PalArch's Journal of Archaeology of Egypt / Egyptology

FINITE COPIES OF BARBELL GRAPH AND ITS HARMONIOUS CHROMATIC NUMBER

Ms.V.Kavitha[1], *Dr.R.Govindarajan*[2]

[1.] Research scholar, University of Madras ,Chennai and Assistant Professor,Vel Tech Multi Tech Dr.RangarajanDr.Sakunthala Engineering College ,Avadi, Chennai 600062,India.

[2.] Associate Professor & Head, PG & Research Department of Mathematics(Retired), D.G. Vaishnav College(Autonomous), Chennai-600106,India.

Ms.V.Kavitha[1], Dr.R.Govindarajan[2], Finite copies of barbell graph and its harmonious chromatic number -- Palarch's Journal Of Archaeology Of Egypt/Egyptology 17(9). ISSN 1567-214x Key words: Barbell graph , complete graphs, Central graphs, HarmoniousColouring.

ABSTRACT:

. In this paper, we find the achromatic number of four copies of **complete** graphwith their harmonious chromatic number.The harmonious Chromatic number $\chi_h(G)$ of a graph is the least number of colours in such a colouring.

Key words:

Barbell graph, complete graphs, Central graphs, HarmoniousColouring.

Introduction

Harmonious colouring number is used in the different families of graph such as trees, cycles, complete bipartite graphs etc. In this topic more than fifty papers are published.

A harmonious Colouring of a simple graph G is a proper vertex colouring such that each pair of colours appears together on at most one edge. The harmonious Chromatic number $\chi_h(G)$ of a graph is the least number of colours in such a colouring, where G is a finite un directed graph with no loops and multiple edges.

The harmonious colouring and line-distinguishing colouring problems seem quite natural thus it is relatively easy to come up with potential applications in communication networks (i.e. transportation networks, computer networks, etc), since requesting each edge to have a unique colour that depends on the assignment of colours on the vertices can be translated as assigning ids to the network nodes such that each communication link can be distinguished.

DEFINITIONS

1.Vertex Colouring

A k-vertex colouring of a graph G, or simply a k- colouring is an assignment of k colours to its vertices. The colouring is proper if no two adjacent vertices are assigned the same colour.

A graph is k- colourable if it has a proper k- colouring.

2.Edge Colouring

A k-edge colouring of a graph G, or simply a k- colouring is an assignment of k colours to its edges. The colouring is proper if no two adjacent edges are assigned the same colour.

A graph is k-edge colourable if it has a proper k-edge colouring.

3.Chromatic Number

The chromatic number of a graph G is the least k for which G is k-vertex colourable and it denoted by $\chi(G)$ .A graph G is k-chromatic if $\chi(G)$ = k.

The chromatic number of a graph G is the least k for which G is k-vertex colourable and it denoted by $\chi^{'}(G)$ .A graph G is k-chromatic if $\chi^{'}(G)$ = k.

4. Line Distinguishing Colouring

Let G(V,E) be a graph. A colouring $\phi : V \rightarrow$ N of the vertices is a line distinguishing colouring iff for every edge (u, v) ε E the edge colour (ϕ (u), ϕ (v)) is unique,(i.e). it appears at most once.

5. Harmonious Colouring and Harmonious Chromatic Number

A harmonious colouring of a graph G(V,E) is a line-distinguishing colouring which is also proper. The harmonious chromatic number of G (denoted by χ_h(G)) is the smallest number k such that there exists a harmonious colouring of G of k colors.

6. Exact coloring

An exact coloring of a graph G(V,E) is a *coloring which is harmonious and co*mplete at the same time.

7. Clique

A clique is a complete sub graph, an independent set is an empty subgraph.

8. (2,n) Barbell graph

The (2,n) - Barbell graph is the simple graph obtained by connecting two copies of a complete graph $K_n$by a bridge and it is denoted by B(K_n,K_n).

From the observation of barbell graph (definition *8)* we define the following definition:

9. (3,n) Barbell graph

The (3,n) - Barbell graph is the simple graph obtained by connecting three copies of a complete graph $K_n$by a bridge and it is denoted by B(K_n ,K_n ,K_n).

10. (N,n) Barbell graph

In general the (N,n) - Barbell graph is the simple graph obtained by connecting N copies of a complete graph $K_n$by a bridge and it is denoted by B(K_n ,K_n,K_n , …N times K_n).

11.Central graph

The central graph of any graph G is obtained by subdividing each edge of G exactly once and joining all the non adjacent vertices of G.

