PalArch's Journal of Archaeology of Egypt / Egyptology

DOMINATING CRITICAL IN BIPOLAR FUZZY GRAPHS

Ambika P & Ramya R

Department of Mathematics,

Bishop Heber college, Trichy-620017, Tamil Nadu, India.

Email id:ambika.ma@bhc.edu.in

Department of Mathematics,

Bannari Amman Institute of Technology, Erode-638401, Tamil Nadu, India.

Email id:ramyark@bitsathy.ac.in

Corresponding Author: **Ambika P**

Ambika P & Ramya R, DOMINATING CRITICAL IN BIPOLAR FUZZY GRAPHS-Palarch's Journal Of Archaeology Of Egypt/Egyptology 17(12), ISSN 1567-214x

Abstract: In this paper, an idea of dominating critical in bipolar fuzzy graphs (BFG) is presented. Further investigate the properties of dominating critical in different types of BFG and operations in BFG.

Keywords: Bipolar fuzzy graphs, Domination number and Dominating critical.

1. Introductionand definition

Akramintroduced the notion of BFG in 2011, anddeliberated the notion of isomorphism of BFG, and studyseveralsignificant properties. Further, he presented the concept of strong BFG and study their characteristics. Samanta and Pal deliberatebipolar fuzzy hyper graphs and irregular BFG.

Again, Akram et al. examinedregular BFG, metric in BFG. Akramdemarcated different operations on BFG. He also presented that the automorphism property of BFG.

A bipolar fuzzy graph (BFG) is of the form $G=(V,E)$ where $V=\{v_1,v_2,....v_n\}$ such that $\mu_1^+ : X \to [0,1]$ and $\mu_1^- : X \to [-1,0]$, $E \subset V \times V$ where $\mu_2^+ : V \times V \to [0,1]$ and $\mu_2^- : V \times V \to [-1,0]$ Such that $\mu_{2_{ij}}^+ = \mu_2^+(v_i v_j) \le \min(\mu_1^+(v_i), \mu_1^+(v_j))$ and $\mu_{2_{ij}}^- = \mu_2^-(v_i v_j) \ge \max(\mu_1^-(v_i), \mu_1^-(v_j))$ for all $(v_i, v_j) \in E$.

A BFG, $G=(V,E)$ is called strong BFG if $\mu_2^+(v_i, v_j) = \min(\mu_1^+(v_i), \mu_1^+(v_j))$ and $\mu_2^-(v_i, v_j) = \max(\mu_1^-(v_i), \mu_1^-(v_j))$ for all $(v_i, v_j) \in E$.

A BFG $G=(V,E)$ is called complete if $\mu_2^+(v_i, v_j) = \min(\mu_1^+(v_i), \mu_1^+(v_j))$ and $\mu_2^-(v_i, v_j) = \max(\mu_1^-(v_i), \mu_1^-(v_j))$ for all $(v_i, v_j) \in V$..

Let $G=(V,E)$ be a BFG. Then the vertex cardinality of $G(V,E)$ is defined by

$$|V| = \sum_{v_i \in V} \left[\frac{1 + \mu_1^+(v_i) + \mu_2^-(v_j)}{2} \right] \text{ for all } v_j \in V .$$

Thesum cardinality of all the vertices inBFGis called the order of BFG and it is denoted by $O(G)$, $O(G) = \sum_{v_i \in V} \left[\frac{1 + \mu_1^+(v_i) + \mu_1^-(v_i)}{2} \right]$

The degree of a vertex v in a BFG, $G(V,E)$ is defined to be sum of the cardinality of strong arcs incident at v .It is denoted by $d_G(v)$. The minimum degree of BFG is $\delta(G) = \min\{d_G(v) / v \in V\}$ The maximum degree of BFGis $\Delta(G) = \max\{d_G(v)/v \in V\}$.

A vertex u be a vertex in BFG, $G(V,E)$ then $N(u) = \{v,\ v \in V \text{ and } (u,\ v) \text{ is a strong arc in G}\}$ is called neighborhood of u in G.

A vertex $u \in V$ of a BFG, $G(V,E)$ is said to be isolated vertex if $\mu_2^{+}(u,v) = 0 \; and \; \mu_2^{-}(u,v) = 0 \; for \; all \; v \in V, u \neq v$. That is $N(u) = 0$. Thus an isolated vertex does not dominate any other vertex of G.

In a BFG, $G(V,E)$. Let $u,v \in V$, we say that u dominates v in G if there exists a strong edge between them.A subset S of V is called dominating set in $G(V,E)$ if for every $v \in V - S$, there exist $u \in S$ such that u dominates v.Adominating set S of a BFG, $G(V,E)$ is said to be minimal dominating set if no proper subset of S is a dominating set. The Minimum cardinality among all minimal dominating set is called the domination number of G and is denoted by $\gamma_{bf}(G)$.

