6°		84	
SH 3	–87/7/23	16.8°	12.9°		60	
SH 3	–87/9/20	18.5°	16.1°		75	49
SH 5 no. 23	–87/12/17	5.3°	13.1°		68	50
SH 5 no. 23	–86/1/16	5.6°	15.6°		79	
SH 3	–86/3/16+	4.8°	10.5°			
	–86/3/17	7.7°	21.0°	late	100	51
SH 3	–86/11/7	10.3°	14.1°		70	
SH 6 no. 88	–84/3/23	7.3°	13.1°		61	
SH 3	–83/7/9	10.7°	16.6°		83	
SH 3	–77/6/4	3.7°	17.0°		90	
SH 3	–77/8/2	15.6°	12.9°		59	
SH 3	–77/9/1	26.3°	13.4°		61	
SH 3	–77/10/30	28.8°	13.9°		82	
SH 3	–73/7/19	21.7°	14.0°		64	

1) Checked: measured lag of 4°, corresponding to astronomically computed 3.75°, between sunrise and moonset on the 14th = –567 May 5/6, resulting in –567/4/22 as 1st and new crescent day as listed in Stern, Table 1. There was no chance of sighting the crescent on –567/4/21, preceding the reported new crescent day, since the moon set together with the sun. – Note that the position of the moon relative to β Virginis which is reported in SH 1 on the 9th and corrected by the editor to 8th, actually occurred on the evening of the 5th = –567/4/26; 2) Checked: positions of moon relative to β Geminorum on 1st and of Venus relative to α Leonis on the 18th. There was a slight

