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Numerical solution of Countercurrent Imbibition in Inclined Homogeneous Porous Medium by using Polynomial based Differential Quadrature Method

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Abstract-In this research paper, we have studied mathematical exemplary of countercurrent imbibition phenomenon arising in slanted oil structured homogeneous porous medium. The governing equation is one dimensional nonlinear partial differential equation. The Polynomial based differential quadrature method used to obtain numerical solution of governing equation with suitable initial and boundary conditions. We used uniform and Chebyshev Gauss Lobatto grid points for obtaining numerical solution. We compared result obtained by Chebyshev and uniform grid points. We used MATLAB coding for Numerical and Graphical representation.

Introduction:

There are many methods to recover oil from oilfield. In general, we are using either primary or secondary oil recovery process to bring oil from oilfield. During natural method of oil recovery, we can recover only 10% to 12% oil from oilfield. So for recovering more oil from oilfield we are applying secondary oil recovery process. We can recover 80% to 85% oil from oilfield by using this process. In this phenomenon the saturated part is drained into a medium by procedures of vessel compressions and

curvaceous crossing point among the saturated and non-saturated part devoid of any exterior strength. Imbibition divide into Cocurrent and Countercurrent based on the flow of direction. In the Cocurrent both the fluids travel in the identical path though during countercurrent both are traveling in the opposite directions. Brownscombe and Dyes have been discussed “water-imbibition displacement as a possibility for the spraberry- Drilling and production practice” [08]. A.E. Scheidegger has been discussed about “the physics of flow through Porous medium” [01]. P. M. Blair obtained “calculation of oil displacement of counter-current imbibition phenomenon by using water injected process” [05]. A.P. Verma and M.N. Mehta have been worked on “composite expansion solution of finger-imbibition in double phase flow through underground porous media” [02]. Tavassoli, Zimmerman & Blunt worked on “analysis of counter-current imbibition with gravity in weakly water-wet systems”. [09]. Bourblaux and Kalaydjian [03] have been talked about “experimental study of cocurrent and countercurrent flows in natural porous media”. Mishra, Pradhan & Mehta [10] obtained “an analytical solution of the countercurrent imbibition phenomenon arising in the fluid flow through homogeneous porous media”. Mehta, Pradhan & Parikh [04] have been studied “the mathematical model and analysis of countercurrent imbibition in vertical downward homogeneous porous media”.
The Polynomial based DQM has been used to obtain numerical result of the governing equation with the help of appropriate initial and boundary conditions.

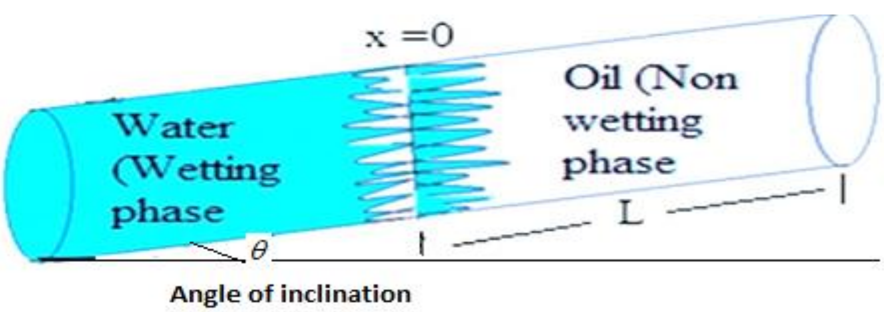

Figure-1: Schematic diagram [12]

Mathematical Model of the Problem

According to Darcy’s law velocity of V_w & V_o can be described as given below in equation (01)

$$V_w = -\frac{K_w}{\delta_w} K\left[\frac{\partial P_w}{\partial x} + \rho_w g \sin\theta\right] \& \; V_o = -\frac{K_o}{\delta_o} K\left[\frac{\partial P_o}{\partial x} + \rho_o g \sin\theta\right] \quad (01)$$

V_w = Velocity of water, V_o = Velocity of oil, K = Permeability of porous medium, K_w = Relative permeability of water, K_o = Relative permeability of oil, δ_w = Constant thickness of water, δ_o = Constant thickness of oil, P_w = Pressure of water, P_o

= Pressure of oil, ρ_w = Density of water, ρ_o = Density of oil, g = Gravity, θ = the slanted angle of the porous medium.

