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# NON-INCIDENCE (NON-END VERTICES) TOPOLOGICAL SPACES ASSOCIATED WITH SIMPLE GRAPHS

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**ABSTRACT**
Let G = (V,E) be a simple graph. In this paper, we associate a topology to G, called non-incidence (non-end vertices of the edge e) topology of G. A sub-basis family to generate the non-incidence topology is introduced on the set of vertices V. Then we investigate some propertiesand discuss the non-incidence topologyon a certain few important types of graphs. Our motivation is to give an fundamental step toward investigation of some properties of simple graphs by their corresponding non-incidence topology which we introduce in this paper.**2010 Mathematics Subject Classification:** 05C99, 54A05.

**Key words and phrases:** simple graphs; non-incidence vertices of the edge e, non-incidence topology.

## 1. INTRODUCTION

In Mathematics graphtheory have a long history, one branch of graph theory is a topological graph theory. The relation between graph theory and topological theory existed before and used many times by researchers to deduce a topology from a given graph. Some of them makes models defined on the set of vertices V of the graph G only and others made it on the set of edges E . They studies graphs as a topologies and have been applied in almost every scientific field. Many excellent basics on the mathematics of graph theory, topological graph theory and some applications may be found in the sources [1-7].

In general graphs divided in two types; directed and undirected graph. To an undirected graph some researchers associate a topological spaces as fellow;
In 2013 [8], Jafarian et al. associate a Graphic Topology with the vertex set of a locally finite graph without isolated vertex, and they defined a sub-basis family for a graphic topology as a sets of all vertices adjacent to the vertex $v$ .
And in 2018 [9], Kilicman and Abdulkalek associate an Incidence Topology with a set of vertices for any simple graph without isolated vertex. where they defined a sub-basis family for an incident topology as a sets of all incident vertices with the edge $e$.

The previous works of topology on graphs was associated with a set of vertices without isolated vertex, these topologies are not appropriate to be associated with graphs that have an isolated vertices. Therefore, these reasons motivate authors in [10], to associate a topology on the vertex set of any undirected graph (not only simple graph or locally finite graph) and which may contain one isolated vertex or more. By introducing a new Sub-basis family defined as a sets of all vertices non-adjacent to the vertex $v$ to induce topology which namedIndependent Topology in 2020. Also the same authors associate a tritopological space with undirected graph, i.e. three different topologies induced from the same graph. These tritopologies are the three topologies mentioned above (Independent Topology, Incidence Topology and Graphic Topology)which named IVIG-Induced Topological Space [11], and they introduced the separation axioms of that topology in [12].

Our motivation or target is to associate a topology on the vertex set of any simple graph and may contain one isolated vertex or more. By introducing a new sub-basis family defined as a setsof all vertices non-incident with the edge e( non-end vertices of the edge e) to induce the new topology (which we named it Non-Incidence Topology), and we present some main properties of simple graphs by their corresponding topologies.

There are various types of graphs depending upon the number of vertices, number of edges, interconnectivity, and their overall structure. We will discuss the Non-Incidence Topologyon only a certain few important types of graphs in this article.

In Section 2 of the article we give some fundamental definitions and preliminaries of graph theory and topology, also In Section 3 we define our new topology (non-incidence topology) on simple graphs by introducing a sub-basis family for the new topology and present some main properties of some types of graphs. In last section,conclusions of this new topology on simple graph are presented.

## 2. PRELIMINARIES

In this section we give some fundamental definitions and preliminaries of graph theory and topology. All this definitions are standard, and can be found for example in sources [2] [ 3] [10].