Example 1

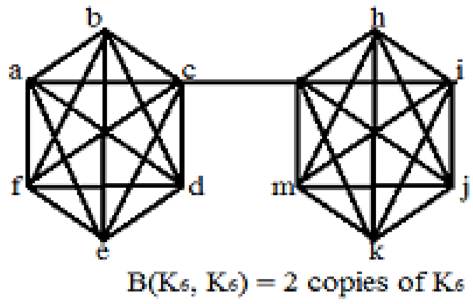

Fig1. Barbell Graph

OBSERVATION

Theorem 1:

For any complete graph K_n, $\chi_h[B(K_n,K_n)]=2n-1$, $n \geq 2$. Here $B(K_n,K_n)$ Satisfies the following properties

(i)The number of vertices in $B(K_n,K_n)$ is 2n. (ii) The number of edges in $B(K_n,K_n)$ is n^2-n+1.

(iii) The maximum degree in $B(K_n,K_n)$ is n. (iv) The minimum degree in $B(K_n, K_n)$ is n-1.

Theorem 2:

For any complete graph K_n, $\chi_h [B(K_n,K_n, K_n)]=3n-2$, $n\geq 2$.

We improved the result in the previous paper for 4 copies of barbell graph by our definition (12) it has given below :

Theorem 3:

For any complete graph K_n, four copies barbell graph and its harmonious chromatic number is$\chi_h [B(K_n,K_n, K_n,K_n)]=4n-3$, $n\geq 2$

***Proof*:**

Let G= $B(K_n,K_n,K_n,K_n)$ be the Barbell graph. By the definition, (4,n) Barbell graph is obtained by connecting four copies of complete graph K_n by a bridge.

Let $U=\{u_1,u_2,u_3,\ldots\ldots,u_n\}$be the vertex set of K^1 ,$V = \{v_1, v_2 , v_3\ldots\ldots ,v_n\}$ be the vertex set of K^2,$W=\{ w_1, w_2 , w_3\ldots\ldots ,w_n\}$ be the vertex set of $K^3$and $X==\{ x_1, x_2 , x_3\ldots\ldots ,x_n\}$ be the vertex set of K^4.

Now the colouring assignments are as follows. Since K^1 contains exactly 'n' vertices ($n \geq 2$) which are mutually adjacent to each other, we should colour all the n vertices of 'K^1' by n different colours B_1^i ,wherei=1,2….n.

For colouring 'K^2', we should use 'n-1' different colours C_1^i apart from B_1^i and any one colour from B_1^i ,$1\leq i\leq n$. andcolouring'K^3'

using 'n-1' different colours D_1^i apart from B_1^i and C_1^i any one colour from C_1^i ,$1\leq i\leq n$

Similarly colour and 'K^4' we should use 'n' different colours apart from B_1^i, C_1^i and any one colour from D_1^i ,$1\leq i\leq n$.Thus the number of colours required =n+(n-1)+(n-1)+(n-1)=4n-3.

Consider the example four copies of complete graph K_6

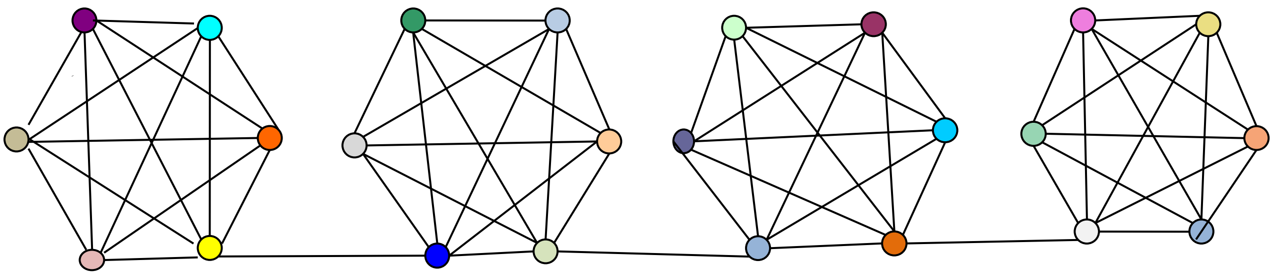

Fig.2 χ_h $[B(K_6,K_6, K_6,K_6)]=4(6)-3= 21$, $n\geq 2$

Theorem 3

The harmonious colouring of central graph of N copies of Barbell graph B(n,N) chromatic number is $\chi_h(C(B(n,N)) = nN - 1$,$n\geq 3$.