Using continuity equation as given below

$$p\frac{\partial S_w}{\partial t}+\frac{\partial V_w}{\partial x}=0, \quad p = \text{Porosity of the medium.} \tag{02}$$

P_C (Capillary pressure) is define as follow

$$P_c = -P_w + P_o \tag{03}$$

Let we used standard relationship as follows

$$P_c = -\lambda S_w, \ \lambda \text{ is constant value.} \tag{04}$$

We assumed ordinary relationship between permeability of water, oil and saturation of water [06].

$$K_w = S_w, \ K_o = 1-\mu S_w \tag{05}$$

According to physical interpretation of this phenomenon, Total velocity of water and oil is zero.

$$V_w + V_o = 0 \tag{06}$$

Using equation (01) in equation (06), we will get

$$\frac{K_w}{\delta_w}K\left[\frac{\partial P_w}{\partial x}+\rho_w g\sin\theta\right]+\frac{K_o}{\delta_o}K\left[\frac{\partial P_o}{\partial x}+\rho_o g\sin\theta\right]=0 \tag{07}$$

From equation (07) and (04)

$$\left(\frac{K_w}{\delta_w}+\frac{K_o}{\delta_o}\right)\frac{\partial P_o}{\partial x}-\frac{K_w}{\delta_w}\frac{\partial P_c}{\partial x}=\left(\frac{K_w}{\delta_w}\rho_w-\frac{K_o}{\delta_o}\rho_o\right)g\sin\theta \tag{08}$$

Solving equation (08)

$$\frac{\partial P_o}{\partial x}=-\left[\frac{\left(\frac{K_w}{\delta_w}\rho_w-\frac{K_o}{\delta_o}\rho_o\right)g\sin\theta-\frac{K_w}{\delta_w}\frac{\partial P_c}{\partial x}}{\left(\frac{K_w}{\delta_w}+\frac{K_o}{\delta_o}\right)}\right] \tag{09}$$

According to equation (1) and (07), we have

$$V_O = -K\frac{K_O}{\delta_O}\left[\frac{g\left(\rho_O+\rho_W\right)\sin\theta-\frac{K_W}{\delta_W}\frac{\partial P_C}{\partial x}}{\left(\frac{K_O}{\delta_O}+\frac{K_W}{\delta_W}\right)}\right], V_W = -\frac{\frac{K_W}{\delta_W}\frac{K_O}{\delta_O}}{\left(\frac{K_O}{\delta_O}+\frac{K_W}{\delta_W}\right)}K\left[\left(\rho_O+\rho_W\right)g\sin\theta-\frac{\partial P_C}{\partial x}\right]$$

(10)

Using values of V_W from equation (10) to (03)

$$P\frac{\partial S_W}{\partial t}-\frac{\partial}{\partial x}\left(\frac{\frac{K_W}{\delta_W}\frac{K_O}{\delta_O}}{\left(\frac{K_O}{\delta_O}+\frac{K_W}{\delta_W}\right)}K\left[\left(\rho_O+\rho_W\right)g\sin\theta\right]\right)+\frac{\partial}{\partial x}\left[K\frac{\frac{K_W}{\delta_W}\frac{K_O}{\delta_O}}{\left(\frac{K_O}{\delta_O}+\frac{K_W}{\delta_W}\right)}\frac{\partial P_C}{\partial x}\right]=0$$

(11)

We are assuming relationship, we have $\frac{\frac{K_O}{\delta_O}\frac{K_W}{\delta_W}}{\left(\frac{K_W}{\delta_W}+\frac{K_O}{\delta_O}\right)}\cong\frac{K_O}{\delta_O}$