Usually the graph is a pair $G = (V, E)$, for more exactly A graph G consist of a non-empty set V of vertices (or nodes), and a set E of edges (or arcs). If $e$ is an edge in G we can write $e = v\,u$ ( $e$ is join each vertex $v$ and $u$ ),where $v$ and $u$ are vertices in V, then ($v$ and $u$) are said adjacent vertices and incident with the edge $e$.If there is no vertex adjacent with a vertex $v$, then $v$ is said isolated vertex. the degree of the vertex $v$ denoted by $d(v)$ is the number of the edges where $v$ incident with $e$, and $\Delta(G)$ is the maximum degree of vertices in $G$. A vertex of degree 0 is isolated. An independent set in a graph $G$is a set of pairwise non-adjacent vertices. The graph $G$ is finite if the number of the vertices in $G$ also the number of the edges in $G$ is finite, then; otherwise it is an infinite graph. If any vertex can be reached from any other vertex in $G$ by travelling along the edges, then $G$ is called connected graph and is called disconnected otherwise.

We use notations $K_n$ ,$K_{m,n}$ ,$P_n$ and $C_n$ for a complete graph with $n$ vertices, the complete bipartite graph when partite sets have sizes $m$ and $n$, the path on $n$ vertices and the cycle on $n$ vertices, respectively.

A topology $\mathcal{T}$ on a set $\mathcal{X}$ is a combination of subsets of $\mathcal{X}$, called open, such that the union of the members of any subset of $\mathcal{T}$ is a member of $\mathcal{T}$, the intersection of the members of any finite subset of $\mathcal{T}$ is a member of $\mathcal{T}$, and both empty set and $\mathcal{X}$ are in $\mathcal{T}$. The ordered pair$(\mathcal{X}, \mathcal{T})$ is called a topological space. When the topology$\mathcal{T} = P(\mathcal{X})$ on $\mathcal{X}$ is called discrete topology while the topology $\mathcal{T} = \{\mathcal{X}, \varphi\}$ on $\mathcal{X}$ is called indiscrete (or trivial)topology. A topology in which arbitrary intersection of open set is open called an Alexandroff space.

## 3. Non-Incidence Topology on Simple Graphs

Now, we define our topology on graphs. Suppose that $G = (V, E)$ is a simple graph (may contain one isolated vertex or more). Remember that $NI_e$ is the set of all vertices non-incident with the edge e( non-end vertices of the edge e) with condition $|E| \geq 3$.

Define $\mathcal{S}_{\mathrm{NI}}$ as follows $\mathcal{S}_{NI} = \{NI_e | \mathrm{e} \in \mathrm{E}\}$, we have $\mathrm{V} = \bigcup_{\mathrm{e}\in\mathrm{E}} NI_{\mathrm{e}}$, then $\mathcal{S}_{\mathrm{NI}}$ forms a sub-basis for a topology $\mathcal{T}_{NI}$ on V, called non-incidence (non-end vertices) topology of $G$.

**Example 3.1.** Let $G=(V,E)$ be a simple graph as in figure 3.1 such that; $V=\{v_1, v_2, v_3, v_4\}$ and $E=\{e_1, e_2, e_3, e_4\}$,

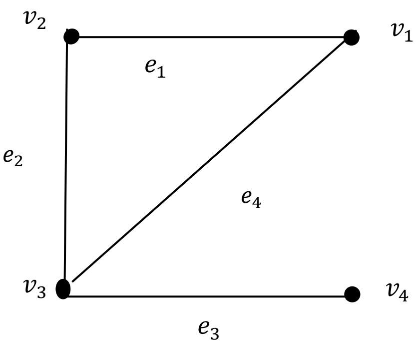



Figure 3.1

we have $NIe_1=\{v_3, v_4\}$, $NIe_2=\{v_1, v_4\}$, $NIe_3=\{v_1, v_2\}$, $NIe_4=\{v_2, v_4\}$.
By taking finitely intersection the base is obtained $\left\{\{v_3, v_4\}, \{v_1, v_4\}, \{v_1, v_2\}, \{v_2, v_4\}, \{v_4\}, \{v_1\}, \{v_2\}, \emptyset\right\}$
Then by taking all unions the non-incidence topology $\mathcal{T}_{NI}$ can be written as :
$= \left\{\emptyset, V, \{v_3, v_4\}, \{v_1, v_4\}, \{v_1, v_2\}, \{v_2, v_4\}, \{v_4\}, \{v_1\}, \{v_2\}, \{v_1, v_3, v_4\}, \{v_1, v_2, v_4\}, \{v_2, v_3, v_4\}\right\}$