Proof:

- Let us consider two copies of 4 vertices complete graph. Now subdivide the each edge into two equal parts.
- The structural properties of central graph of barbell graph is defined as follows. The number of vertices is 2n and the number of edges is n^2-n+1. The maximum degree is n. Also the number of vertices for central graph is n^2+ n+1and the edges in the central graph is $3n^2$-2n+1.
- Colour the first copy of complete graph in the clockwise direction and the second copy in the anticlockwise direction.
- Let $u_i$and $u_i^1$where $1\leq i \leq n$ be the vertices of the edge connecting the two complete graphs. Consider threecolour class $C=\{C_1,C_2,\ldots,C_n\}$,$1\leq i \leq n$ and $C^/=\{C_{n+1},C_{n+2},\ldots C_{2n-1}\}$, $n+1\leq i \leq 2n-1$. Using minimum colours assign C_i to u_i andassign $C_i^{\,/}$ to $u_i^{\,/}$ for the harmonious colouring.

Example 2:

The harmonious colouring of central graph of 2 copies of Barbell graph B(n,N) chromatic number is $\chi_h(C(B(2,4)) = 2*4-1=7$,$n\geq 3$.

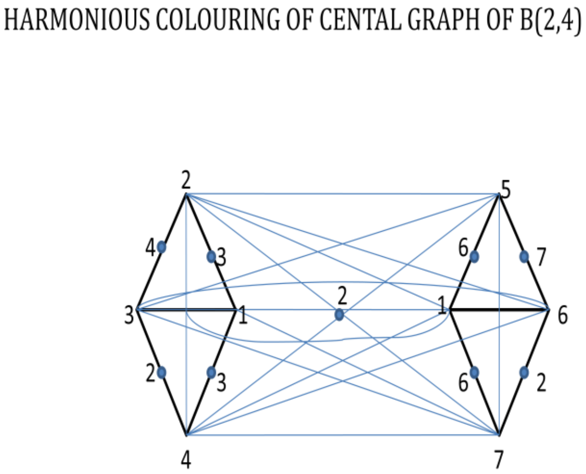

Fig.3 $\chi_h(C(B(2,4)) = 7$

***Conclusion*:**

In this research paper, we improved four copiesof complete graph for barbell graph and their harmonious chromatic number of graphs.Also N copies central graph of Barbell graph and its chromatic number. These types of colouring have several applications networks and communication field.Also it has to be used in data compression (design of minimal hash functions) and clustering.

REFERENCES

[1] Colin Mc Diarmid Luo Xinhua , “Upper Bounds for Harmonious Colourings Journal of Graph Theory”. Vol 15, No.6, 629-636 (1991).

[2] Donald G. Beare Norman L.Biggs Brain J.Wilson “The Growth rate of the Harmonious Chromatic Number” Journal of Graph theory Vol 13, No.3 291-299 (1989)

[3] JAYENTHI, A., and A. KULANDAI THERESE. "ON ECCENTRIC CONNECTIVITY INDEX OF SUBDIVISION GRAPHS."International Journal of Mathematics and Computer Applications Research (IJMCAR)4.5, Oct 2014, 41-46

[4] J. Hopcroft and M.S. Krishnomoorthy, “On the harmonious colouring of graphs” , SIAM J.Algebraic Discrete methods 4(1983) 306-311.

[5] K.Thilagavathi And A.Sangeethadevi, “Harmonious colouring of $C[B(K_n,K_n)]$ And $C[F_{2,k}]$,Proceedings of the international conference on mathematics and computer science-2009.

[6] K.Thilagavathiand J.V.Vivin, “Harmonious chromatic number of line graph of central graph. Proceedings of the international conference on mathematics and computer science-2006.

[7] Kesavarao, Yenda, Ch Ramakrishna, and AineelkamalArji. "Stress Analysis of Laminated Graphite/Epoxy Composite Plate Using FEM." *International Journal of Mechanical Engineering (IJME)* 4 (2015): 5.

[8] K.P.Thilagavathy and A.Santha, “The Achromatic and B-Chromatic Number of Sun Graph ,Barbell Graph and some Named Graphs” , International journal of pure and Applied mathematics , Volume 116 No. 12, 2017, 147-155 ,ISSN 1311 8080[Printed version].
[9] Sharma, Ashwani, and Murtaza MA. "Modeling and Finite Element Analysis of Vertical Axis Wind Turbine Rotor Configurations." *International Journal of Mechanical and Production Engineering Research and Development (IJMPERD) ISSN (P)* (2016): 2249-6890.
[10] Venkatesulu, M., and C. H. Sambaiah. "Modelling and Analysis of Static and Dynamic of Connecting Lugs and Mechanical Bracket using Abaqus." *International Journal of Mechanical and Production Engineering Research and Development,(IJMPERD) ISSN (P)* (2018): 2249-6890.