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Other generalized forms of separation Axioms in Topological Spaces via E-open and δ-ß-open sets

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ABSTRACT

New generalized classes of separation axioms in topological spaces namely, E and δ-ß-separation axioms by utilizing E -open and δ-ß-open sets respectively are introduced and studied. Several of their fundamental characterizations and their relationships with other corresponding kinds of spaces are discussed. Moreover, New forms of Regularity and Normality namely, E (resp.δ-ß)-Regularity and E (resp.δ-ß)-Normality are investigated in the context of these new classes of E -open and δ-ß-open sets respectively. As well as several of interesting properties which are concerning of E (resp.δ-ß)-Regularity and E (resp.δ-ß)-Normality are established.

Mathematics Subject Classification: 54C08, 54D20, 54A05, 54C05.

1. Introduction

The class of generalized open and closed sets has an important role in general topology and it applications, especially its suggestion new generalized of separation axioms which are useful in digital topology. The investigation on generalization of open and closed sets has lead to significant contribution to the theory of separation axioms. Indeed a significant theme in General Topology, Real analysis and many other branches of mathematics concerns the variously modified forms of separation axioms by utilizing generalized open and closed sets.

"Furthermore, in recent literature, we find many topologists worldwide are focusing their researches in the direction of investigating different types of separation axioms. Some of these have been found to be useful in computer science and digital topology [see for example [1, 2]". Dontcheve and Ganster [1] proved that the digital line is T3/4 space but not T1. Also, Navalagi [3] introduced semi generalized- Ti spaces, i= 0, 1, 2.

In addition, in 2011, Ahu Açıkgöz [4] defined two new separation axioms called ß*T1/2 and ß** T1/2 spaces as applications of ß*g−closed sets.
Hariwan Z. Ibrahim in [5] presented and investigated some weak of separation axioms by using the concepts of Bc-open sets and the Bc-closure operator. Also, in the same year Hussein A. Khaleefah[6] studied new types of separation axioms termed by, generalized b- Ri, i= 0, 1 and generalized b-Ti, i= 0, 1, 2 by using generalized b-open sets, Relations among these types are investigated, and several properties and characterizations are provided.
"In addition, Regularity and Normality are important topological properties and hence they are significance both from intrinsic interest as well as from applications view point to obtain factorizations of Regularity and Normality in terms of weaker topological properties".
Many authors have studied several forms of normality and regularity [7, 8, 9, 10]
Recently, A.I. EL-Maghrabi and M.A. AL-Juhani [11] introduced and investigated a new class of separation axioms called M-Ti-spaces, i = 0, 1, 2. Also, the M-Regularity and the M-Normality are examined in the context of these new concepts. B. K. Tyagi and H. V. Chauhan in [12] explained the relationships among several separation axioms such as, μ-T_o,μ-T_1,μ-T_2,μ-T_(1⁄2), μ-regularity,μ-normality,μ-R_o,μ-R_1,μ-D_o,μ-D_1,μ-T_2,μ-R_δ,μ-π_o,μ-weakly regular,μ-R_(D_δ) ,μ-R_(d_δ) ,μ-R_D,μ-R_d,μ-D_(D^*) and μ-D_(d^*), in the framework of generalized topological spaces, also they discussed the relationship of some of the above axioms with θ-generalized topology.
As well as, in [13] S. H. Abdulwahid and A. M. F. AL. Jumaili, introduced and studied some new types of separation axioms called, E_c (resp. 〖 δ-ß〗 _c)-separation axioms and some of their fundamental properties and relationships with other types of spaces are discussed.
The main goal of the present paper is to consider and study new classes of generalized separation axioms called,E and δ-ß-separation axiomsby using E -open and δ-ß-open sets respectively. Several basic properties and preservation properties concerning of these kinds of generalized separation axioms are presented. Also, the relationships among these types of separation axioms and other kinds of spaces are investigated. Furthermore,
E (resp.δ-ß)-Regularity and E and δ-ß-Normality are studied in the context of these new concepts. Additional, some of basic interesting properties of them are provided.

2. PRELIMINARIES

Throughout this paper, $(\mathcal{X},\mathcal{T})$, $(\mathcal{Y},\ \mathcal{T}^*)$ and $(\mathcal{Z},\mathcal{T}^{**})$(or simply $\mathcal{X},\mathcal{Y}$ and $\mathcal{Z}$) meantopological spaces on which no separation axioms are assumed unless explicitly stated. For anysubset $\mathcal{A}$ of $\mathcal{X}$, the closure and interior of A are denoted by Cl$(\mathcal{A})$ and Int$(\mathcal{A})$, respectively. We recall the following required definitions and the fundamental concepts, which will be used often throughout this paper.

Definition 2.1: Let $(\mathcal{X},\mathcal{T})$ be a topological space. A subset $\mathcal{A}\ of\ \mathcal{X}$ is said to be:

a) Regular open (resp. regular closed) [**14**]if $\mathcal{A} = Int(Cl(\mathcal{A}))(resp.\ \mathcal{A} = Cl(Int(\mathcal{A})))$.

b) $\delta -$ open [**15**] if for each $x \in \mathcal{A}$ there exists a regular open set $\mathcal{V}$ such that $x \in \mathcal{V} \subseteq \mathcal{A}$. The δ-interior of $\mathcal{A}$ is the union of all regular open sets contained in $\mathcal{A}$ and is denoted by $Int_\delta(\mathcal{A})$. The subset A is called $\delta -$ open [**15**] if $\mathcal{A} = Int_\delta(\mathcal{A})$. A point $x \in \mathcal{X}$ is called a $\delta -$ cluster points of $\mathcal{A}$ [**15**] if $\mathcal{A} \cap \mathrm{Int}(\mathrm{Cl}(\mathcal{V})) \neq \emptyset$, for each open set $\mathcal{V}$ containing x. The set of all δ-cluster points of A is called the δ-closure of $\mathcal{A}$ and is denoted by $Cl_\delta(\mathcal{A})$. If $\mathcal{A} = Cl_\delta(\mathcal{A})$, then $\mathcal{A}$ is said to be $\delta -$ closed [**15**]. The complement of $\delta -$ closed set is said to be $\delta -$ open set. A subset $\mathcal{A}$ of a Topological space $\mathcal{X}$ is called $\delta -$ open [**15**] if for each $x \in \mathcal{A}$ there exists an open set $\mathcal{G}$ such that, $x \in \mathcal{G} \subseteq \mathrm{Int}(\mathrm{Cl}(\mathcal{G})) \subseteq \mathcal{A}$. The family of all $\delta -$ open sets in $\mathcal{X}$ is denoted by. $\delta\Sigma(\mathcal{X}, \mathcal{T})$.

Definition 2.2: Let$(\mathcal{X}, \mathcal{T})$be a Topological space. Then:

a) A subset $\mathcal{A}$ of a space $\mathcal{X}$ is called $E -$ open[**16**]if $\mathcal{A} \subseteq \mathrm{Cl}(\delta - Int(\mathcal{A})) \cup Int(\delta - Cl(\mathcal{A}))$. The complement of an $E -$ open set is called $E -$ closed. The intersection of all $E -$ closed sets containing $\mathcal{A}$ is called the $E -$ closure of $\mathcal{A}$ [**16**] and is denoted by$E - Cl(\mathcal{A})$. The union of all $E -$ open sets of $\mathcal{X}$ contained in $\mathcal{A}$ is called the$E -$ interior[**16**] of $\mathcal{A}$ and is denoted by $E - \mathrm{Int}(\mathcal{A})$.

b) A subset $\mathcal{A}$ of a space $\mathcal{X}$ is called $\delta -$ ß $-$ open [**17**] or $e^* -$ open [**18**], if $\mathcal{A} \subseteq \mathrm{Cl}(\mathrm{Int}(\delta - \mathrm{Cl}(\mathcal{A})))$, the complement of $\delta -$ ß $-$ open set is called$\delta -$ ß $-$ closed.

The intersection of all $\delta -$ ß $-$ closedsets containing $\mathcal{A}$is called the $\delta -$ ß $-$ closure of $\mathcal{A}$ [**17**] and is denoted by $\delta -$ ß $- \mathrm{Cl}(\mathcal{A})$. The union of all $\delta -$ ß $-$ open sets of $\mathcal{X}$ $\mathcal{A}$ is called contained in the $\delta -$ ß $-$ interior [**17**] of $\mathcal{A}$ and is denoted by$\delta -$ ß $- Int(\mathcal{A})$.

Remark 2.3: The family of all $E -$ open (resp. $E -$ closed, $\delta -$ ß $-$ open, $\delta -$ ß $-$ closed) subsets of $\mathcal{X}$ containing a point $x \in \mathcal{X}$ is denoted by $E\Sigma(\mathcal{X}, x)$ (resp. $EC(\mathcal{X}, x), \delta -$ ß$\Sigma(\mathcal{X}, x), \delta -$ ß$C(\mathcal{X}, x)$). The family of all $E -$ open (resp. $E -$ closed, $\delta -$ ß $-$ open, $\delta -$ ß $-$ closed)sets in $\mathcal{X}$ are denoted by $E\Sigma(\mathcal{X}, \mathcal{T})$ $(resp. EC(\mathcal{X}, \mathcal{T}), \delta -$ ß$\Sigma(\mathcal{X}, \mathcal{T}), \delta -$ ß$C(\mathcal{X}, \mathcal{T}))$.

Proposition 2.4: [**16, 19**] the following properties hold for a space $\mathcal{X}$:

a) The Arbitrary union of any family of $E - (resp.\ \delta -$ ß$) -$ open sets in$\mathcal{X}$, is an $E - (resp.\ \delta -$ ß$) -$ open set.