**Example 3.2.** Let $G = (V,E)$ be a simple graph as in figure 3.2, let the set of vertices $V=\left\{v_1, v_2, v_3, v_4, v_5\right\}$ and E=$\{e_1, e_2, e_3, e_4, e_5, e_6\}$,

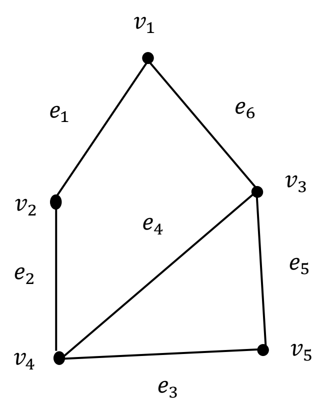



Figure 3.2

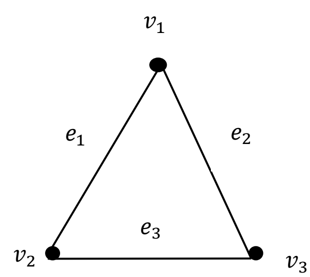
We have $NIe_1 = \{v_3, v_4, v_5\}$, $NIe_2 = \{v_1, v_3, v_5\}$, $NIe_3 = \{v_1, v_2, v_3\}$, $NIe_4 = \{v_1, v_2, v_5\}$, $NIe_5 = \{v_1, v_2, v_4\}$, $NIe_6=\{v_2, v_4, v_5\}$
By taking finitely intersection the basis is obtained
$\{\emptyset, \{v_3, v_4, v_5\}, \{v_1, v_3, v_5\}, \{v_1, v_2, v_3\}, \{v_1, v_2, v_4\}, \{v_1, v_2, v_5\}, \{v_3, v_5\}, \{v_3\}, \{v_4\}, \{v_5\},$
$\{v_1, v_3\}, \{v_1\}, \{v_1, v_5\}, \{v_1, v_2\}, \{v_2\}, \{v_2, v_5\}\}$

By taking all unions the non-incidence topology $\mathcal{T}_{NI}$can be written as
$=\{\emptyset, V, \{v_3, v_4, v_5\}, \{v_1, v_3, v_5\}, \{v_1, v_2, v_3\}, , \{v_1, v_2, v_4\}, \{v_2, v_4\}, \{v_1, v_2, v_5\}, \{v_3, v_5\},$
$\{v_3\}, \{v_4\}, \{v_5\}, \{v_1, v_3\}, \{v_1\}, \{v_1, v_5\}, \{v_1, v_2\}, \{v_1, v_3, v_4, v_5\}, \{v_1, v_2, v_4, v_5\}, \{v_1, v_2, v_3, v_5\}$
$, \{v_3, v_4\}, \{v_4, v_5\}, \{v_1, v_3, v_4\}, \{v_1, v_4\}, \{v_2, v_4, v_5\}, \{v_1, v_4, v_5\}, \{v_2, v_3, v_4, v_5\}, \{v_2\}, \{v_2, v_5\}$
$, \{v_2, v_3, v_5\}, \{v_2, v_3\}, \{v_2, v_5\}, \{v_1, v_2, v_3, v_4\}\}.$

Though, there are a lot of different types of graphs depending upon the number of vertices, number of edges, interconnectivity, and their overall structure,in this section we will discuss the non-incidence topologyonsome of such common types of graphs are as follows:

**Remark 3.3.**It easy to see that the non-incidence topology of cycle $C_n$when $n \geq 3$ is discrete topological space, since each vertex non incident with at least one edges. Then the finitely intersection of the sub-basis give all singleton sub sets of $V$ and this basis generates the discrete topology.

**Example 3.4.** Let G=$(V, E)$ be cycle $C_3$as in figure 3.3




We have ;$NIe_1$={

Figure 3.3

By taking finitely ,$\{v_3\}\}$

By taking all unions the non-incidence topology can be written as
:$\mathcal{T}_{NI}=\{\emptyset, V, \{v_1\}, \{v_2\}, \{v_3\}, \{v_2, v_3\}, \{v_1, v_3\}, \{v_1, v_2\}\}$.