(12)

Using equations (13), (06) into equation (12), we have

$$P\frac{\partial S_W}{\partial t}=\left(\frac{K\left(\rho_W+\rho_O\right)g\sin\theta}{\delta_O}\frac{\partial}{\partial x}\left(1-\mu S_W\right)\right)+\frac{K\beta}{\delta_O}\frac{\partial}{\partial x}\left[\left(1-\mu S_W\right)\frac{\partial S_W}{\partial x}\right]$$

(13)

We will use dimensionless variables [15], $X=\frac{x}{L}$, $T=\frac{K\beta t}{\delta_O L^2 P}$

(14)

Equation (11) reduces to

$$\frac{\partial S_W}{\partial T}=\frac{\partial^2 S_W}{\partial X^2}-\mu S_W\frac{\partial^2 S_W}{\partial X^2}-\mu\left(\frac{\partial S_W}{\partial X}\right)^2-\mu G\frac{\partial S_W}{\partial X}\Rightarrow\frac{\partial S}{\partial T}=\frac{\partial^2 S}{\partial X^2}-\mu S\frac{\partial^2 S}{\partial X^2}-\mu\left(\frac{\partial S}{\partial X}\right)^2-\mu G\frac{\partial S}{\partial X}$$

Where $G=\frac{L\left(\rho_W+\rho_O\right)g\sin\theta}{\lambda}$ *and* $S=S_W\left(X,T\right)$

(15)

Initial and Boundary condition selected as follow

$S(X,0)=1-X^2,\ 0<X<1$

(16) $S(0,T)=a_1,\ T>0$ $S(1,T)=a_2,\ T>0$, where a_1 and a_2 are arbitrary constants.

(17)

Numerical result by Polynomial based DQM: Differential quadrature process is an analytical method for obtaining solution of differential equations. We discretize the partial derivatives of 1st and 2nd order on a point X_i as follows

$$\left(\frac{\partial S}{\partial X}\right)_{\left(X=X_i\right)}=\sum_{i=1}^{N} A_{ij} S(X_i)\ ,\&\ \left(\frac{\partial^2 S}{\partial X^2}\right)_{\left(X=X_i\right)}=\sum_{i=1}^{N} B_{ij} S(X_i)\ ,\ for\ \ j=1,2,3...,N for\ \ j=1,2,3...,N$$

A_{ij} & B_{ij} are the weighting coefficients of 1st & 2nd order partial derivatives correspondingly.

$$A_{ij}=\frac{T^{(1)}\left(X_i\right)}{\left(X_i-X_j\right)T^{(1)}\left(X_j\right)},\ i\neq j,\ i,\ j=1,2,3...,N\ \ \&\ A_{ii}=-\sum_{j=1}^{N} A_{ij}\ ,\ \ i=1,2,3....,N$$

(18)

Second order partial derivatives weighting coefficients B_{ij} is given by

$$B_{ij}=2A_{ij}\left[A_{ii}-\frac{1}{X_i-X_j}\right],\ for\ i\neq j,\ i,\ j=1,2,3,...,N\ \&\ B_{ii}=-\sum_{j=1,\ j\neq i}^{N} B_{ij},\ i=1,2,3,...,N$$

(19)

We are using following grid points for the numerical result.

(a) **Uniform grid points**: In uniform grid points, we are using same step sizes.

$$X_i=X_1+ih,\ \ i=1,2,3,...,N,\ where\ \ X_1=a\ \ and\ \ h=\frac{b-a}{N} \tag{20}$$

(b) **Non-uniform grid points (Chebyshev-Gauss-Lobatto)**: Non-uniform grid points are defined by

$$X_i=c+(0.5)*\left(1-\cos\frac{\left(\left(i-1\right)*\pi\right)}{N-1}\right)(d-c),\ \ i=1,2,3,...,N \tag{21}$$

Here we discretize our governing equation (18) by using standard form as follows [07]

