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APPLICATIONS OF GRAPH COLORING

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Abstract: Graph theory begins with very simple geometric ideas and has many powerful applications. Semigraphs are the generalization of graphs.The studies of semigraphs with new variants in dominations and coloring may yield a lot of applications in many practical situations. In this paper, a real time application of coloring in semigraphs are discussed.

Keywords: Semigraph, Edge Coloring, Chromatic Number, Simple Chromatic Number, Total Chromatic Number.

AMS Subject Classification: 05C69

1. Introduction

Semigraphs are introduced by E. Sampathkumar [5] in the year 2000. The graph coloring problems have been applied in a variety of practical problems in which colors may have almost any meaning. For example, if the graph represents a connected grid of cities, each city can be marked with the name of the airline having the most flights to and from that city. In this case, the vertices are cities and the colors are the airline names.

A coloring of a semigraph $S=(V,X)$ is an assignment of colors to its vertices such that not all vertices in an edge are colored the same. A strong coloring of S is a coloring of vertices, so that no two adjacent vertices are colored the same. An $n-$ coloring uses n colors, and partition V into n respective color classes, each class consisting of vertices of the same color. The chromatic number $\psi=\psi(S)$ of S is the minimum number of colors needed in any coloring of S. Similarly we define the strong chromatic number χ_s and $n-$chromatic number χ_n.

N. Murugesan and D. Narmatha studied different types of dominations in [3]. C. Berge [1] discussed the theory of graphs and its applications extensively.Dominations colorings in graphs are discussed in [2]. Coloring in semigraphs are discussed in [4].

2. Simple Coloring and Total Coloring in Semigraphs

2.1 Simple Coloring

Two edges in a semigraph are said to be adjacent edges if they have a common end vertex. On the other hand, two edges are said to be associate edges if they have a common vertex, which may be either middle or middle-end, or end vertex. Two edges are said to be separate edges if they do not have any common vertex. Two separate edges are said to be alternate if there is an edge which is adjacent to both of them.

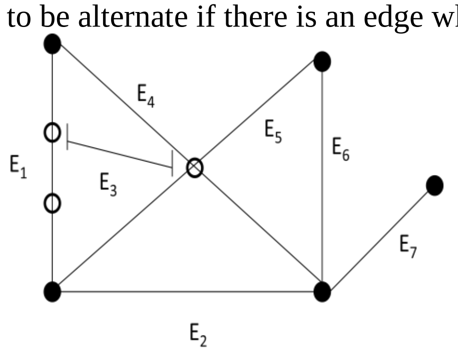

Fig. 2.1 Semigraph

In the above semigraph $E_1,\ E_2$ are adjacent edges and $E_1,\ E_3$ are associate edges. Also note that $E_4,\ E_5$ are associate edges. $E_1,\ E_7$ are associate edges. $E_5,\ E_7$ are separate edges

2.2 Remarks

1. Every pair of adjacent edges are associate edges but not the converse.
2. The path semigraphs and cycle semigraphs have no associate edges. i.e., the path semigraphs and cycle semigraphs are associate edge free semigraph.

2.3 Definition

An edge coloring of a semigraph is said to be simple, if no two adjacent edges have the same color. The minimum
number of colors required for such a coloring in the semigraph S is called simple chromatic number of S and it is denoted as $\psi_s(S)$, or simply ψ_s.

2.4 Total Coloring

An edge coloring of a semigraph is said to be total if no two associate edges have the same color. The minimum number of colors required for such a coloring in the semigraph is called total chromatic number and it is denoted as $\psi_t(S)$, or simply it is denoted as ψ_t.

In the semigraph S given in the following example $\psi_s = 3$ and $\psi_t = 4$.