**Remark 3.5.** Complete graph$K_n$ verify the non-incidence topology, and is discrete topology when $n \geq 3$ . ( by the same reason of remark 3.3 above )

**Example 3.6.** G=$(V, E)$ be a complete graph $K_4$ as in figure 3.4 .

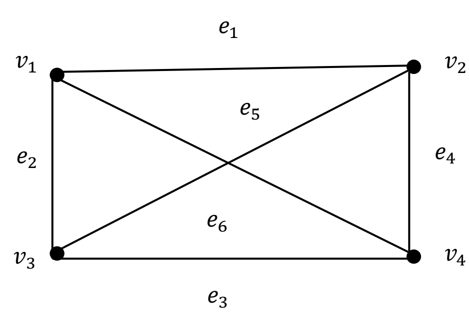



Figure 3.4

We have: $NIe_1$={$v_3, v_4$}, $NIe_2$={$v_2, v_4$}, $NIe_3$={$v_1, v_2$}, $NIe_4$={$v_1, v_3$}, $NIe_5$={$v_2, v_3$}, $NIe_6$={$v_1, v_4$}
By taking finitely intersection the base is obtained :
$\{\emptyset, \{v_3\}, \{v_4\}, \{v_2\}, \{v_1\}, \{v_3, v_4\}, \{v_2, v_4\}, \{v_1, v_2\}, \{v_1, v_3\}, \{v_2, v_3\}, \{v_1, v_4\}\}$
By taking all unions the non- incidence topology can be written as :
$\mathcal{T}_{NI}$ ={Ø, $V$, {$v_1$} , {$v_2$} , {$v_3$} , {$v_4$} , {$v_3, v_4$} , {$v_2, v_4$} , {$v_1, v_2$} , {$v_1, v_3$} , {$v_2, v_3$} , {$v_1, v_4$} , {$v_2, v_3, v_4$},{$v_1, v_3, v_4$},{$v_1, v_2, v_4$},{$v_1, v_2, v_3$}}

**Remark 3.7.** The non-incidence topologyis not verify in the trivial andnull graphs.

**Example 3.8.** Let G=(V, E) bea null graph is also called empty graphhas n=2 as in figure3.5


Figure 3.5

The non-incidence topologyis not verify because there is no edge in the trivial and empty graphs.

**Remark 3.9.** The path $P_n$ when $n \geq 3$ is represent a non-incidence topologybut not discrete.

**Example 3.10.**Let G=$(V, E)$be a path $P_3$ as in figure 3.6

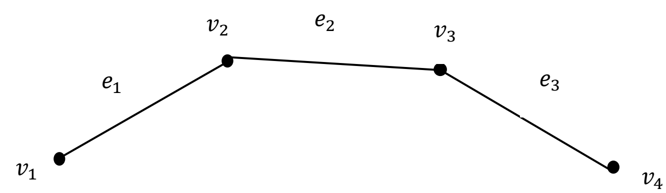



Figure 3.6

$NIe_1$={$v_3, v_4$} ,$NIe_2$={$v_1, v_4$} ,$NIe_3$={$v_1, v_2$}
By finite intersection the base is obtained $\{\emptyset, \{v_1\}, \{v_4\}, \{v_3, v_4\}, \{v_1, v_4\}, \{v_1, v_2\}\}$ , then by taking all unions the non- incidence topology can be written as
$\mathcal{T}_{NI}=\{\emptyset, V, \{v_1\}, \{v_4\}, \{v_3, v_4\}, \{v_1, v_4\}, \{v_1, v_2\}, \{v_1, v_2, v_4\}, \{v_1, v_3, v_4\}\}$

**Remark 3.11.** The non-incidence topology of incomplete bipartite,where A and B are bipartite sets of$K_{n,m}$: $n, m > 1$is not discrete.