$\frac{\partial S}{\partial T}=\frac{dS_i}{dT},\ \frac{\partial^2 S}{\partial X^2}=\sum_{j=1}^{N} B_{ij}\ S_j\ ,\ \frac{\partial S}{\partial X}=\sum_{j=1}^{N} A_{ij} S_j\ ,\ i,\ j=1,2,3,...,N$, Where

$S_i=S\left(x_i\right), S_j=S\left(x_j\right)$ (22)

Replacing equation (25) in equation (18), we get the systems of the first order ODE in the form.

$$\frac{dS_i}{dT} = \sum_{j=1}^{N} B_{ij} S_j - \mu S_i \sum_{j=1}^{N} B_{ij} S_j - \mu \left(\sum_{j=1}^{N} A_{ij} S_j \right)^2 - \mu G \sum_{j=1}^{N} A_{ij} S_j \quad (23)$$

The value of some constants is used as follows:

$L = 1, g = 9.8, \rho_n = 0.3, \rho_i = 0.1, \lambda = 0.1, \mu = 1.11 \Rightarrow G \approx 0.43$

Here solving this problem, we are using uniform grid points. The weighting coefficients A_{ij} & B_{ij} of 1st and 2nd order derivatives can be obtained by using the formula given in equation (23) and (25) respectively.

TABLE-1: Numerical values of saturation vs distance(X) at different time level t for fixed angle $\theta = 0^\circ$ [11]

Time T	0.5		0.8		1	
Distance X	Chebyshev	Uniform	Chebyshev	Uniform	Chebyshev	Uniform
0	0.5701	0.5901	0.3502	0.3789	0.2932	0.3192
0.1	0.4788	0.5128	0.2782	0.3082	0.2125	0.2525
0.2	0.4088	0.4588	0.2208	0.2608	0.1601	0.2001
0.3	0.3316	0.3879	0.1779	0.2102	0.1226	0.1606
0.4	0.2702	0.3302	0.1393	0.1693	0.1001	0.1201
0.5	0.2192	0.2656	0.1101	0.1351	0.0826	0.09806
0.6	0.1642	0.2202	0.0805	0.1105	0.0601	0.0701
0.7	0.1209	0.1659	0.0608	0.0799	0.04006	0.0526
0.8	0.0808	0.1208	0.04208	0.0558	0.0258	0.03901
0.9	0.06052	0.08062	0.03512	0.0422	0.02002	0.02405
1	0.0502	0.06002	0.02698	0.03301	0.01058	0.01998