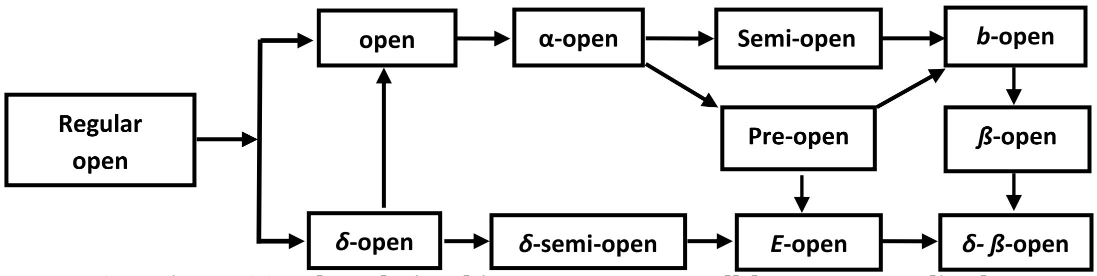
b) The Arbitrary intersection of any family of $E - (resp.\ \delta -$ ß$) -$ closed sets in $\mathcal{X}$, is an $E - (resp.\ \delta -$ ß$) -$ closed set.

Theorem 2.5: [**13**] The following properties hold for a topological space $(\mathcal{X}, \mathcal{T})$:

a) Every regular closed subset in a space $\mathcal{X}$ is $\delta -$ ß $-$ open set.

b) if $\mathcal{X}$ is regular space. Then every open set is an E and $\delta -$ ß $-$ open set.

Remark 2.6: "We have the following figure in which the converses of implications need not be true, see the examples in [**19**], [**16**] and [**18**]".



a) Figure (1): The relationships among some well-known generalized open sets in topological spaces

3. CHARACTERIZATIONS OF E ($\boldsymbol{resp.\delta}$ $-$ ß) $-\boldsymbol{\mathcal{T}_i}-$ **SPACES** ($\boldsymbol{i = 0, 1, 2}$)

Our motivation in this section is to provide several characterizations and some basic properties concerning of other kinds of separationaxioms namely, E ($resp. \delta -$ ß) $-$ separation axioms such as E $- \mathcal{T}_0 -$ ($resp. \delta -$ ß $- \mathcal{T}_0$), E $- \mathcal{T}_1 -$ ($resp. \delta -$ ß $- \mathcal{T}_1$) and E $- \mathcal{T}_2 -$ ($resp. \delta -$ ß $- \mathcal{T}_2$) $-$ Spaces, as well as to discussion the relationships among these kinds of spaces and other well $-$ known spaces.

Definition 3.1: A mapping $f: (\mathcal{X}, \mathcal{T}) \longrightarrow (\mathcal{Y}, \mathcal{T}^*)$ is said to be:

i) E $-$ Irresolute, [**20**] if $f^{-1}(\mathcal{V})$ is E $-$ open in $\mathcal{X}$ $\forall$ E $-$ open sub $-$ set $\mathcal{V}$ of $\mathcal{Y}$.

ii) $\delta -$ ß $-$ Irresolute, [**20**] if $f^{-1}(\mathcal{V})$ is $\delta -$ ß $-$ open in $\mathcal{X}$ $\forall$ $\delta -$ ß $-$ open set $\mathcal{V}$ of $\mathcal{Y}$.

iii) E $-$ open, if the image of each open set of $(\mathcal{X}, \mathcal{T})$ is E $-$ open of $(\mathcal{Y}, \mathcal{T}^*)$.

iv) $\delta -$ ß $-$ open, if the image of each open set of $(\mathcal{X}, \mathcal{T})$ is $\delta -$ ß $-$ open of $(\mathcal{Y}, \mathcal{T}^*)$.

v) E $-$ closed, if the image of each closed set of $(\mathcal{X}, \mathcal{T})$ is E $-$ closed of $(\mathcal{Y}, \mathcal{T}^*)$.

vi) $\delta -$ ß $-$ closed, if the image of each closed set of $(\mathcal{X}, \mathcal{T})$ is $\delta -$ ß $-$ closedof $(\mathcal{Y}, \mathcal{T}^*)$.

vii) E $-$ continuous, [**16**] if $f^{-1}(\mathcal{V})$ is E $-$ open in $\mathcal{X}$ for every open subset $\mathcal{V}$ of $\mathcal{Y}$.

viii) $\delta -$ ß $-$ continuous, [**19**] if $f^{-1}(\mathcal{V})$ is $\delta -$ ß $-$ open in $\mathcal{X}$ $\forall$ open subset $\mathcal{V}$ of $\mathcal{Y}$.

ix) Strongly $-$ E $-$ open, if the image of each E $-$ open set of $\mathcal{X}$ is E $-$ open of $\mathcal{Y}$.

x) Strongly $- \delta -$ ß $-$ open, if the image of each $\delta -$ ß $-$ open set of $\mathcal{X}$ is $\delta -$ ß $-$ open of $\mathcal{Y}$.

xi) E $-$ Homeomrphism, if f is bijective, E $-$ irresolute and strongly $-$ E $-$ open.

xii) $\delta -$ ß $-$ Homeomrphism, if f is bijective, $\delta -$ ß $-$ irresolute and strongly $- \delta -$ ß $-$ open.

Definition 3.2: [20]A topological space$(\mathcal{X}, \mathcal{T})$ is said to be:

a) $E(resp. \delta - ß) - \mathcal{T}_0 -$ Space if for each distinct points x and y of $\mathcal{X}$, there is E $(resp. \delta - ß) -$ open set containing one of them but not the other.

b) $E(resp. \delta - ß) - \mathcal{T}_1 - Space$ if for each pair of distinct points x, y $(x \neq y) \in \mathcal{X}$, there exist two E $(resp. \delta - ß) -$ open sets $\mathcal{U}$ and $\mathcal{V}$ such that $x \in \mathcal{U}$ $but\ y \notin \mathcal{U}\ and\ y \in \mathcal{V}\ but\ x \notin \mathcal{V}$.

c) $E(resp. \delta - ß) - \mathcal{T}_2 - Space\ or\ E(resp. \delta - ß) - Hausdorff\ Space$ if for each pair of distinct pointsx, y $(x \neq y) \in \mathcal{X}, \exists$ two disjoint E $(resp. \delta - ß) -$ open sets $\mathcal{U}$ and $\mathcal{V}$ such that$x \in \mathcal{U}\ and\ y \in \mathcal{V}$.

Theorem 3.3: The following conditions are hold in a topological space$(\mathcal{X}, \mathcal{T})$:

a) Every $E(resp. \delta - ß) - \mathcal{T}_2 - Space$ is $E(resp. \delta - ß) - \mathcal{T}_1 - Space$

b) Every $E(resp. \delta - ß) - \mathcal{T}_1 - Space$ is $E(resp. \delta - ß) - \mathcal{T}_0 - Space$

c) Every $E - \mathcal{T}_2 - Space$ is $\delta - ß - \mathcal{T}_2 - Space$.

d) Every $E - \mathcal{T}_1 - Space$ is $\delta - ß - \mathcal{T}_1 - Space$.

e) Every $E - \mathcal{T}_0 - Space$ is $\delta - ß - \mathcal{T}_0 - Space$.

Proof: The proof is clear it is followsdirectly from their respective definitions.

Remark 3.4: From the respective definitions, the relationships among E $(resp. \delta - ß) - \mathcal{T}_i - spaces$ $(i = 0, 1, 2)$and some other well − knowntypes of spaces shown in the following figure:

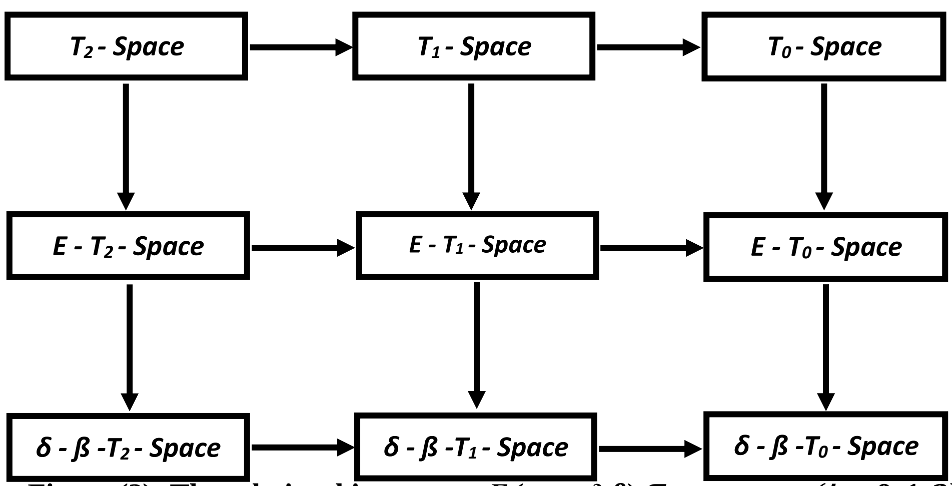


Figure (2): The relationships among E (resp. δ-ß)-$\mathcal{T}_i - spaces$ $(i = 0, 1, 2)$ and some other well-known types of spaces

However noneof these implicationsis reversible as shown in the following examples.

Examples 3.5: **(1)** − Let $\mathcal{X} = \{a, b, c, d\}$ with a topology $\mathcal{T} = \{\emptyset, \{a\}, \{b\}, \{a, b\}, \{a, b, c\}, \{a, b, d\}, \mathcal{X}\}$. Then, $\mathcal{X}$ is $E - \mathcal{T}_0 - Space$, but it is neither $E - \mathcal{T}_1 - Space$ nor $E - \mathcal{T}_2 - Space$.