**Example 3.12.** Let G=$(V,E)$ be an incomplete bipartite $K_{2,3}$ as in figure 3.7

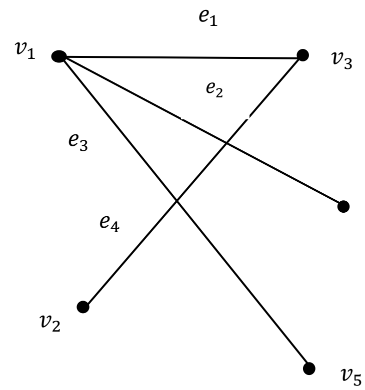



Figure 3.7

$NIe_1$={$v_2, v_4, v_5$},$NIe_2$={$v_2, v_3, v_5$},$NIe_3$={$v_2, v_3, v_4$},$NIe_4$={$v_1, v_4, v_5$}
By taking finitely intersection the base is obtained
{Ø,{$v_2, v_4, v_5$},{$v_2, v_3, v_5$},{$v_2, v_3, v_4$},{$v_1, v_4, v_5$},{$v_2, v_5$},{$v_2, v_4$},{$v_2, v_3$},{$v_4, v_5$}, {$v_5$}, {$v_4$}}
By taking all unions the non-incidence topology can be written as :
$\mathcal{T}_{NI}$ ={Ø, $V$ , {$v_2, v_4, v_5$} , {$v_2, v_3, v_5$} , {$v_2, v_3, v_4$} , {$v_1, v_4, v_5$} , {$v_2, v_5$} , {$v_2, v_4$} , {$v_4, v_5$} , {$v_2, v_3$},{$v_4$},{$v_5$} ,{$v_2, v_3, v_4, v_5$},{$v_1, v_2, v_4, v_5$},{$v_2, v_4, v_5$},{$v_2, v_3, v_5$},{$v_2, v_3, v_4$}}

**Remark 3.13.** The non-incidence topology of complete bipartite $K_{n,m}: n, m > 1$ is not discrete topological space.

**Example 3.14.** Let G =$(V,E)$be a complete bipartite $K_{2,3}$as in figure 3.8.

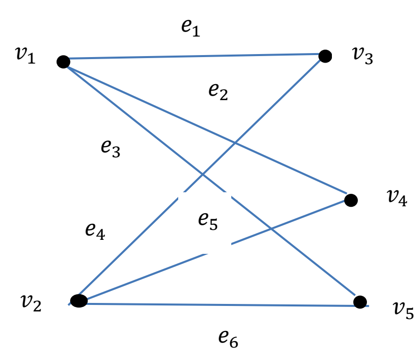



Figure 3.8

$NIe_1 = \{v_4, v_2, v_5\}$ , $NIe_2 = \{v_2, v_3, v_5\}$ , $NIe_3 = \{v_2, v_4, v_3\}$ , $NIe_4 = \{v_1, v_4, v_5\}$ , $NIe_5 = \{v_1, v_3, v_5\}$,$NIe_6$={$v_1, v_3, v_4$},
by taking finitely intersection the base is obtained {Ø, {$v_2, v_5$} , {$v_2, v_4$} , {$v_4, v_5$} , ,{$v_5$},{$v_3$},{$v_4$}, {$v_2, v_3$},{$v_3, v_5$},{$v_3, v_4$},{$v_1, v_5$},{$v_1, v_4$},{$v_1, v_3$},{$v_4, v_2, v_5$},{$v_2, v_3, v_5$}, {$v_2, v_4, v_3$},{$v_1, v_4, v_5$},{$v_1, v_3, v_5$},{$v_1, v_3, v_4$}}