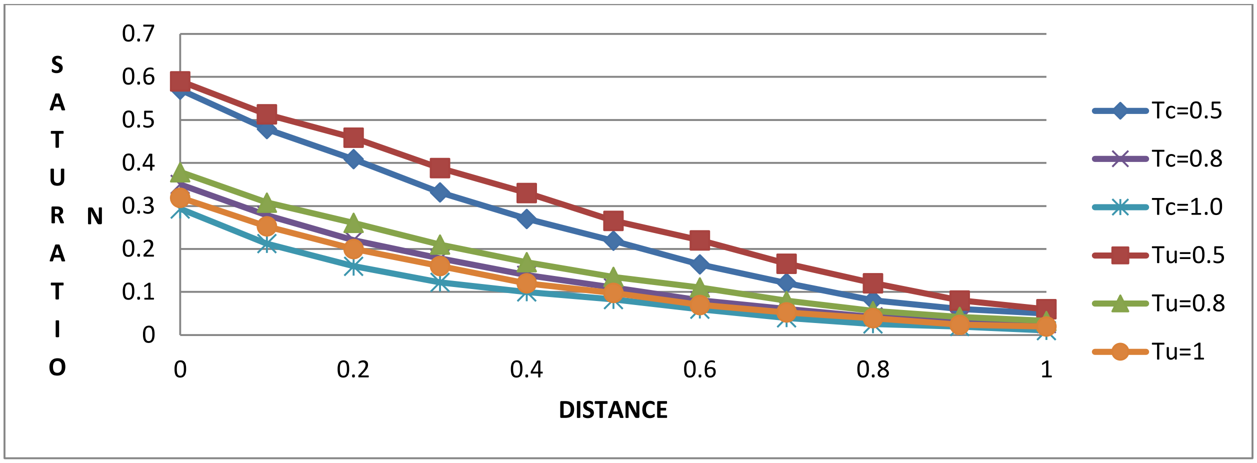

FIGURE-2: SATURATION VS DISTANCE

TABLE-2: Numerical values of saturation vs distance(X) at different time level t for fixed angle $\theta = 40^{\circ}$)

Time T	0.5		0.8		1	
Distance X	Chebyshev	Uniform	Chebyshev	Uniform	Chebyshev	Uniform
0	0.6401	0.6852	0.4482	0.4788	0.3892	0.4092
0.1	0.5228	0.5928	0.3682	0.4082	0.2905	0.3325
0.2	0.4508	0.5188	0.3108	0.3508	0.2401	0.2801
0.3	0.3802	0.4479	0.2507	0.2979	0.1926	0.2326
0.4	0.318	0.3852	0.2093	0.2493	0.1602	0.1901
0.5	0.2656	0.3156	0.1651	0.2051	0.1326	0.1526
0.6	0.2142	0.2542	0.1402	0.1653	0.1051	0.1301
0.7	0.1659	0.2059	0.1152	0.1399	0.08006	0.1026
0.8	0.1308	0.1588	0.0908	0.1188	0.06202	0.0801
0.9	0.1058	0.1188	0.0752	0.0922	0.0525	0.0625
1	0.0902	0.1002	0.0604	0.08084	0.0408	0.0498

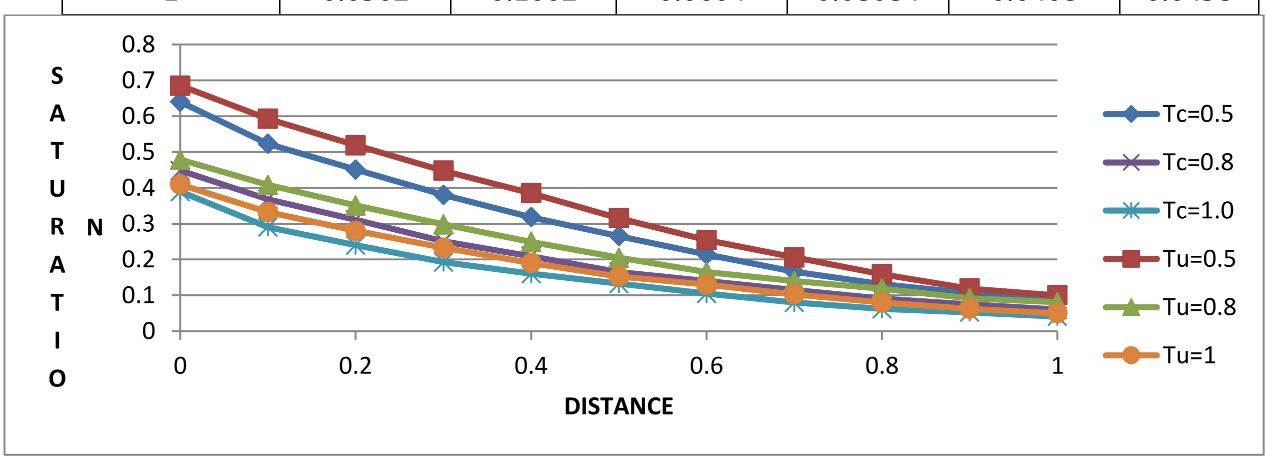

FIGURE-3: SATURATION VS DISTANCE

TABLE-3: Numerical values of saturation vs distance(X) at different time level t for fixed angle $\theta = 90^{\circ}$)