$(\mathbf{2})$ − Let $\mathcal{X} = \{x, y, w, z\}$with a topology $\mathcal{T} = \{\emptyset, \{x\}, \{w\}, \{x, y\}, \{x, w\}, \{x, y, w\}, \{x, w, z\}, \mathcal{X}\}$. Then, $\mathcal{X}$ is $\delta - ß - \mathcal{T}_0 - Space$, but it is neither $\delta - ß - \mathcal{T}_1 - Space$ nor $\delta - ß - \mathcal{T}_2 - Space$.

Example 3.6: Consider $\mathcal{X}$ any infinite set with the co − finite topology $\mathcal{T}_c$ (such that the closedsets are $\mathcal{X}$ and the finite subsets).

Since $\mathcal{X} \setminus \{x\}$ is E $(resp. \delta - ß)$ − open set, therefore $\mathcal{X}$ is, $E - \mathcal{T}_1$ and $\delta - ß - \mathcal{T}_1$ Space.

But there is no non emptyE and $\delta - ß -$ open sets are disjoint, so $\mathcal{X}$ cannot be neither $E - \mathcal{T}_2$ nor $\delta - ß - \mathcal{T}_2$ space.

Theorem 3.7: For a space $(\mathcal{X}, \mathcal{T})$the following properties are equivalent:

a) $\mathcal{X}$ is an $E(resp. \delta - ß) - \mathcal{T}_0 - Space$;

b) For every two distinct points $x, y\ (x \neq y) \in \mathcal{X}$,
$E - Cl(\{x\})(resp. \delta - ß - Cl(\{x\})) \neq E - Cl(\{y\})(resp. \delta - ß - Cl(\{y\}))$.

Proof: **Necessity**. Assume that$\mathcal{X}$ *is* $E(resp. \delta - ß) - \mathcal{T}_0 - Space$ and $\forall\ x, y\ (x \neq y) \in \mathcal{X}$, there exists an E $(resp. \delta - ß)$ − open set $\mathcal{U}\ (s.t)\ x \in \mathcal{U}, y \notin \mathcal{U} \Longrightarrow y \in \mathcal{X} \setminus \mathcal{U}$,
where $\mathcal{X} \setminus \mathcal{U}$ is E $(resp. \delta - ß)$ − closed which does not contain x but contains y.
Since $E - Cl(\{y\})(resp. \delta - ß - Cl(\{y\}))$ is the smallestE $(resp. \delta - ß)$ − closed set containing y, thus $E - Cl(\{y\})(resp. \delta - ß - Cl(\{y\})) \subseteq \mathcal{X} \setminus \mathcal{U}$ and hence $x \notin E - Cl(\{y\})(resp. \delta - ß - Cl(\{y\}))$.
So $E - Cl(\{x\})(resp. \delta - ß - Cl(\{x\})) \neq E - Cl(\{y\})(resp. \delta - ß - Cl(\{y\}))$.
(**Sufficiency**), suppose that $x, y\ (x \neq y) \in \mathcal{X}$, and
$E - Cl(\{x\})(resp. \delta - ß - Cl(\{x\})) \neq E - Cl(\{y\})(resp. \delta - ß - Cl(\{y\}))$.
Let $z \in \mathcal{X}$ such that $z \in E - Cl(\{x\})(resp. \delta - ß - Cl(\{x\}))$ but $z \notin E - Cl(\{y\})(resp. \delta - ß - Cl(\{y\}))$.
We prove that$x \notin E - Cl(\{y\})(resp. \delta - ß - Cl(\{y\}))$. Suppose that, $x \in E - Cl(\{y\})(resp. \delta - ß - Cl(\{y\}))$,
consequently$\{x\} \subseteq E - Cl(\{y\})(resp. \delta - ß - Cl(\{y\}))$, which implies that,
$E - Cl(\{x\})(resp. \delta - ß - Cl(\{x\})) \subseteq Cl(\{y\})\big(resp. \delta - ß - Cl(\{y\})\big)$ and hence
$z \in E - Cl(\{y\})(resp. \delta - ß - Cl(\{y\}))$which is a contradiction with the fact of $z \notin E - Cl(\{y\})(resp. \delta - ß - Cl(\{y\}))$. Therefore, $x \notin E - Cl(\{y\})(resp. \delta - ß - Cl(\{y\}))$,
which implies that, $x \in \mathcal{X} \setminus E - Cl(\{y\})\big(resp. \delta - ß - Cl(\{y\})\big)$. So
$\mathcal{X} \setminus E - Cl(\{y\})\big(resp. \delta - ß - Cl(\{y\})\big)$is anE $(resp. \delta - ß)$ − open set containing x but not y. Therefore, $\mathcal{X}$ is an $E(resp. \delta - ß) - \mathcal{T}_0 - Space$.

Theorem 3.8: Let $\mathcal{X}$ be a topological space. Then the following properties are equivalent:

a) $\mathcal{X}$ is an $E(resp. \delta - ß) - \mathcal{T}_1 - Space$.

b) For each point $x \in \mathcal{X}$ the singleton set $\{x\}$ isE $(resp. \delta - ß)$ − closed set,

c) For each point $x \in \mathcal{X}$, $\text{E} - D(\{x\})(resp.\, \delta - \text{ß} - D(\{x\})) = \emptyset$.

Proof: $(\boldsymbol{a}) \Longrightarrow (\boldsymbol{b})$ Let X be $E(resp.\, \delta - \text{ß}) - \mathcal{T}_1 - Space$. For each $x, y\ (x \neq y) \in \mathcal{X}$, there exists E $(resp.\, \delta - \text{ß}) -$ open set $\mathcal{U}\ (s.t)\ y \in \mathcal{U}\ but\ x \notin \mathcal{U}$. Consequently, $y \in \mathcal{U} \subseteq \mathcal{X} \setminus \{x\}$.

Thus $\mathcal{X} \setminus \{x\} = \cup\{\mathcal{U}: y \in \mathcal{X} \setminus \{x\}\}$ which is the union of an E $(resp.\, \delta - \text{ß}) -$ open sets. Then, $\mathcal{X} \setminus \{x\}$ is an E $(resp.\, \delta - \text{ß}) -$ open set. Thus $\{x\}$ *is* E $(resp.\, \delta - \text{ß}) -$ closed set.

$(\boldsymbol{b}) \Longrightarrow (\boldsymbol{a})$ Suppose that $\{\mathcal{P}\}$ is E $(resp.\, \delta - \text{ß}) -$ closed for each $\mathcal{P} \in \mathcal{X}$.

So via supposition for each $x, y\ (x \neq y) \in \mathcal{X}$, $\{x\}, \{y\}$ are E $(resp.\, \delta - \text{ß}) -$ closed sets.

Hence, $\mathcal{X} \setminus \{x\}, \mathcal{X} \setminus \{y\}$ are E $(resp.\, \delta - \text{ß}) -$ open sets such that, $x \in \mathcal{X} \setminus \{y\}, y \notin \mathcal{X} \setminus \{y\}$ and $y \in \mathcal{X} \setminus \{x\}, x \notin \mathcal{X} \setminus \{x\}$.

Therefore, $\mathcal{X}$ is an $E(resp.\, \delta - \text{ß}) - \mathcal{T}_1 -$ Space.

$(\boldsymbol{b}) \Longrightarrow (\boldsymbol{c})$ Let $\{x\}$ be E $(resp.\, \delta - \text{ß}) -$ closed set for each $x \in \mathcal{X}$. Thus,

$\{x\} = \text{E} - Cl(\{x\})(resp.\, \delta - \text{ß} - Cl(\{x\})) = \{x\} \cup \text{E} - D(\{x\})(resp.\, \delta - \text{ß} - D(\{x\}))$.

Therefore, $\text{E} - D(\{x\})(resp.\, \delta - \text{ß} - D(\{x\})) = \emptyset$.

$(\boldsymbol{c}) \Longrightarrow (\boldsymbol{b})$ Let $\text{E} - D(\{x\})(resp.\, \delta - \text{ß} - D(\{x\})) = \emptyset$, for each $x \in \mathcal{X}$.

Since, $\text{E} - Cl(\{x\})\ (resp.\, \delta - \text{ß} - Cl(\{x\}))$

$$= \{x\} \cup \text{E} - D(\{x\})(resp.\, \delta - \text{ß} - D(\{x\})).$$

Thus, $\text{E} - Cl(\{x\})\ (resp.\, \delta - \text{ß} - Cl(\{x\})) = \{x\}\ iff \{x\}\ is$ E $(resp.\, \delta - \text{ß}) -$ closed set.

$(\boldsymbol{a}) \Longrightarrow (\boldsymbol{c})$ Assume that $\mathcal{X}$ is an $E(resp.\, \delta - \text{ß}) - \mathcal{T}_1 - Space$ and suppose that, $\text{E} - D(\{x\})(resp.\, \delta - \text{ß} - D(\{x\})) \neq \varphi$ for some $x \in \mathcal{X}$, *then* $\exists\ y \in \text{E} - D(\{x\})$ $(resp.\, \delta - \text{ß} - D(\{x\}))$ and $(x \neq y)$. Since, $\mathcal{X}$ is an $E(resp.\, \delta - \text{ß}) - \mathcal{T}_1 - Space$,

so $\exists$ E $(resp.\, \delta - \text{ß}) -$ open set $\mathcal{U}\ (s.t)\ y \in \mathcal{U}\ and\ x \notin \mathcal{U}$ which implies, $\mathcal{U} \cap \{x\} = \emptyset$,

and thus $y \notin \text{E} - D(\{x\})(resp.\, \delta - \text{ß} - D(\{x\}))$, which a contradiction with the assumption. Hence, $\forall\ x \in \mathcal{X}$, $\text{E} - D(\{x\})(resp.\, \delta - \text{ß} - D(\{x\})) = \emptyset$.