By taking all unions the non- incidence topology can be written as :
$\mathcal{T}_{NI}$ ={Ø,$V$ ,$\{v_2, v_5\}$,$\{v_2, v_4\}$,$\{v_4, v_5\}$,$\{v_5\}$,$\{v_4\}$,$\{v_3\}$,$\{v_2, v_3\}$,$\{v_3, v_5\}$,$\{v_3, v_4\}$,$\{v_1, v_5\}$, $\{v_1, v_4\}$,$\{v_1, v_3\}$,$\{v_4, v_2, v_5\}$,$\{v_2, v_3, v_5\}$,$\{v_2, v_4, v_3\}$,$\{v_1, v_4, v_5\}$,$\{v_1, v_3, v_5\}$,$\{v_1, v_3, v_4\}$, $\{v_2, v_4, v_5\}$ , $\{v_2, v_3, v_4, v_5\}$ , $\{v_1, v_2, v_5\}$ , $\{v_1, v_2, v_4, v_5\}$ , $\{v_1, v_2, v_3, v_5\}$ , $\{v_2, v_4, v_5\}$ , $\{v_1, v_2, v_4\}$,$\{v_1, v_2, v_3, v_4\}$,$\{v_3, v_4, v_5\}$,$\{v_1, v_3, v_4, v_5\}$,$\{v_1, v_2, v_3\}$}

**Remark 3.15.** The non-incidence topology of tree is not discrete.

**Example 3.16.** Let G=$(V, E)$be a treeas in figure 3.9 .

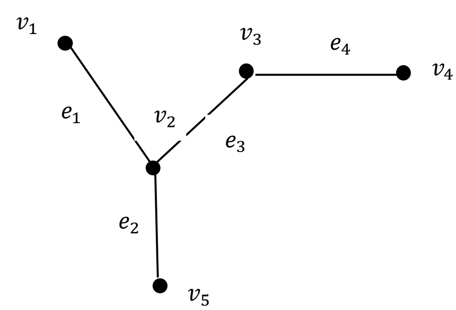



Figure 3.9

$NIe_1$=$\{v_3, v_4, v_5\}$,$NIe_2$=$\{v_1, v_3, v_4\}$ ,$NIe_3$=$\{v_1, v_4, v_5\}$,$NIe_4$=$\{v_1, v_2, v_5\}$
By finitely intersection the base is obtained
{Ø,$\{v_3, v_4, v_5\}$,$\{v_{1,}, v_3, v_4\}$,$\{v_1, v_4, v_5\}$,$\{v_1, v_2, v_5\}$,$\{v_3, v_4\}$,$\{v_{4,}, v_5\}$,$\{v_5\}$,$\{v_1\}$,$\{v_1, v_4\}$, $\{v_1, v_5\}$}
$\mathcal{T}_{NI}$ ={Ø,$V$ ,$\{v_3, v_4, v_5\}$,$\{v_1, v_3, v_4\}$,$\{v_1, v_4, v_5\}$,$\{v_1, v_2, v_5\}$,$\{v_3, v_4\}$,$\{v_4, v_5\}$,$\{v_5\}$,$\{v_1\}$, $\{v_1, v_4\}$,$\{v_1, v_5\}$,$\{v_1, v_3, v_4, v_5\}$ ,$\{v_1, v_2, v_4, v_5\}$}

**Remark 3.17.** The non-incidence topology of graph has two or more components (not connected) is not discrete.

**Example 3.18.** Let G=$(V, E)$ be a graph has two component as in figure 3.10such that; the set of vertices$V$=$\{v_{1,} v_2, v_3, v_4, v_5\}$ and E=$\{e_1, e_2, e_3, e_4\}$, then