chance of sighting the crescent on –567/5/21, preceding the reported new crescent day. The remark "it was thick" will relate to geocentric w = 1.5 arcminutes as crescent width which is relatively large, by comparison to w = 0.75 arcminutes of the new crescent on –264/9/26 and w = 0.17 arcminutes of the old crescent on –248/10/27, both close to the borderline of visibility; *cf.* also the values for crescent width in Yallop (1997: Table 4; 3) Checked: position of the moon relative to β Librae on the 8th and positions of Mars and Mercury relative to [α Leonis ...] on the reported new crescent day; in the latter case the distance is reported as 4 cubits, when is was actually just above 3 cubits. The remark "it was thick" will relate to geocentric w = 1.0 arcminute as width of the new crescent; 4) Checked: measured lag of 25°, corresponding to astronomically computed 26° on –566/3/14. There was a slight chance of sighting the crescent on –566/3/13, preceding the reported new crescent day; 5) The text does not state clearly that a new crescent was sighted or expected, since only a lag of 18° is mentioned without further specification; the latter does not compare well with the astronomically computed lag of 12.5° (*cf.* the remarks on lag in Appendix 1). Nevertheless, the identification of the 1st day as –463/9/8 seems to be confirmed by 3°30' as reported lag between sunset to moonrise on the 14th, corresponding to astronomically computed 3°; 6) Stern lists this case in his Table 2 as a predicted, not as a sighted new crescent. Since the observer reported: "I did not see it," I have used the case above in the main article as a non-sighted crescent; 7) Checked: position of moon relative to Saturn on the 10th, corresponding to –418/10/20 as new crescent day and 1st day. There was a medium chance of sighting the crescent on the evening preceding the reported new crescent day; Stern classifies the case as "late" ; 8) The text does not clearly state that a new crescent was sighted. In Rev. 5, it says: "sunset to moonset: 13° 40'; mi[st ...]"; the latter compares well with astronomically computed 10.75°. The lag may be predicted or not and the remark "mi[st]" may indicate that the crescent was invisible because of mist. The chance to sight the crescent on –381/8731 was slight; Stern classifies the case as "early"; 9) The report states: "I did not see the moon; sunset to moonset : 14°; in Borsippa it was s[een ? ...]"; the predicted lag of 14° corresponds to astronomically computed 11°. If the values of Table 7 in the main article apply, then there was in Babylon as well as in neighbouring Borsippa a medium chance for seeing the crescent on –366/5/19; 10) The report does not clearly indicate that new crescent was sighted; the text says Obv., 15: "sunset to moonset: 5°? ...". In any case, new crescent day as 1st day of the month was –345/1/1 as shown for example by the position of the moon on the 20th relative to Scorpii; Stern classifies the case as "late". There was a slight chance of sighting the crescent on –346/12/31, preceding the reported new crescent day; 11) Checked: position of moon on 6th relative to Jupiter. There was a medium chance of sighting the crescent on the day preceding the reported new crescent day; Stern classifies the case as "late"; 12) Stern questions the sighting of the new crescent. – SH 5 no. 36, Obv. III 7 says indeed nothing about seeing the moon, but the parallel text SH 1 no. -321, Rev. 2 states correctly that "the moon was 2 cubits above Jupiter" on new crescent day; 13) Checked: measured lag of 27° on 1st, corresponding to astronomically computed 28°. There was a medium chance of sighting the crescent on the day preceding the reported new crescent day; Stern classifies the case as "late"; 14) According to Stern the text is "not sound". The 1st and new crescent day is identifiable as –286/6/4, since for the 7th the position of the moon relative to Saturn is reported. On the evening preceding the reported new crescent, the moon stood below the seasonal uncertainty zone; 15) Checked: position of moon on 8th relative to Mars which allows the identification of the 1st and new crescent day as –286/10/1. There was a medium chance of sighting the moon on the preceding evening; Stern classifies the case as "late or okay"; 16) Checked: reported lag of 17° which corresponds to astronomically computed 17.25° on –283/10/27. On the preceding evening the moon stood far below the seasonal uncertainy zone; Stern classifies the case as "possibly late"; 17) The report seems to have stated that new crescent was sighted on –264/9/26, since it says Obv. 11: "(... sunset to moonset:) 9° 30'; a little mist; the moon [...]".). The date corresponds to no. 63 in the list of Fatoohi *et al.* It is one of the crescents with which I define above the minimum visibility line; Stern's classifies the case as "early"; 18) New crescent was sighted ("it was bright"), but the lag of 20° is not specified as "measured". The respective new crescent day is identifiable as –250/8/24 on the basis of 7° as measured lag between sunrise and moonset, corresponding to astronomically computed 7.25° on –250/9/7 as 14th (morning). There was a medium chance of sighting the moon on the evening preceding the reported new crescent; Stern classifies the case as "late"; 19) There was a medium chance to sight the crescent on –249/8/12, preceding the reported new crescent day; Stern classifies the case as "late"; 20) There was a medium chance to sight the crescent on –246/5/13; preceding the reported new crescent day; Stern classifies this new crescent as "late"; 21) There was a slight chance to sight the crescent on –234/9/25, preceding the reported new crescent day; sighting was first reported on the following day; 22) The measured lag of 18° between sunset and moonset, corresponding to astronomically computed 19.5°, identifies the reported 1st day as –233/10/15. There was a medium chance of sighting the crescent on the preceding evening; Stern classifies the case as "late". – Note that in SH 6 no. 10 the numbering of the months does not refer to the year of observation, rather to the Goal Year, see Brack-Bernsen 1999, 29-37; 23) There was a slight to medium chance to sight the crescent on –232/10/2, preceding the reported new crescent day; Stern classifies the case as "late ?"; 24) Checked: position of moon relative to η Tauri on 21st. There was a medium chance of sighting the crescent on –225/6/19, preceding the reported new crescent day; Stern classifies the case as "late"; 25) Checked: measured lags between sunset and moonset on the reported 1st = –224/6/8 and between moonrise and sunset on the 14th. On –224/6/7 there was a slight chance of sighting the crescent, preceding the reported new crescent day; Stern classifies the case as "possibly late"; 26) There was a medium chance of sighting the crescent on –210/7/3, preceding the reported new crescent day; Stern classifies the case as "late"; 27) Stern: –206/1/**13; 28) SH 2, No. –200, Obv. 1-2 comments on the new crescent situation: "[...] was seen? at sunset; clouds were in the sky". It seems that the sighting of the moon was noted in the lacuna of line 1, the chances of sighting the crescent on –199/2/3 being medium; Stern classifies the case as "early". – The reported positions of moon and Venus on the following evenings show clearly that the first day of the month fell indeed on –199/2/3; 29) There was a medium chance of sighting the crescent on –197/10/6, preceding the reported new crescent day; Stern classifies the case as "late"; 30) According to Stern, an early new crescent was reported on June 6 in –194. A new crescent was under no circumstances visible on June 6, since