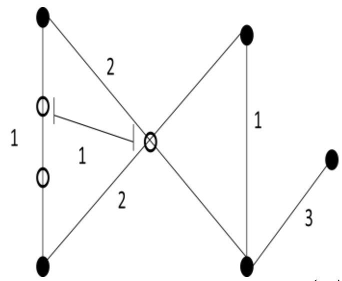

Fig. 2.2 $\psi_s(S) = 3$

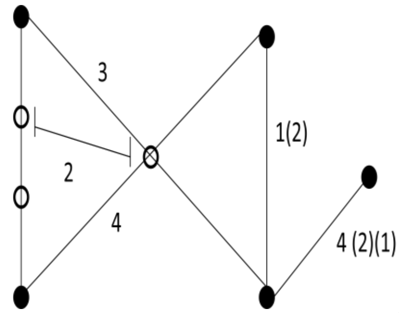

Fig. 2.3 $\psi_t(S) = 4$

3. Applications of Graph Coloring

Graph coloring is an useful technique to model many real time situations involving conflict of interest. In particular, it has wide applications in programming schedule. For example, it is often a challenging task to prepare courses schedule in a college when students opt different elective courses according to their need. Two elective courses cannot be handled at a time when different groups of students opt the same elective course. This kind of problem can be dealt with the coloring of a graph in which a vertex is created for each elective course, and two vertices are adjacent iff there are students opted these two elective courses as their choice. Hence, colors are assigned to vertices so that no two adjacent vertices receive the same color. Each color represents an elective course.

Assume that there are 150 students in a class room and they are allowed to choose two elective courses according to their choice, among the following five courses.

1. Philosophy
2. Sociology
3. Literature
4. Commerce
5. Law

Now each student has selected exactly two colors, and each course is alloted to equal number of students, say 60. This is pictorially represented as given in the fig. 3.1.

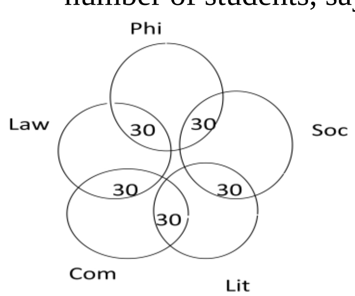

Fig. 3.1 : Students distribution diagram

The graphical representation G of the above problem given in fig 3.2.

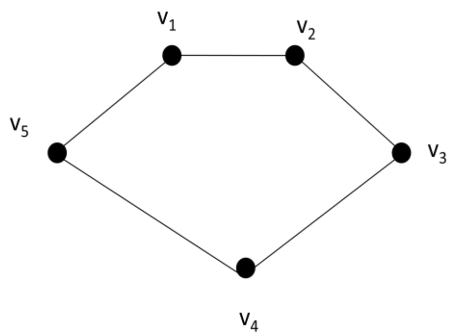

Fig. 3.2 : Graph of elective courses

In the graph G given in fig. 3.2, the elective courses philosophy, sociology, literature, commerce and law are respectively represented by the vertices v_1, v_2, v_3, v_4, v_5. An edge between two vertices exists when there are students common to the respective courses. The graph G is an odd cycle C_5, and it is easy to observe that the chromatic number of C_5 is 3. Therefore, these elective course can be scheduled in 3 periods, instead of 5 periods, one for each elective course. The vertices colored with same color represent that the corresponding course can be scheduled simultaneously so that no student is left without getting the possibility of the attending any course. The graph G with proper vertex coloring is given in fig. 3.3.

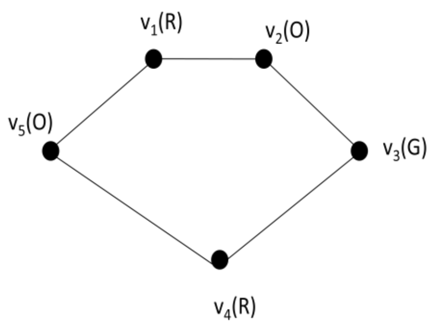

Fig. 3.3 The graph G with vertex coloring.

The programming schedule is given in the table 3.1.

Class Rooms	**Periods**		
	I	II	III
A	Philosophy	Sociology	Literature
B	Commerce	Law	-

Table : 3.1 A course programming schedule

Suppose, consider the above class room schedule with more complexity, as a student of a class can take any number of elective courses, apart from the one which may be compulsory for all the students of a class. Moreover, the number of courses are plenty, and some smart students does not require special classes for their courses at initial stage, but the individual course required in the later stage. So two separate programming schedule required for each-one for initial stage, and another one for the later stage.

This problem can be illustrated with edge coloring of semigraphs. Simple coloring technique of semigraph is suitable to schedule the programming at the initial stage, and total coloring technique of semigraph is helpful to schedule the program of second stage. The each elective course is represented by vertices, and classes by edge of a semigraph. Two edges are adjacent iff there are students opted common elective course, which are represented by end or middle-end vertices of the semigraph. The middle vertices represent the specialized courses which are meant only for the particular class.

Assume there are 6 classes each consisting of 30 students, and 12 elective courses. The distribution of elective courses for each class are given in the table 3.2.