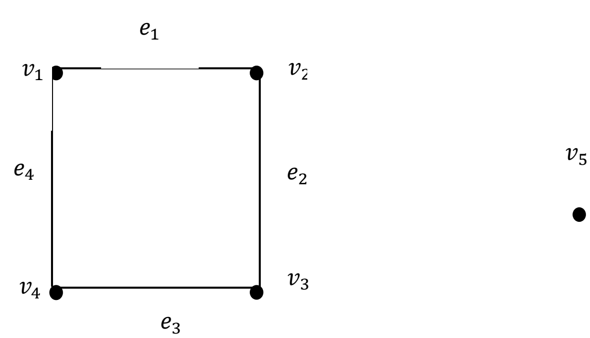



Figure. 3.10

$NIe_1$=$\{v_3, v_4, v_5\}$,$NIe_2$=$\{v_1, v_4, v_5\}$,$NIe_3$=$\{v_1, v_2, v_5\}$,$NIe_4$=$\{v_{2,} v_3, v_5\}$
BY taking finitely intersection the base is obtained
$\{\emptyset,\{v_3, v_4, v_5\},\{v_1, v_4, v_5\},\{v_1, v_2, v_5\},\{v_2, v_3, v_5\},\{v_4, v_5\},\{v_5\},\{v_3, v_5\},\{v_1, v_5\},\{v_2, v_5\}\}$
By taking all unions the non-incidence topology can be written as :
$\mathcal{T}_{NI}$=$\{\emptyset, V, \{v_3, v_4, v_5\},\{v_1, v_4, v_5\},\{v_1, v_2, v_5\},\{v_2, v_3, v_5\},\{v_4, v_5\},\{v_5\},\{v_3, v_5\},\{v_1, v_5\},$ $\{v_2, v_5\}$ , $\{v_1, v_3, v_4, v_5\}$ , $\{v_2, v_3, v_4, v_5\}$ , $\{v_1, v_2, v_4, v_5\}$ , $\{v_1, v_2, v_3, v_5\}$ , $\{v_3, v_4, v_5\}$ , $\{v_2, v_4, v_5\},\{v_1, v_3, v_5\}\}$

**Example 3.19**. Let G=$(V, E)$ be a graph has more than two component as in figure 3.11

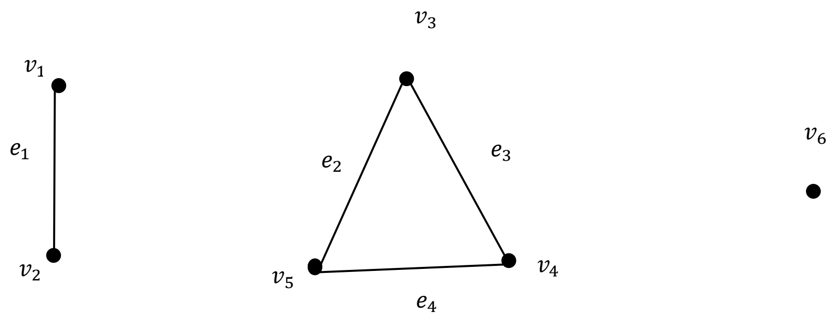



Figure 3.11

$NIe_1$=$\{v_3, v_4, v_5, v_6\}$,$NIe_2$=$\{v_1, v_2, v_4, v_6\}$,$NIe_3$=$\{v_1, v_2, v_5, v_6\}$,$NIe_4$=$\{v_1, v_2, v_3, v_6\}$
By taking finitely intersection the base is obtained,$\{\emptyset,\{v_4, v_6\},\{v_5, v_6\},\{v_3, v_6\},\{v_1, v_2, v_6\},\{v_3, v_4, v_5, v_6\},\{v_1, v_2, v_4, v_6\},$ $\{v_1, v_2, v_5, v_6\},\{v_1, v_2, v_3, v_6\}\}$
By taking all unions the non-incidence topology can be written as :
$\mathcal{T}_{NI}$ =$\{\emptyset, V$ , $\{v_3, v_4, v_5, v_6\}$ , $\{v_1, v_2, v_4, v_6\}$ , $\{v_1, v_2, v_5, v_6\}$ , $\{v_1, v_2, v_3, v_6\}$ , $\{v_4, v_5, v_6\}$ , $\{v_3, v_4, v_6\},\{v_1, v_2, v_4, v_6\},\{v_1, v_2, v_4, v_5, v_6\},\{v_1, v_2, v_3, v_4\},\{v_1, v_2, v_3, v_5, v_6\}\}$

## Conclusions :

A synthesis between graph theory and topology has been made. A topology with the set of vertices for any simple graphs has been associated, called non-incidence topology. The study of some properties of this new model of topology has been presentedon a certain few important types of graphs. It has been shown that the non-incidence topologies of the cycle $C_n$ and the complete graph $K_n$; where $n \geq 3$are discrete, but the non-incidence topology of $P_n$ is not discrete also the tree, complete bipartite, incomplete bipartite and the graph has two or more component. this article can be lead to a significant applications in the future.

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