an outlier of more than 5° below the seasonal visibility line (ca. 5° below the impossibility line in the Caldwell-Laney diagram) is out of the question. With regard to this new crescent, the source (SH 2, No. –194, Obv. 6) remarks: "sunset to moonset: 19°; it was bright, earthshine, measured; it could be seen while the sun stood there; it was low to the sun". A measured lag of 19° is incompatible with astronomically computed 5° for –194/6/6. By contrast, the astronomically computed lag on –194/6/7 amounted to 22.75° which is close enough to the reported 19°. Furthermore, the positions of the moon relative to various stars as reported for days 4, 7, 9, 15, 23, 25, 26 and 27 are only correct if the observer counted June 7 in –194 as day 1 of the month. Evidently June 6 is mistakenly identified by Stern and new crescent was observed on June 7 in –194 under unexceptional circumstances; 31) This case may be added to Stern's list as a new crescent which became visible under unexceptionable circumstances; 32) There was a medium chance of sighting the crescent on –194/10/2, preceding the reported new crescent day; Stern classifies the case as "late"; 33) There was a slight to medium chance of sighting the crescent on 192/1/19, preceding the reported new crescent day; Stern classifies the case as "late"; 34) Checked: measured lags on 1st and 13th. There was a slight chance of sighting the crescent on –185/4/29, preceding the reported new crescent day; 35) There was a slight chance of sighting the crescent on –178/8/7, preceding the reported new crescent day; Stern classifies the case as "late ?"; 36) There was a medium chance of sighting the crescent on –175/5/8, preceding the reported new crescent day; Stern classifies the case as "late"; 37) There was a slight to medium chance of sighting the crescent on –168/3/22, preceding the reported new crescent day; 38) Fatoohi *et al.* identify –163/5/26 as new crescent day in SH 3; Parker-Dubberstein, 1st edition, following Schoch's age-of-the-moon criterion also give –163/5/26. Stern comments „but this is impossible and contradicts the previous month on 25 April" implying that there would be a 31-day month between –163/4/25 and –163/5/26 as new crescent days. On the other hand, there is no doubt about the Julian calendar days of the reported new crescents. On –163/4/25 the lag amounted to 70 m which compares well enough with the reported "13° [= 52 m], measured (despite) clouds". Furthermore, the positions of the moon relative to β Geminorum on the 4th and relative to β Virginis on the 9th confirm ¬–163/4/25 as new crescent day and beginning of the lunar month. One month later, on –163/5/26 the observer reported a lag of 26° (104 minutes), "measured (despite) clouds" which compares well with astronomically computed 30° (120 minutes). But there are also entries which refer to the following days and on the basis of the reported positions of the moon relative to α Leonis on the 4th, on the 6th relative to θ Leonis and on the 7th relative to β Virginis the 1st day of the month is reckoned as –163/5/25. Is it possible that somebody had realized that –163/5/26 was not correct and that the count of the lunar days was changed to –163/5/25 as first day? Probably the crescent was obscured by clouds on May 25, since the observer reported clouds on the preceding 29th lunar day and on the following 2nd lunar day and a "very overcast" sky on the 3rd lunar day. – The lunar positions on –163 June 25 & 26 are indicated in Fig.s 14 & 19; 39) There was a slight chance of sighting the crescent on –162/7/12, preceding the reported new crescent day; Stern classifies the case as "late ?"; 40) The report in SH 3 no. –161 shows that the moon was observed, although the details remain unclear: "when the moon [came out] of a cloud". The qualification "measured" of the 20° lag between sunset and moonset on the 1st day might have stood in a lacuna; in the parallel text SH 6 no. 50, the measured lag amounts to 21°. Since on –161/9/28 the astronomically computed lag amounted to 11.5°, and to 20° on –161/9/29, the observer apparently reckoned –161/9/29 as 1st day. In both sources and throughout the month the positions of the moon are mostly lost in lacunae. If the restoration "[Night of the 2]4th ... the moon was below γ Vir[ginis]" in SH 3 is correct, then the 1st day was indeed counted as –161/9/29. There was a slight chance of sighting the crescent on the preceding day; Stern classifies the case as "late"; 41) Stern, Table 1, lists –145/2/8 as a late new crescent and the beginning of month XI. His interpretation seems to depend on the restored length of the preceding month. Month X began on time on –145/1/9 and Hunger restores its length as follows: "[Month XI, the 1st (of which followed the 30th of the preceding month), ... measu]red". If the restoration be correct, the 1st day of month XI would have fallen on –145/2/8. On the other hand the text states about new crescent of month XI: "... [measu]red": "it was high to the sun; the moon stood 1 cubit behind Venus to the east, the moon being 1 c[ubit? ...]". Such a relative position of Venus and moon occurred on the evening of –145/2/7; on the following evening the two were separated by about 16° ≈ 7 cubits. Therefore new crescent was observed on –145/2/7 and not late on the following evening; the length of month X ought to be restored as 29 days; 42) Checked: position of moon relative to θ Ophiuchi on the 3rd = –144/9/23 and in the preceding (sic) lunar month the position of the moon relative to β Virginis on the 26th = –144/9/17. The observer reported new crescent on –144/9/21 under a lag of 18° (time degrees), corresponding to astronomically computed 20.75°. – There is no report about the preceding evening which would have been on a 30th lunar day, although there is a remark about noon of that same Julian calendar day: "The 29th, at noon, clouds [....] the sky [.... bar]ley in the beginning of the month". It must remain open whether clouds or high extinction caused the non-sighting on –144/9/20; there was a sizeable to about certain chance of sighting the crescent on that day. – Stern classifies the case as "late"; 43) According to the position of the moon on the 5th relative to Capricorni, the 1st day was –141/10/17. – There was a medium chance of sighting the crescent on –141/10/16, preceding the reported new crescent day; Stern classifies the case as "late"; 44) According to the positions of the moon on the 3rd relative to α Scorpii and on the 11th relative to α Tauri, the 1st day was –124/12/7. There was a medium chance of sighting the crescent on –124/12/6, preceding the reported new crescent day; Stern classifies the case as "late"; 45) This new crescent figures as no. 184 in the list of Fatoohi *et al.*, though not in Stern's Table 1. According to Hunger SH 3, 317 (no. -119 D) the text dates either to SE 192 = –119 or SE 250 = –61. In both cases the new crescent would have been about 1 cubit in front of Saturn. A decision in favor of SE 192 is possible on the basis of the lag of 1° 40` = 6.6 minutes between moonrise and sunset which was measured on the evening of the 15th. The latter day corresponded in SE 192 to –119/4/19 when the astronomically computed lag amounted to 3 minutes. By contrast, the 15th corresponded in SE 250 to –61/4/23, when the respective lag would have been negative, since the sun set about an hour before the moon; 46) Checked: measured lag of 19° between sunset and moonset on the 1st and between moonset and sunrise on the 13th. The lag of 19° on the 1st corresponds to astronomically computed 20.5°. There was a medium chance on –118/8/4 of sighting the crescent; Stern classifies the date as "late"; 47) Checked:

measured lag between sunset and moonset on the reported 1st. There was a sizeable to about certain chance of sighting the crescent on –118/9/3, preceding the reported new crescent day; Stern classifies the case as "late"; 48) Stern indicates the sighting of the crescent, though SH 5 no. 23 Rev. V 11 gives no further information about the lag between sunset and moonset than "10 + [x° ...]" which may indicate prediction or observation; 49) Corrected from –86/12/6 in Stern's Table 1; *cf.* the comments of Christopher Walker cited in SH 5, 70; 50) Corrected from –85/1/5 in Stern's Table 1; *cf.* preceding note; 51) According to the position of the moon relative to β Tauri on the 4th, the reported 1st day was –86/3/17. There was a medium chance of sighting new crescent on –86/3/16, preceding the reported new crescent day; Stern classifies the case as "late".

Abbreviations and Glossary

(For most of the astronomical terms below *cf.* the glossary of "The Astronomical Almanac" at http://asa.usno.navy.mil/SecM/Section_M.html.)

Age of the moon: positive age: time elapsed since conjunction; negative age – time remaining until conjunction;

Air mass (or airmass): the optical path length through the earth's atmosphere for light coming from a celestial object like the moon. As it passes through the atmosphere, the light is dimmed. By definition, the sea-level air mass at the zenith is 1. Air mass increases as the angle between the source and the zenith increases, reaching a value of approximately 38 at the horizon;

Alex(andrian): Alexandrian calendar, introduced in Egypt by Augustus. It corresponds to the Julian Calendar insofar as a common year comprises 365 days, and every fourth year is a leap year comprising 366 days;

Altitude: the angular distance of a celestial body above or below the horizon, measured along the great circle passing through the body and the zenith. Positive numbers indicate values of altitude above the horizon, and negative numbers indicate positions below the horizon; for example, the boundary between civil twilight and nautical twilight is when the sun reaches an altitude of –6°;

ARCL : Arc of Light or elongation; for explanation, *cf.* 1.2;

Arc minutes: There are 60 minutes (denoted as 60‘) of arc in 1 degree or 1°;

ARCV = Arc of Visibility; for explanation, *cf.* 1.2;

Artaba: measure of dry capacity; Graeco-Roman period;

Azimuth: the angular distance measured eastward along the horizon from a specified reference point (usually north). Azimuth is measured to the point where the great circle determining the altitude of an object meets the horizon;

Calendar, Julian: the calendar introduced by Julius Caesar in 45 B.C. to replace the Roman calendar then in use. In the Julian calendar, a common year comprises 365 days, and every fourth year is a leap year with an additional day, or 366 days all told. The Julian calendar was superseded in 1582 by the Gregorian calendar;