$(\boldsymbol{c}) \Longrightarrow (\boldsymbol{a})$ Let $\text{E} - D(\{x\})(resp.\, \delta - \text{ß} - D(\{x\})) = \emptyset, \forall\ x \in \mathcal{X}$, consequently,

$\text{E} - Cl(\{x\})\ (resp.\, \delta - \text{ß} - Cl(\{x\})) = \{x\} \cup \text{E} - D(\{x\})(resp.\, \delta - \text{ß} - D(\{x\})) = \{x\}$. Which implies, $\{x\}$ is E $(resp.\, \delta - \text{ß}) -$ closed set and thus via (part (a) & (b)),

we have $\mathcal{X}$ is an $E(resp.\, \delta - \text{ß}) - \mathcal{T}_1 -$ Space.

Theorem 3.9: If $\mathcal{X}$ is a topological space, then the followingproperties are equivalent:

a) $\mathcal{X}$ is an $E(resp.\, \delta - \text{ß}) - \mathcal{T}_2 - Space$.

b) If $x \in \mathcal{X}$, then $\forall\ (x \neq y)$, $\exists$ an E $(resp.\, \delta - \text{ß}) -$ open set $\mathcal{U}$ *containing* $x\ (s.t)$, $y \notin \text{E} - Cl(\{\mathcal{U}\})\ (resp.\, \delta - \text{ß} - Cl(\{\mathcal{U}\}))$.

Proof: $(\boldsymbol{a}) \Longrightarrow (\boldsymbol{b})$ since $\mathcal{X}$ is an $E(resp.\, \delta - \text{ß}) - \mathcal{T}_2 - Space$, so $\forall$

$(x \neq y)\ \exists$ E $(resp. \delta -$ ß$) -$ open sets $\mathcal{U}$ & V such that $x \in \mathcal{U}$ and $y \in V\ and\ \mathcal{U} \cap \mathcal{V} = \emptyset$.

Thus, $x \in \mathcal{U} \subseteq \mathcal{X} \setminus \mathcal{V}$, put $\mathcal{X} \setminus \mathcal{V} = \mathcal{F}$, then $\mathcal{F}$ is E $(resp. \delta -$ ß$) -$ closed set, $\mathcal{U} \subseteq \mathcal{F}\ and\ y \notin \mathcal{F} \Longrightarrow y \notin \cap \{\mathcal{F}: \mathcal{F}$ is E $(resp. \delta -$ ß$) -$ closed set and $\mathcal{U} \subseteq \mathcal{F}\}$
$=$ E $- Cl(\{\mathcal{U}\})\ (resp. \delta -$ ß $- Cl(\{\mathcal{U}\}))$.

$(\boldsymbol{b}) \Longrightarrow (\boldsymbol{a})$ Suppose that $x, y\ (x \neq y) \in \mathcal{X}$, by supposition, there exists E $(resp. \delta -$ ß$) -$ open set $\mathcal{U}$ $containing\ x$ such that $y \notin$ E $- Cl(\{\mathcal{U}\})(resp. \delta -$ ß $- Cl(\{\mathcal{U}\}))$. Hence,
$y \in \mathcal{X} \setminus$ E $- Cl(\{\mathcal{U}\})(resp. \delta -$ ß $- Cl(\{\mathcal{U}\}))$ which is E $(resp. \delta -$ ß$) -$ open and
$x \notin X \setminus ($E $- Cl(\{\mathcal{U}\})(resp. \delta -$ ß $- Cl(\{\mathcal{U}\})))$. As well as,
$\mathcal{U} \cap (X \setminus$ E $- Cl(\{\mathcal{U}\})(resp. \delta -$ ß $- Cl(\{\mathcal{U}\}))) = \emptyset$. So, $\mathcal{X}$ is $E(resp. \delta -$ ß$) - \mathcal{T}_2 - Space$.,

Definition 3.10: Let $(\mathcal{X}, \mathcal{T})$ be a topological space and $\mathcal{A} \subseteq \mathcal{X}$. Then, the intersection of all E $(resp. \delta -$ ß$) -$ open subsets of $\mathcal{X}$ containing $\mathcal{A}$ is called the E $- kernal\ (resp. \delta -$ ß $- kernal)$ of $\mathcal{A}$ and it's denoted via E $- ker(\mathcal{A})(resp. \delta -$ ß $- ker(\mathcal{A}))\ of\ \mathcal{A}\ (i.e)$:

E $- ker(\mathcal{A})(resp. \delta -$ ß $- ker(\mathcal{A})) = \cap \{\mathcal{U} \in E\Sigma(\mathcal{X})(resp. \delta -$ ß$\Sigma(\mathcal{X})): \mathcal{A} \subseteq \mathcal{U}\}$.

Theorem 3.11: Let $(\mathcal{X}, \mathcal{T})$ be a topological space and $x \in \mathcal{X}$. then,
$y \in$ E $- ker(\{x\})(resp. \delta -$ ß $- ker(\{x\})\ iff x \in$ E $- Cl(\{y\})\big(resp. \delta -$ ß $- Cl(\{y\})\big)$.

Proof: Suppose that $y \notin$ E $- ker(\{x\})(resp. \delta -$ ß $- ker(\{x\})$. So, there exists E $(resp. \delta -$ ß$) -$ open set $\mathcal{U}$ $containing\ x\ (s.t)\ y \notin \mathcal{U}$. Thus we obtain,
$x \notin$ E $- Cl(\{y\})\big(resp. \delta -$ ß $- Cl(\{y\})\big)$.

In the same method we can prove the converse case.

Theorem 3.12: Let $\mathcal{A}$ $be\ a\ sub\ set\ of$ a space $\mathcal{X}$. Then,
E $- ker(\mathcal{A})(resp. \delta -$ ß $- ker(\mathcal{A})) = \{x \in \mathcal{X}$: E $- Cl(\{x\})$
$\big(resp. \delta -$ ß $- Cl(\{x\})\big) \cap \mathcal{A} \neq \emptyset\}$.

Proof: Let $x \in$ E $- ker(\mathcal{A})(resp. \delta -$ ß $- ker(\mathcal{A})$ and
E $- Cl(\{x\})\big(resp. \delta -$ ß $- Cl(\{x\})\big) \cap \mathcal{A} = \emptyset$. Therefore,
$x \notin \mathcal{X} \setminus ($E $- Cl(\{x\})(resp. \delta -$ ß $- Cl(\{x\})))$, Which is E $(resp. \delta -$ ß$) -$ open containing $\mathcal{A}$. This case is not possible, since $x \in$ E $- ker(\mathcal{A})(resp. \delta -$ ß $- ker(\mathcal{A})$.

so, E $- Cl(\{x\})\big(resp. \delta -$ ß $- Cl(\{x\})\big) \cap \mathcal{A} \neq \emptyset$. Now assume that, $x \in \mathcal{X}$ such that.
E $- Cl(\{x\})\big(resp. \delta -$ ß $- Cl(\{x\})\big) \cap \mathcal{A} = \emptyset$, and $x \notin$ E $- ker(\mathcal{A})\big(resp. \delta -$ ß $- ker(\mathcal{A})\big)$.

So, there exists an E $(resp. \delta - ß) -$ open set $\mathcal{U}$ $containing\ \mathcal{A}\ and\ x \notin \mathcal{U}$.
Let $y \in$ E $- Cl(\{x\})\big(resp. \delta - ß - Cl(\{x\})\big) \cap \mathcal{A}$. Thus,
$\mathcal{U}$ is an E $(resp. \delta - ß)$ $-$ Neighbourhood of y which does not contain x. Hence via this contradiction we obtain, $x \in$ E $- ker(\mathcal{A})(resp. \delta - ß - ker(\mathcal{A}))$ and this is the request.

Theorem 3.13: For the subsets $\mathcal{A}\ and\ \mathcal{B}$ of a space $(\mathcal{X}, \mathcal{T})$, the following properties hold :

a) $\mathcal{A} \subseteq$ E $- ker(\mathcal{A})(resp. \delta - ß - ker(\mathcal{A}))$.

b) $If\ \mathcal{A} \subseteq B \Longrightarrow$ E $- ker(\mathcal{A})(resp. \delta - ß - ker(\mathcal{A})) \subseteq$ E $- ker(\mathcal{B})(resp. \delta - ß - ker(\mathcal{B}))$

c) If $\mathcal{A}$ is E $(resp. \delta - ß) -$ open of $\mathcal{X}$, then $\mathcal{A} =$ E $- ker(\mathcal{A})(resp. \delta - ß - ker(\mathcal{A}))$.

d) E $- ker\big($E $- ker(\mathcal{A})\big)\{resp. \delta - ß - ker\big(\delta - ß - ker(\mathcal{A})\big)\} =$ E $- ker(\mathcal{A})(resp. \delta - ß - ker(\mathcal{A}))$.

Proof: The proof of sections (**a**), (**b**) and (**c**), are immediately consequences of definition(**3.10**). Now we prove section(**d**), first by sections (**a**) and (**b**) we have:
E $- ker(\mathcal{A})\big(resp. \delta - ß - ker(\mathcal{A})\big)$
$\subseteq$ E $- ker\big($E $- ker(\mathcal{A})\big)\big\{resp. \delta - ß - ker\big(\delta - ß - ker(\mathcal{A})\big)\big\}$.
If $x \notin$ E $- ker(\mathcal{A})(resp. \delta - ß - ker(\mathcal{A}))$. $So\ \exists\ \mathcal{U} \in E\Sigma(\mathcal{X})$ $(resp. \delta - ß\Sigma(\mathcal{X}))$ (s.t)
$\mathcal{A} \subseteq \mathcal{U}\ and\ x \notin \mathcal{U}$. Thus, E $- ker(\mathcal{A})(resp. \delta - ß - ker(\mathcal{A})) \subseteq \mathcal{U}$, and so we obtain:
$x \notin$ E $- ker\big($E $- ker(\mathcal{A})\big)\big\{resp. \delta - ß - ker\big(\delta - ß - ker(\mathcal{A})\big)\big\}$. Therefore,
E $- ker\big($E $- ker(\mathcal{A})\big)\big\{resp. \delta - ß - ker\big(\delta - ß - ker(\mathcal{A})\big)\big\} =$ E $- ker(\mathcal{A})(resp. \delta - ß - ker(\mathcal{A}))$.