Time T	0.5		0.8		1	
Distance X	Chebyshev	Uniform	Chebyshev	Uniform	Chebyshev	Uniform
0	0.8102	0.8362	0.5705	0.5909	0.41	0.4252
0.1	0.6281	0.7188	0.4353	0.4853	0.3094	0.3394
0.2	0.526	0.6245	0.3562	0.4162	0.2501	0.283
0.3	0.4307	0.5207	0.2826	0.3426	0.2003	0.2303
0.4	0.3426	0.4226	0.2239	0.2739	0.1514	0.1914
0.5	0.2613	0.3313	0.1704	0.22	0.1156	0.1456
0.6	0.1961	0.2461	0.1208	0.1604	0.0831	0.1131
0.7	0.1401	0.187	0.091	0.1148	0.0535	0.0735
0.8	0.1006	0.1401	0.0602	0.0804	0.0266	0.0466
0.9	0.0601	0.0982	0.0361	0.0501	0.010	0.0222
1	0.0401	0.0501	0.025	0.0301	0.007	0.1

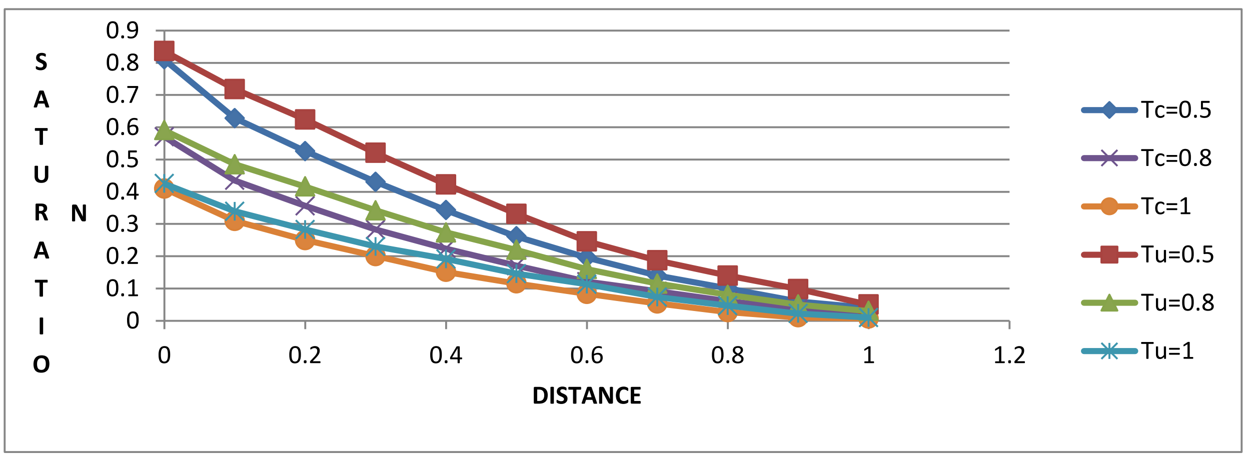

FIGURE-4: SATURATION VS DISTANCE

Conclusion:

In present study, we have discussed Counter-current imbibition phenomenon in inclined homogeneous porous medium. We have obtained numerical result of the governing non-linear partial differential equation by using Polynomial based Differential Quadrature Method with the help of suitable initial and boundary conditions. In numerical and graphical result, we have shown the comparison between

Uniform and Chebyshev Gauss Lobatto grid points at the different angles $\left(\theta=0^{\circ}, 40^{\circ} \,\&\, 90^{\circ}\right)$. Here we have studied the effect of two different types of grid points on the saturation of injected water during counter current imbibition phenomenon and observed that, for fixed time $t>0$, saturation of injected water is decreasing with respect to the distance and also it can be observed from Table-1, 2 and 3 that the saturation of injected water in Chebyshev-Gauss-Lobatto grid points is slightly less in compare to the Uniform grid points. Comparing the results with the available solutions and looking to the physical nature of the problem, we can conclude that the Chebyshev-Gauss-Lobatto grid point gives improved accurate solution compare to uniform grid points.

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