Classes	**Elective Courses**
A	Philosophy, Commerce, Sociology, Law
B	Philosophy, Literature, Arts, Mathematics
C	Mathematics, Physics
D	Physics, Chemistry, law
E	Law, Biology, Mathematics
SMART CLASS	Arts, Boilogy, Chemistry, Information Technology

Table 3.2 Distribution of elective courses

The specialized courses for each courses are given in the table 3.3.

Classes	**Specialized Elective Courses**
A	Commerce, Sociology
B	Literature
C	_
D	_
E	Computer Science
SMART CLASS	Information Technology

Table : 3.3 Distribution of specialized courses

The above problem can be represented in a semigraph S as given in fig. 3.4.

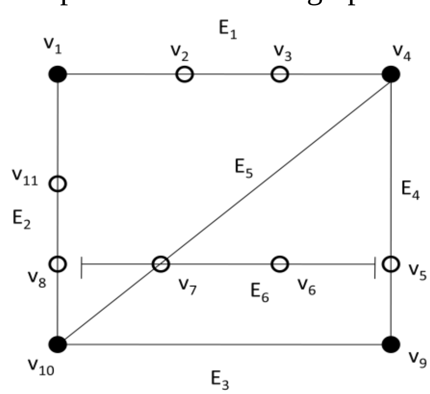

Fig. 3.4 :Semigraph of elective courses distribution

The classes A, B, C, D, E and SMART CLASS respectively represented as edges $E_1, E_2, E_3, E_4, E_5, E_6$ in the semigraph, and each vertex represents an elective course as in the table 3.4.

Vertex	Elective Courses
v_1	Philosophy
v_2	Commerce
v_3	Sociology
v_4	Law
v_5	Chemistry
v_6	Information Technology
v_7	Biology
v_8	Arts
v_9	Physics
v_{10}	Mathematics
v_{11}	Literature

Table : 3.4 Vertices of elective courses

Now we apply greedy algorithm to color the edges. Following the procedure given below.

Step 1 : Arrange the edges in any order.

Step 2 : Color the edge E_1 by the color C_1.

Step 3 : Color the edge E_i by the smallest indexed color not already used on its lower indexed adjacent edges.

Step 4 : If an edge E_i is not adjacent to E_j ; $j=1,2,......., i-1$, then color E_i with any of the colors already used.

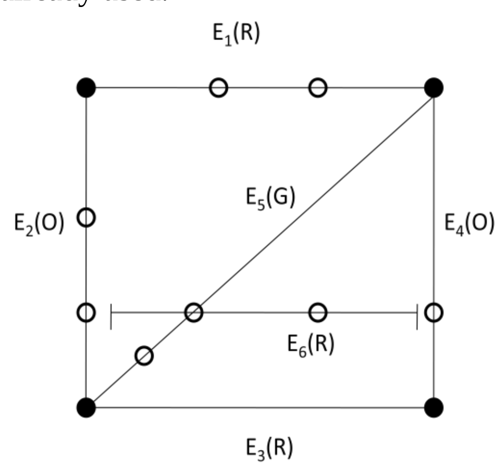

Fig. 3.5 :Semigraph with simple edge coloring

The coloring of edges of S is done in the following way.

E_1 - Red ; E_2 - Orange ; E_3 - Red ; E_4 - Orange ; E_5 - Green ; E_6 - Red

Thus the schedule of elective courses for the first stage of the program in which there is no need for conducting separate class for smart class students.

Since $\psi_s(S)=3$. These periods are enough to conduct these programs. The complete course schedule for the first stage of the program is given in the table 3.6.

Class Rooms	**Classes**		
I	E_1	E_2	E_5
II	E_3	E_4	___
III	E_6	___	___

Table 3.6 Classroom Schedule

Suppose, if the smart class students also wanted to attend specialized courses commonly chosen with the students of other classes then we apply total coloring of edges in the semigraph S , in which no two associate edges receive the same color. By applying greedy algorithm the coloring of edges can be done in the following way.

E_1 - Red ; E_2 - Orange ; E_3 - Red ; E_4 - Orange ; E_5 - Green ; E_6 - Yellow

The schedule corresponding to this coloring is given the table 3.7.

Class rooms	**Classes**			
I	E_1	E_2	E_5	E_6
II	E_3	E_4	—	—

Table 3.7 Classroom Schedule

4. Conclusion

In the field of graph theory, the variants in dominations and coloring of graphs have been introduced and studied extensively. The researches in the new type of domination and a new way of coloring the graph play an important role in the development of graph theory. In this paper, a real time application of graph coloring was discussed.

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