Theorem 3.14: The following properties areequivalent for any two distinct points x and y in a topological space $(\mathcal{X}, \mathcal{T})$:

a) E $- ker(\{x\})(resp. \delta - ß - ker(\{x\}) \neq$ E $- ker(\{y\})(resp. \delta - ß - ker(\{y\}))$.

b) E $- Cl(\{x\})\big(resp. \delta - ß - Cl(\{x\})\big) \neq$ E $- Cl(\{y\})\big(resp. \delta - ß - Cl(\{y\})\big)$

Proof: $(\boldsymbol{a}) \Longrightarrow (\boldsymbol{b})$ Suppose that E $- ker(\{x\})(resp. \delta - ß - ker(\{x\}) \neq$ E $- ker(\{y\})$ $\big(resp. \delta - ß - ker(\{y\})\big)$. So there exists a point $z \in \mathcal{X}$ such that $\in$ E $- ker(\{x\})(resp. \delta - ß - ker(\{x\})$ and $z \notin$ E $- ker(\{y\})(resp. \delta - ß - ker(\{y\}))$.
Since, $z \in$ E $- ker(\{x\})(resp. \delta - ß - ker(\{x\})$. Consequently that ,
$\{x\} \cap$ E $- Cl(\{z\})(resp. \delta - ß - Cl(\{z\})) \neq \emptyset \Longrightarrow x \in$ E $- Cl(\{z\})(resp. \delta - ß - Cl(\{z\}))$.
By using, $z \notin$ E $- ker(\{y\})\big(resp. \delta - ß - ker(\{y\})\big)$. We obtain,

$\{y\} \cap E - Cl(\{z\})(resp. \delta - ß - Cl(\{z\})) = \emptyset$. Since $x \in E - Cl(\{z\})(resp. \delta - ß - Cl(\{z\}))$.

So, $E - Cl(\{x\})(resp. \delta - ß - Cl(\{x\})) \subseteq E - Cl(\{z\})(resp. \delta - ß - Cl(\{z\}))$, and $\{y\} \cap E - Cl(\{x\})(resp. \delta - ß - Cl(\{x\})) = \emptyset$. Thus, it follows that

$E - Cl(\{x\})(resp. \delta - ß - Cl(\{x\})) \neq E - Cl(\{y\})(resp. \delta - ß - Cl(\{y\}))$. Therefore,

$E - ker(\{x\})(resp. \delta - ß - ker(\{x\}) \neq E - ker(\{y\})(resp. \delta - ß - ker(\{y\}))$. implies that,

$E - Cl(\{x\})(resp. \delta - ß - Cl(\{x\})) \neq E - Cl(\{y\})(resp. \delta - ß - Cl(\{y\}))$.

$(\boldsymbol{b}) \Longrightarrow (\boldsymbol{a})$ Assume that, $E - Cl(\{x\})(resp. \delta - ß - Cl(\{x\})) \neq E - Cl(\{y\})(resp. \delta - ß - Cl(\{y\}))$. So thereexists a point $z \in \mathcal{X}$ such that,

$z \in E - Cl(\{x\})(resp. \delta - ß - Cl(\{x\}))$ and $z \notin E - Cl(\{y\})(resp. \delta - ß - Cl(\{y\}))$.

Then, there exists an E $(resp. \delta - ß) -$ open set containing z and x but not y, namely,

$y \notin E - ker(\{x\})(resp. \delta - ß - ker(\{x\}))$, and therefore

$E - ker(\{x\})(resp. \delta - ß - ker(\{x\}) \neq E - ker(\{y\})(resp. \delta - ß - ker(\{y\}))$.

Theorem 3.15: If $f: (\mathcal{X}, \mathcal{T}) \longrightarrow (\mathcal{Y}, \mathcal{T}^*)$ is an injective E $(resp. \delta - ß) -$ continuous mapping and $\mathcal{Y}$ is $\mathcal{T}_i - space$, then $\mathcal{X}$ is E $(resp. \delta - ß) - \mathcal{T}_i -$ space, where $(i = 0, 1, 2)$.

Proof: Suppose that x, y $(x \neq y) \in \mathcal{X}$, since f is injective, then $f(x) \neq f(y)$ in $\mathcal{Y}$.

But $\mathcal{Y}$ is $\mathcal{T}_0 -$ space, then there exist an open set $\mathcal{U}$ such that $f(x) \in \mathcal{U}, f(y) \notin \mathcal{U}$ OR

$f(y) \in \mathcal{U}, f(x) \notin \mathcal{U}$, since f is E $(resp. \delta - ß) -$ continuous, so $f^{-1}(\mathcal{U})$ is E $(resp. \delta - ß) -$ open set of $\mathcal{X}$ such that: $x \in f^{-1}(\mathcal{U}), y \notin f^{-1}(\mathcal{U})$ or $y \in f^{-1}(\mathcal{U}), x \notin f^{-1}(\mathcal{U})$. So

$\mathcal{X}$ is E $(resp. \delta - ß) - \mathcal{T}_0 -$ space.

The prove of other spaces such as $E(resp. \delta - ß) - \mathcal{T}_1 - Space$ and $E(resp. \delta - ß) - \mathcal{T}_2 - Space$ is similar to the proof of theorem (**3.15**) thus omitted.

Theorem 3.16: Let $f: (\mathcal{X}, \mathcal{T}) \longrightarrow (\mathcal{Y}, \mathcal{T}^*)$ be injective E $(resp. \delta - ß) -$ Irresolute map and $\mathcal{Y}$ is E $(resp. \delta - ß) - \mathcal{T}_i -$ space, then $\mathcal{X}$ is E $(resp. \delta - ß) - \mathcal{T}_i -$ space, $(i = 0, 1, 2)$.

Proof: Assume that x, y $(x \neq y) \in \mathcal{X}$, since f is injective, then $f(x) \neq f(y)$ in $\mathcal{Y}$.

But $\mathcal{Y}$ is an E $(resp. \delta - ß) - \mathcal{T}_2 -$ space, so there exist two disjoint E $(resp. \delta - ß) -$ open sets $\mathcal{U}$ and $\mathcal{V}$ such that $f(x) \in \mathcal{U}$ and $f(y) \in \mathcal{V}$.

Now, by using E $(resp. \delta - ß) -$ Irresoluteof f we obtain,

$f^{-1}(\mathcal{U})$ and $f^{-1}(\mathcal{V})$ are E $(resp. \delta - ß) -$ openset of $\mathcal{X}$ such that:

$x \in f^{-1}(\mathcal{U}), y \in f^{-1}(\mathcal{V})$ and $f^{-1}(\mathcal{U}) \cap f^{-1}(\mathcal{V}) = \emptyset$. So, $\mathcal{X}$ is E $(resp. \delta - ß) - \mathcal{T}_2 - Space$.

The prove of other spaces such as $E(resp. \delta - ß) - \mathcal{T}_0$ − Space and $E(resp. \delta - ß) - \mathcal{T}_1$ − Space is similarto the proof of theorem (**3.16**) thus omitted.

Theorem 3.17: If $f: (\mathcal{X}, \mathcal{T}) \longrightarrow (\mathcal{Y}, \mathcal{T}^*)$ is bijective E $(resp. \delta - ß)$ − open mapping and $\mathcal{X}$ is $\mathcal{T}_i - space$, then $\mathcal{Y}$ isE $(resp. \delta - ß) - \mathcal{T}_i$ − space, where $(i = 0, 1, 2)$.

Proof: Let y_1, y_2 $(y_1 \neq y_2) \in \mathcal{Y}$. since f is bijective, so there exist x_1, x_2 $(x_1 \neq x_2) \in \mathcal{X}$. such that $f(x_1) = y_1$ and $f(x_2) = y_2$. Since $\mathcal{X}$ is $\mathcal{T}_2$, then there exist two disjoint open sets $\mathcal{U}$ and $\mathcal{V}$ of $\mathcal{X}$ such that $x_1 \in \mathcal{U}$ and $x_2 \in \mathcal{V}$. Since f is E $(resp. \delta - ß)$ − open mapping,

then$f(\mathcal{U})$ and $f(\mathcal{V})$ areE $(resp. \delta - ß)$ − open sets of $\mathcal{Y}$ with $y_1 \in f(\mathcal{U})$ and $y_2 \in f(\mathcal{V})$.

Therefore, $\mathcal{Y}$ isE $(resp. \delta - ß) - \mathcal{T}_2 - Space$.

The prove of other spaces such as $E(resp. \delta - ß) - \mathcal{T}_0$ − Space and $E(resp. \delta - ß) - \mathcal{T}_1$ − Space is similar to the proof of theorem (**3.17**) thus omitted.

Theorem 3.18: An E $(resp. \delta - ß) - \mathcal{T}_0$ − Space is a topological property.

Proof: Suppose that, $f: (\mathcal{X}, \mathcal{T}) \longrightarrow (\mathcal{Y}, \mathcal{T}^*)$ is an $E(resp. \delta - ß)$ − Homeomrphism, and $x, y \in \mathcal{X}$ such that$(x \neq y)$, since f is injective, so $f(x) \neq f(y)$. Since $\mathcal{X}$ is $E(resp. \delta - ß) - \mathcal{T}_0$ − Space, ⟹ ∃ anE $(resp. \delta - ß)$ − open set $\mathcal{U}$ such that $x \in \mathcal{U}, y \notin \mathcal{U}$.

Since f is strongly – E($resp.$ strongly – $\delta - ß$) − open then, $f(\mathcal{U})$ is E $(resp. \delta - ß)$ − open set in $\mathcal{Y}$ such that $f(x) \in f(\mathcal{U}), f(y) \notin f(\mathcal{U})$. Thus $\mathcal{Y}$ is $E(resp. \delta - ß) - \mathcal{T}_0$ − Space.