Celestial sphere: an imaginary sphere of arbitrary radius for representing the relative positions of celestial bodies. As circumstances require, the celestial sphere may be centered at the observer, at the earth's center, or at any other location;

Civ(il) : civil calendar, referring to the Egyptian year of 365 days;

Conjunction : the moment when the ecliptical longitudes of sun and moon are identical;

Crescent width: angular width of the illuminated part of a lunar crescent measured along the lunar equator as seen from earth;

DALT: difference in altitude; for explanation, cf. 1. 7;

DAZ: difference in azimuth; for explanation, cf. 1.2;

declination: angular distance on the celestial sphere north or south of the celestial equator;

Delta t or fft: the difference between Terrestrial Time (Delta T) and Universal Time (UT);

Degree (°): 360th part of a circle;

Ecliptic: The apparent path of the sun as seen from earth against the background of the celestial sphere;

Elongation (or ARCL): the geocentric angle between two celestial objects;

Epagomena: last five days of the Egyptian 365-day year;

Extinction: dimming by absorption and scattering of the light that reaches an observer through the atmosphere. Its amount depends on the altitude of the celestial body, being lowest at the zenith and at a maximum near the horizon as a result of a short or long path through the atmosphere

Geocentric: with reference to, or pertaining to, the center of the earth;

Geocentric coordinates: 1. The latitude and longitude of a point on the earth's surface relative to its center. 2. Celestial coordinates given with respect to the center of the earth;

GMT: Greenwich Mean Time, *cf*. Universal Time (UT);

Height: the distance above or below a reference surface such as mean sea level on the earth;

Horizon, astronomical: the plane perpendicular to the line from an observer to the zenith that passes through the point of observation;

Horizon, geocentric: the plane perpendicular to the line from an observer to the geocentriczenith passing through the center of the earth;

LAG or lag: time difference between sunset and moonset or moonrise and sunrise respectively;

Latitude, geographic: The latitude ϕ of a point on the earth's surface is the angular distance between the equator and that point, either north or south of the equator;

Latitude, ecliptic: angular distance on the celestial sphere measured north or south of the ecliptic;

Latitude, lunar: ecliptic latitude of the moon;

LD: day of a lunar month of 29 or 30 days counted in Egyptian fashion beginning with the first day of invisibility after sighting old or last crescent;

Longitude, geographical: The longitude λ of a point on the earth's surface is the angular distance east or west from the zero meridian of Greenwich;

Longitude, ecliptic: angular distance on the celestial sphere measured eastward along the ecliptic from the dynamical equinox to the great circle passing through the poles of the ecliptic and the celestial object;

Longitude, lunar: ecliptic longitude of the moon;

Magnitude (m), stellar: a measure on a logarithmic scale of the brightness of a celestial object. For example, the bright star Sirius has a magnitude of –1.46 whereas the faintest stars detectable with an unaided eye under ideal conditions have magnitudes of about +6.0. Further examples are: m sun = – 26.86; m full moon (mean) = –12.72;

Month, synodic: the period between successive new crescents, as if seen from the center of the earth. The mean length of the synodic month is approximately 29.531 days;

New crescent (colloquial: "new moon"): first visibility of the waxing moon.

Old crescent (old moon): last visibility of the waning moon;

Parallax, lunar horizontal: the angular difference between the topocentric and a geocentric direction toward the moon when on the astronomical horizon;

Phase: the name applied to the apparent degree of illumination of the disk of the moon or of a planet as seen from earth;

Phase angle: the angle measured at the center of an illuminated body (moon) between the light source (sun) and the observer on earth; the brightness of the moon is a function of the phase angle;

Phyle: term for one of the four (or five) rotating groups of priests responsible for the temple service in ancient Egypt;

Refraction: the change in direction (bending) of a ray of light as it passes through the atmosphere. Refraction causes the observed altitude of a celestial object to be greater than its geometric altitude. The amount of refraction depends on the altitude of the object, on temperature, and on air pressure;

Semidiameter of the moon, the sun, or a planet: the angle subtended by the equatorial radius of the moon, the sun or a planet as seen by the observer on earth;

Terrestrial Time (TT): an idealized form of International Atomic Time (TAI); it is strictly uniform;

Topocentric: with reference to, or pertaining to, a point on the surface of the earth;

Universal Time (UT): a time scale based on the rotation of the earth which is not uniform. It approximates the mean diurnal motion of the sun and is loosely spoken of as mean solar time on the Greenwich meridian, previously referred to as Greenwich Mean Time;

zenith: in general, the point directly overhead on the celestial sphere.