Theorem 3.19: An E $(resp. \delta - ß) - \mathcal{T}_1$ − Space is atopological property.

Proof: Suppose that, $f: (\mathcal{X}, \mathcal{T}) \longrightarrow (\mathcal{Y}, \mathcal{T}^*)$ is an$E(resp. \delta - ß)$ − Homeomrphism, and $x, y \in \mathcal{X}$ such that$(x \neq y)$, since f is injective, so $f(x) \neq f(y)$. Since $\mathcal{X}$ is $E(resp. \delta - ß) - \mathcal{T}_1$ − Space, ⟹ ∃ two E $(resp. \delta - ß)$ − open sets $\mathcal{U}$ and Vsuch that $x \in \mathcal{U}$ & $y \notin U$ & $y \in V$& $x \notin V$. Since f is strongly – E($resp.$ strongly – $\delta - ß$) − open then, $f(\mathcal{U})$ and $f(\mathcal{V})$ are E $(resp. \delta - ß)$ − open set in $\mathcal{Y}$such that: $f(x) \in f(\mathcal{U}), f(y) \notin f(\mathcal{U})$ and $f(x) \notin f(\mathcal{V}), f(y) \in f(\mathcal{V})$. Thus, $\mathcal{Y}$ is E $(resp. \delta - ß) - \mathcal{T}_1$ − Space.

Theorem 3.20: An $E(resp. \delta - ß) - \mathcal{T}_2$ − Space is a topological property.

Proof: Assume that, $f: (\mathcal{X}, \mathcal{T}) \longrightarrow (\mathcal{Y}, \mathcal{T}^*)$ is an$E$ $(resp. \delta - ß)$ − Homeomrphism, and $x, y \in \mathcal{X}$ such that$(x \neq y)$, since f is injective, so $f(x) \neq f(y)$. Since $\mathcal{X}$ is

$E(resp.\delta-ß)-\mathcal{T}_2-$ Space,

$\Longrightarrow \exists$ two disjointE $(resp.\delta-ß)-$ open sets $\mathcal{U}$ and $\mathcal{V}$ such that $x\in\mathcal{U}$ and $y\in\mathcal{V}$. Since f is strongly – E $(resp.$ strongly – $\delta-ß)-$ open then, $f(\mathcal{U})$ and $f(\mathcal{V})$ are two disjoint E $(resp.\delta-ß)-$ open sets in $\mathcal{Y}$such that: $f(x)\in f(\mathcal{U})$ and $f(y)\in f(\mathcal{V})$. Thus, $\mathcal{Y}$ is $E(resp.\delta-ß)-\mathcal{T}_2-$ Space.

4. FUNDAMENTAL PROPERTIES OF E $(resp.\delta-ß)-$ REGULARITY AND E $(resp.\delta-ß)-$ NORMALITY

In this part, the presentation ofE $(resp.\delta-ß)-$ Regular spaces and E $(resp.\delta-ß)-$ Normal spaces, and explores a portion of some important their characterizations and several of their fundamental properties.

Definition 4.1: A Topological space$(\mathcal{X},\mathcal{T})$is said to beE $(resp.\delta-ß)-$ Regular space if for each closed set $\mathcal{F}\subseteq\mathcal{X}$and each point $x\in\mathcal{X}$ such that $x\in\mathcal{X}\backslash\mathcal{F}$, there exist two disjoint E $(resp.\delta-ß)-$ open sets $\mathcal{U}$ and $\mathcal{V}$such that$\mathcal{F}\subseteq\mathcal{U}$, $x\in\mathcal{V}$.

Theorem 4.2: For a space $(\mathcal{X},\mathcal{T})$ the following statements are equivalent:

i) $\mathcal{X}$ is E $(resp.\delta-ß)-$ Regular,

ii) For each closed set $\mathcal{F}\subseteq\mathcal{X}$ and $x\in\mathcal{X}\backslash\mathcal{F}$, E $(resp.\delta-ß)-$ open set $\mathcal{U}$ such that $x\in\mathcal{U}\subseteq \text{E}-Cl(\{\mathcal{U}\})$ $(resp.$ $\delta-ß-Cl(\{\mathcal{U}\}))\subseteq\mathcal{X}\backslash\mathcal{F}$.

Proof: $(\boldsymbol{i})\Longrightarrow(\boldsymbol{ii})$Let $\mathcal{X}$ be an E $(resp.\delta-ß)-$ Regular space, $\mathcal{F}\subseteq\mathcal{X}$and $x\notin\mathcal{F}$, there exist two disjoint E $(resp.\delta-ß)-$ open sets $\mathcal{U}$ and $\mathcal{V}$ such that$x\in\mathcal{U}$ and $\mathcal{F}\subseteq\mathcal{V}=\mathcal{X}\setminus \text{E}-Cl(\{\mathcal{U}\})$ $(resp.\delta-ß-Cl(\{\mathcal{U}\}))$. Since $\mathcal{F}\subseteq\mathcal{X}\backslash \text{E}-Cl(\{\mathcal{U}\})\big(resp.\delta-ß-Cl(\{\mathcal{U}\})\big)$, soE $-Cl(\{\mathcal{U}\})\big(resp.\delta-ß-Cl(\{\mathcal{U}\})\big)\subseteq\mathcal{X}\backslash\mathcal{F}$. Thus, $x\in\mathcal{U}\subseteq \text{E}-Cl(\{\mathcal{U}\})(resp.\delta-ß-Cl(\{\mathcal{U}\}))\subseteq\mathcal{X}\backslash\mathcal{F}$.

$(\boldsymbol{ii})\Longrightarrow(\boldsymbol{i})$ Let $x\in\mathcal{X}$ and $\mathcal{F}\subseteq\mathcal{X}\backslash\{x\}$ be closed set such that, $x\in\mathcal{U}\subseteq \text{E}-Cl(\{\mathcal{U}\})(resp.\delta-ß-Cl(\{\mathcal{U}\}))\subseteq\mathcal{X}\backslash\mathcal{F}$. So $\mathcal{F}\subseteq\mathcal{X}\backslash \text{E}-Cl(\{\mathcal{U}\})$ $(resp.\delta-ß-Cl(\{\mathcal{U}\}))$, which is anE $(resp.\delta-ß)-$ open set and disjoint with $\mathcal{U}$. Thus $\mathcal{X}$ is E $(resp.\delta-ß)-$ Regular.

Theorem 4.3: Let $\mathcal{X}$ be an E $(resp.\delta-ß)-$ Regular space, for any two points $x,y\in\mathcal{X}$, then either: E $-Cl(\{x\})(resp.\delta-ß-Cl(\{x\}))$ $=\text{E}-Cl(\{y\})(resp.\delta-ß-Cl(\{y\}))$ OR E $-Cl(\{x\})$ $(resp.\delta-ß-Cl(\{x\}))\cap \text{E}-Cl(\{y\})(resp.\delta-ß-Cl(\{y\}))=\emptyset$.

Proof: Suppose that E $-Cl(\{x\})\big(resp.\delta-ß-Cl(\{x\})\big)\neq$ E $-Cl(\{y\})\big(resp.\delta-ß-Cl(\{y\})\big)$ then either $x\notin \text{E}-Cl(\{y\})\big(resp.\delta-ß-Cl(\{y\})\big)$ OR $y\notin$ E $-Cl(\{x\})\big(resp.\delta-ß-Cl(\{x\})\big)$. Assume that

$y \notin$ E $- Cl(\{x\})(resp. \delta -$ ß $- Cl(\{x\}))$. Since $\mathcal{X}$ is E $(resp. \delta -$ ß$)$ $-$ Regular, then there exists an E $(resp. \delta -$ ß$) -$ open set $\mathcal{U}$ such that E $- Cl(\{x\})(resp. \delta -$ ß $- Cl(\{x\}))$ $\subseteq \mathcal{U}$and $y \in \mathcal{X}\backslash\mathcal{U}$. Where $\mathcal{X}\backslash\mathcal{U}$ is E $(resp. \delta -$ ß$) -$ closed and E $- Cl(\{y\})(resp. \delta -$ ß $- Cl(\{y\})) \subseteq \mathcal{X}\backslash\mathcal{U}$. Thus, E $- Cl(\{x\})(resp. \delta -$ ß $- Cl(\{x\})) \cap$ E $- Cl(\{y\})(resp. \delta -$ ß $- Cl(\{y\})) \subseteq \mathcal{U} \cap (\mathcal{X}\backslash\mathcal{U}) = \emptyset$.

Theorem 4.4: Suppose that $f: (\mathcal{X}, \mathcal{T}) \longrightarrow (\mathcal{Y}, \mathcal{T}^*)$ is a bijective continuous and strongly $-$E$(resp. \delta -$ ß$) -$ open mapping and $\mathcal{X}$ isE $(resp. \delta -$ ß$)$ $-$ Regular space, then $\mathcal{Y}$is E $(resp. \delta -$ ß$) -$ Regular.

Proof: Assume that $\mathcal{F} \subseteq \mathcal{Y}$ is a closed set and $y \in \mathcal{Y}\backslash \mathcal{F}$. Since f is bijective continuous, So $f^{-1}(\mathcal{F})$ is closed of $\mathcal{X}$. Put $f(x) = y$, then $x \in \mathcal{X}\backslash f^{-1}(F)$. Since $\mathcal{X}$ is E $(resp. \delta -$ ß$) -$ Regular space, so there exist two disjoint E $(resp. \delta -$ ß$) -$ opensets $\mathcal{U}$ *and* $\mathcal{V}$ such that $x \in \mathcal{U}$ and $f^{-1}(\mathcal{F}) \subseteq \mathcal{V}$. Since f is bijective and strongly $-$ E$(resp. \delta -$ ß$) -$ open mapping Therefore, $y \in f(\mathcal{U})$ and $\mathcal{F} \subseteq f(\mathcal{V})$ and $f(\mathcal{U}) \cap f(\mathcal{V}) = \emptyset$. Thus $\mathcal{Y}$ is E$(resp. \delta -$ ß$) -$ Regular space.

Theorem 4.5: Let $f: \mathcal{X} \longrightarrow \mathcal{Y}$ be an injectiveE $(resp. \delta -$ ß$) -$ irresolute and closed mapping and $\mathcal{Y}$ is an E $(resp. \delta -$ ß$) -$ Regular space, then $\mathcal{X}$ isE $(resp. \delta -$ ß$)$ $-$ Regular.

Proof: Suppose that $\mathcal{F} \subseteq \mathcal{X}$ is a closed set and $x \notin \mathcal{F}$. Since f is injective closed mapping, so$f(\mathcal{F})$is closed of $\mathcal{Y}$ and $f(x) \notin f(\mathcal{F})$, thus $f(x) \in \mathcal{Y}\backslash f(\mathcal{F})$. Since $\mathcal{Y}$ is E $(resp. \delta -$ ß$) -$ Regular space, so there exist two disjoint E $(resp. \delta -$ ß$) -$ opensets $\mathcal{U}$ *and* $\mathcal{V}$ $(s.t)$ $f(x) \in \mathcal{V}$ and $f(\mathcal{F}) \subseteq \mathcal{U}$. Since f is E $(resp. \delta -$ ß$) -$ irresolute mapping, therefore $\mathcal{F} \subseteq f^{-1}(\mathcal{U})$ and $x \in f^{-1}(\mathcal{V})$ & $f^{-1}(\mathcal{U}) \cap f^{-1}(\mathcal{V}) = \emptyset$. Thus $\mathcal{X}$ isE $(resp. \delta -$ ß$) -$ Regular.

Theorem 4.6: A E $(resp. \delta -$ ß$) -$ Regular spaceis a topological property.

Proof: Suppose that $f: (\mathcal{X}, \mathcal{T}) \longrightarrow (\mathcal{Y}, \mathcal{T}^*)$ is $E(resp. \delta -$ ß$) -$ Homeomrphism. Then f is a bijective strongly $-$ E$(resp. \delta -$ ß$) -$ open continuous mapping. Let $\mathcal{F} \subseteq \mathcal{Y}$ be a closed set and $y \in \mathcal{Y}\backslash\mathcal{F}$, so $f^{-1}(\mathcal{F})$is closed set of$\mathcal{X}$ & $x \in X\backslash f^{-1}(\mathcal{F})$. Since $\mathcal{X}$ is E $(resp. \delta -$ ß$) -$

Regular space, so there exist two disjoint E $(resp. \delta - ß) -$ open sets $\mathcal{U}$ *and* $\mathcal{V}$ such that $x \in \mathcal{U}$ and $f^{-1}(\mathcal{F}) \subseteq \mathcal{V}$. Since f is strongly $-$ E$(resp. \delta - ß) -$ open, then $y \in f(\mathcal{U})$ *and* $\mathcal{F} \subseteq f(\mathcal{V})$ such that $f(\mathcal{U}) \cap f(\mathcal{V}) = \emptyset$. So $\mathcal{Y}$ is E $(resp. \delta - ß) -$ Regular.

Definition 4.7: A Topological space$(\mathcal{X}, \mathcal{T})$is said to be E $(resp. \delta - ß) -$ Normal if for each pair of disjoint closed sets $\mathcal{F}_1$ *and* $\mathcal{F}_2$ there exist two disjoint E $(resp. \delta - ß) -$ open sets $\mathcal{U}$ *and* $\mathcal{V}$ such that, $\mathcal{F}_1 \subseteq \mathcal{U}, \mathcal{F}_2 \subseteq \mathcal{V}$.

Theorem 4.8: the following statements are equivalent for a space$(\mathcal{X}, \mathcal{T})$:

i) $\mathcal{X}$ is E $(resp. \delta - ß) -$ Normal,

ii) For every pair of open sets $\mathcal{U}$ and $\mathcal{V}$ such that $\mathcal{U} \cup \mathcal{V} = \mathcal{X}$, there exists anE $(resp. \delta - ß) -$ closed sets $\mathcal{A}$ *and* $\mathcal{B}$ such that $\mathcal{A} \subseteq \mathcal{U}, \mathcal{B} \subseteq \mathcal{V}$ and $\mathcal{A} \cup \mathcal{B} = \mathcal{X}$,

iii) For every closed set $\mathcal{F}$and every open set H containing $\mathcal{F}$, there exists E $(resp. \delta - ß) -$ open set $\mathcal{U}$ such that, $\mathcal{F} \subseteq \mathcal{U} \subseteq \mathrm{E} - Cl(\{\mathcal{U}\})(resp. \delta - ß - Cl(\{\mathcal{U}\})) \subseteq H$.

Proof: $(i) \Longrightarrow (ii)$ Suppose that $\mathcal{U}$ and $\mathcal{V}$ are two open sets in E $(resp. \delta - ß) -$ Normal space $\mathcal{X}$ (s.t) $\mathcal{U} \cup \mathcal{V} = \mathcal{X}$. So $\mathcal{X} \backslash \mathcal{U}$ and $\mathcal{X} \backslash \mathcal{V}$ are disjoint closed sets. Since $\mathcal{X}$ is E $(resp. \delta - ß) -$ Normal space, so there exist two disjoint E $(resp. \delta - ß) -$ open sets $\mathcal{U}_1$ *and* $\mathcal{V}_1$ such that $\mathcal{X} \backslash \mathcal{U} \subseteq \mathcal{U}_1$ *and* $\mathcal{X} \backslash \mathcal{V} \subseteq \mathcal{V}_1$. Assume that $\mathcal{A} = \mathcal{X} \backslash \mathcal{U}_1$ *and* $B = \mathcal{X} \backslash \mathcal{V}_1$.

Therefore, $\mathcal{A}$ *and* B are E $(resp. \delta - ß) -$ closed sets $(s.t)$ $\mathcal{A} \subseteq \mathcal{U}, B \subseteq \mathcal{V}$ *and* $\mathcal{A} \cup B = \mathcal{X}$.

$(ii) \Longrightarrow (iii)$ Suppose that $\mathcal{F} \subseteq \mathcal{X}$ is a closed set and H be an open set containing $\mathcal{F}$. So $\mathcal{X} \backslash \mathcal{F}$ and H are open sets such that $\mathcal{X} \backslash \mathcal{F} \cup H = \mathcal{X}$. Consequently via part (**ii**) there exist twoE $(resp. \delta - ß) -$ closed sets K_1 and K_2 such that $K_1 \subseteq \mathcal{X} \backslash \mathcal{F}$ and $K_2 \subseteq H$ and $K_1 \cup K_2 = \mathcal{X}$. Then, $\mathcal{F} \subseteq \mathcal{X} \backslash K_1$ and $X \backslash H \subseteq \mathcal{X} \backslash K_2$ and $(\mathcal{X} \backslash K_1) \cap (\mathcal{X} \backslash K_2) = \emptyset$. Let $\mathcal{U} = \mathcal{X} \backslash K_1$ and $\mathcal{V} = \mathcal{X} \backslash K_2$. Thus $\mathcal{U}$ and $\mathcal{V}$ are disjoint E $(resp. \delta - ß) -$ open sets such that $\mathcal{F} \subseteq \mathcal{U} \subseteq \mathcal{X} \backslash \mathcal{V} \subseteq H$. So $\mathcal{F} \subseteq \mathcal{U} \subseteq \mathrm{E} - Cl(\{\mathcal{U}\})(resp. \delta - ß - Cl(\{\mathcal{U}\})) \subseteq H$.

(**iii**) $\Longrightarrow$ (**i**) Let $\mathcal{F}_1$ and $\mathcal{F}_2$be two disjoint closed sets such that$\mathcal{F}_1$ and $\mathcal{F}_2 \subseteq \mathcal{X}$. Put $H = X \backslash F_2$, so $\mathcal{F}_1 \subseteq H$ where H is an open set. Via part (**iii**) $\exists$ E $(resp. \delta - ß) -$ openset $\mathcal{U} \subseteq \mathcal{X}$ such that$\mathcal{F}_1 \subseteq \mathcal{U} \subseteq \mathrm{E} - Cl(\{\mathcal{U}\})(resp. \delta - ß - Cl(\{\mathcal{U}\})) \subseteq H$. Consequently that $\mathcal{F}_2 \subseteq \mathcal{X} \backslash H \subseteq \mathcal{X} \backslash \mathrm{E} - Cl(\{\mathcal{U}\})(resp. \delta - ß - Cl(\{\mathcal{U}\})) = \mathcal{V}$. Then, there exist two E $(resp. \delta - ß) -$ open sets $\mathcal{U}$ and $\mathcal{V}$ such that$\mathcal{F}_1 \subseteq \mathcal{U}$ *and* $\mathcal{F}_2 \subseteq \mathcal{V}$ and $\mathcal{U} \cap \mathcal{V} = \emptyset$.

So, $\mathcal{X}$ is E $(resp. \delta - ß) -$ Normal.

Theorem 4.9: Let $f: (\mathcal{X}, \mathcal{T}) \longrightarrow (\mathcal{Y}, \mathcal{T}^*)$ be a surjective strongly $-$ E ($resp. \delta -$ ß) $-$ open continuous and E ($resp. \delta -$ ß) $-$ irresolute mapping from an E ($resp. \delta -$ ß) $-$ Normal space $\mathcal{X}$ onto $\mathcal{Y}$, then $\mathcal{Y}$ is E ($resp. \delta -$ ß) $-$ Normal.

Proof: Suppose that $\mathcal{F} \subseteq \mathcal{Y}$ is a closed set and $\mathcal{A}$ be an open set containing $\mathcal{F}$. So via continuity of f, we get $f^{-1}(\mathcal{F})$ is closed and $f^{-1}(\mathcal{A})$is open of $\mathcal{X}$ (s.t) $f^{-1}(\mathcal{F}) \subseteq f^{-1}(\mathcal{A})$.

Via E ($resp. \delta -$ ß) $-$ Normality of $\mathcal{X}$ and via(**Theorem 4.8**), $\exists$ an E ($resp. \delta -$ ß) $-$ open set $\mathcal{U} \subseteq \mathcal{X}$ (s.t) $f^{-1}(\mathcal{F}) \subseteq \mathcal{U} \subseteq \text{E} - Cl(\{\mathcal{U}\})$ ($resp. \delta -$ ß $- Cl(\{\mathcal{U}\})$) $\subseteq f^{-1}(\mathcal{A})$.

Then, $f(f^{-1}(\mathcal{F})) \subseteq f(\mathcal{U}) \subseteq f\left(\text{E} - Cl(\{\mathcal{U}\})(resp. \delta - \text{ß} - Cl(\{\mathcal{U}\}))\right) \subseteq f(f^{-1}(\mathcal{A}))$.

Since f is surjective strongly $-$ E ($resp. \delta -$ ß) $-$ open andE ($resp. \delta -$ ß) $-$ irresolute mapping, so we get $\mathcal{F} \subseteq f(\mathcal{U}) \subseteq \text{E} - Cl(\{f(\mathcal{U})\})$ $\left(resp. \delta - \text{ß} - Cl(\{f(\mathcal{U})\})\right) \subseteq \mathcal{A}$.

So, $\mathcal{Y}$ is E ($resp. \delta -$ ß) $-$ Normal space.

Theorem 4.10: Let $f: (\mathcal{X}, \mathcal{T}) \longrightarrow (\mathcal{Y}, \mathcal{T}^*)$ be a bijective continuous and strongly $-$ E ($resp. \delta -$ ß) $-$ open mapping from aE ($resp. \delta -$ ß) $-$ Normal space $\mathcal{X}$ onto $\mathcal{Y}$, then $\mathcal{Y}$ is E ($resp. \delta -$ ß) $-$ Normal.

Proof: Assume that $\mathcal{F}_1$ and $\mathcal{F}_2$ are two disjoint closed sets of $\mathcal{Y}$. Since f is continuous, so $f^{-1}(\mathcal{F}_1)$ and$f^{-1}(\mathcal{F}_2)$are disjoint closed sets of $\mathcal{X}$. Since $\mathcal{X}$ is E ($resp. \delta -$ ß) $-$ Normal space, then there exist two disjoint E ($resp. \delta -$ ß) $-$ open sets $\mathcal{U}$ *and* $\mathcal{V}$ such that $f^{-1}(\mathcal{F}_1) \subseteq \mathcal{U}$ and $f^{-1}(\mathcal{F}_2) \subseteq \mathcal{V}$. Via bijective and strongly $-$ E ($resp. \delta -$ ß) $-$ openof a mapping f, we obtain $\mathcal{F}_1 \subseteq f(\mathcal{U})$ & $\mathcal{F}_2 \subseteq f(\mathcal{V})$& $f(\mathcal{U}) \cap f(\mathcal{V}) = \emptyset$.

So, $\mathcal{Y}$ is E ($resp. \delta -$ ß) $-$ Normal space.

Theorem 4.11: Let $f: (\mathcal{X}, \mathcal{T}) \longrightarrow (\mathcal{Y}, \mathcal{T}^*)$ be an injective closed and E ($resp. \delta -$ ß) $-$ irresolute mapping and $\mathcal{Y}$ beE ($resp. \delta -$ ß) $-$ Normal space, then $\mathcal{X}$ *is* E ($resp. \delta -$ ß) $-$ Normal.

Proof: Suppose that $\mathcal{F}_1$ *and* $\mathcal{F}_2$ are two disjoint closed sets of $\mathcal{X}$. Since f is closed mapping, so $f(\mathcal{F}_1)$ *and* $f(\mathcal{F}_2)$ are disjoint closed sets of $\mathcal{Y}$. Since $\mathcal{Y}$ is E ($resp. \delta -$ ß) $-$ Normal space, there exist twodisjoint E ($resp. \delta -$ ß) $-$ open sets $\mathcal{U}$ *and* $\mathcal{V}$ $(s.t)$ $f(\mathcal{F}_1) \subseteq \mathcal{U}$ & $f(\mathcal{F}_2) \subseteq \mathcal{V}$.

Via injective and E ($resp. \delta -$ ß) $-$ irresolute of a mapping f, we obtain $\mathcal{F}_1 \subseteq f^{-1}(\mathcal{U})$ and

$\mathcal{F}_2 \subseteq f^{-1}(\mathcal{V})$ and $f^{-1}(\mathcal{U}) \cap f^{-1}(\mathcal{V}) = \emptyset$. So, $\mathcal{X}$ isE $(resp. \delta - ß) -$ Normal.

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CONCLUSION

" The class of generalized open sets has an essential role in general topology, especially its suggestion of new separation axioms which are useful in digital topology. "Many topologists worldwide are focusing their researches on these topics Indeed a significant theme in General Topology, Real analysis and many other branches of mathematics concerns the variously modified forms of separation axioms by utilizing generalized open sets". One of the well-known concepts and that expected it will has a wide applying in physics and topology and their applications is the notion of *E* and *δ-ß*-open sets. "In this work we introduced and studied new generalized types of separation axioms namely, *E* and *δ-ß*-separation axioms. Several fundamental properties concerning of these classes of generalized separation axioms are obtained. Furthermore, *E (resp. δ-ß)*-Regularity and *E (resp. δ-ß)*- Normality are investigated in the context of these new concepts. Also the fuzzy topological version of the concepts and results introduced in this paper are very important, since El-Naschie has shown that the notion of fuzzy topology has very important applications in quantum particle physics especially in related to superstring theory, string theory and ε^{∞} theory **[21, 22, 23]**".

REFERENCES

[1] J. Dontchev and M. Ganster. On δ-generalized closed sets and T3/4 -spaces. Mem. Fac.
Sci. Kochi Univ. Ser. A Math., 17, (1996), 15-31.
[2] V. Kovalevsky and R. Kopperman. Some topology-based image processing algorithms.
Ann NY Acad. Sci., 728, (1994), 174-182.
[3] G. Navalagi. Semi-generalized separation axioms in topology, IJMCA, 3(1-2), (2011), 23- 31.
[4] A. Açıkgöz. On ß*g−closed Sets and New Separation Axioms. European journal of pure and applied mathematics. 4(1), (2011), 20-33.
[5] H. Z. Ibrahim. Bc-Separation Axioms in Topological Spaces. Gen. Math. Notes, 17(1), (2013), 45-62.
[6] H. A. Khaleefah. New Types of Separation Axioms via Generalized B-Open Sets. Research Journal of Mathematical and Statistical Sciences. 1(8), (2013), 16-24.
[7] M. K. Singal and S. P. Arya. On almost normal and almost completely regular spaces, Glasnik Mat., 5 (5) (1970), 141-152.
[8] M. Ganster, S. Jafari and G. B. Navalagi. On semi-g-regular and semi-g-normal spaces, Demonstratio Math., 35 (2) (2002), 415-421.
[9] J. K. Kohli and A. K. Das, New normality axioms and decompositions of normality, Glasnik Mat., 37 (57) (2002), 163-173.

[10] E. Ekici. On γ-normal spaces, Bull. Math. Soc. Sci. Math. Roumanie, Tome 50(98) No. 3, 2007, 259–272.

[11] A. I. EL-Maghrabi and M. A. AL-Juhani. New separation axioms by M-Open sets, International Journal of Mathematical Archive., 4 (6) (2013), 93-100.

[12] B. K. Tyagi and H. V. Chauhan. On some separation axioms in generalized topological spaces. Questions and Answers in General Topology, (36), (2018), 9-29.

[13] S. H. Abdulwahid and A. M. F. AL. Jumaili. On Ec-(δ-ßc)-Compact Spaces and Ec-(δ-ßc)-Separation Axioms, SCIREA Journal of Mathematics, 5 (4) (2020), 44-60.

[14] M. H. Stone, Applications of the theory Boolean rings to general topology, Trans. Amer. Math. Soc., 41 (1937), 375–381.

[15] N. V. Velicko, H-closed topological spaces, Amer. Math. Soc. Transl., 2 (78), 103–118 (1968)

[16] E. Ekici, on e-open sets, DP*-sets and DPE*-sets and decompositions of continuity, The Arabian J. for Sci. Eng., 33 (2A) (2008), 269–282.

[17] E. Hatir, and T. Noiri. Decompositions of continuity and complete continuity, Acta, Math. Hungar, 113 (4), 281-287, (2006)

[18] E. Ekici, On e*-Open Sets and (D, S)*-Sets, Math. Moravica, 13 (1) 29-36, (2009).

[19] E. Hatir and T. Noiri. On δ-ß-continuous functions, Chaos, Solitons and Fractals, 42, 205-211, (2009).

[20] E. Ekici, New forms of contra-continuity, Carpathian J. Math. 24(1) (2008) 37–45.

[21] M. S. El-Naschie, Topics in the mathematical physics of E-infinity theory, Chaos, Solitons & Fractals. 30 (3), 656-663, (2006).

[22] M. S. El-Naschie. On the unification of heterotic strings, M theory and ε^{∞} theory. Chaos, Solitons & Fractals. 11, 2397–408, (2000).

[23] M. S. El-Naschie. A review of E-infinity theory and the mass spectrum of high energy particle physics. Chaos, Solitons & Fractals. 19, 209–36, (2004).