In this paper, we presented an idea of dominating critical in BFG. Further investigate the properties of dominating critical in different types of BFG.

2. DOMINATING CRITICAL ONBFG

In this segment, the definition of dominating critical is presented and investigates the properties of dominating critical in different types of BFG.

Definition 2.1. In a BFG $G(V,E)$, the set of all vertices $V^0 = \{u \mid \gamma_{bf}(G-u) = \gamma_{bf}(G)\}$ is called a null dominating critical in $G(V,E)$.The set of all vertices $V^{+} = \{u | \gamma_{bf}(G-u) > \gamma_{bf}(G)\}$,is called a positive dominating critical in $G(V,E)$.The set of all vertices $V^{-} = \{ u \mid \gamma_{bf}(G-u) < \gamma_{bf}(G)\}$, is called a negative dominating critical in $G(V,E)$.

Remarks: In BFG graphs $G(V,E)$ the vertex set $V = V^{0} \cup V^{+} \cup V^{-}$

Theorem 2.1: Ina complete BFG, $G(V,E)$. D is a dominating set of $G(V,E)$. Then

(i) $D = V^{+}$

(ii) $V - D = V^{0}$

Proof: (i). In a complete BFG, $G(V,E)$. D is a dominating set of $G(V,E)$. Therefore $D = \{v\}$, v is the vertex having the minimum cardinality in $G(V,E)$. This implies we get $< G - v >$ is also a complete BFG and the cardinality of all the vertices $u \in V - \{v\}$ is greater than cardinality of the vertex $v \in D$. Since D is a γ_{bf} set of $G(V,E)$. Therefore $\gamma_{bf}(G-v) > \gamma_{bf}(G)$. This implies we get $D = V^{+}$.

(ii). Let $u \notin D$,the sub graph $< G - u >$ is also a complete BFG. Therefore $D = \{v\}$, v is the vertex having the minimum cardinality in $G(V,E)$. This implies $\gamma_{bf}(G-u) = \gamma_{bf}(G)$, every vertex $u \notin D$ is an element of V^{0}. This implies $V - D = V^{0}$.

Example 2.1

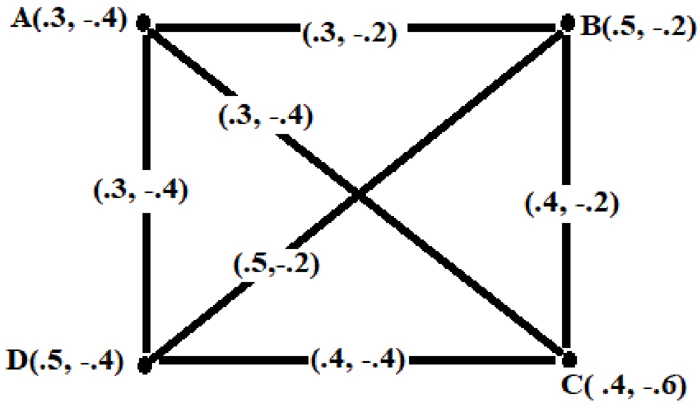


Figure 2.1: $G(V,E)$ Complete BFG

In the Complete BFG, $G(V,E)$, degree of the vertices are $|a| = 0.45, |b| = 0.65, |c| = 0.4$ *and* $|d| = 0.55$. The null dominating critical $V^0 = \{a,b,d\}$ and positive dominating critical $V^+ = \{c\}$.

Theorem 2.2: In a BFG $G(V,E)$ if $d_N(v) = \Delta_N(G)$. Then $v \in V^+$.

Proof: In a BFG $G(V,E)$, if $d_N(v) = \Delta_N(G)$. This implies $v \in D$, D is a $\gamma_{bf}(G)$ set of $G(V,E)$. Note that $D - v$ is not a dominating set of $G(V,E)$. There are some vertices $u_i \in N(v), i = 1,2,...n$ not dominated by the set $D - v$. Therefore $(D - v) \cup \{u_i\}, i = 1,2...n$ is a dominating set of $< G - v >$. This implies $\gamma_{bf}(G - v) > \gamma_{bf}(G)$, we get $v \in V^+$.

Theorem 2.3: In a BFG $G(V,E)$, D is a γ_{bf} set of $G(V,E)$. Then $D \subseteq V^+$.

Proof: In a BFG $G(V,E)$, D is a γ_{bf} set of $G(V,E)$. This implies $D - \{v\}$ is not a dominating set of $G(V,E)$. There exist some vertex u in BFG $< G - v >$ is not dominated by the set $D - \{v\}$. There is a vertex $w \in V$ is adjacent to a vertex u such that $(D - \{v\}) \cup \{w\}$ is a minimal dominating set of $< G - v >$.

Clearly $\gamma_{bf}(G-v) > \gamma_{bf}(G)$ this suggests every vertex $v \in D$ belongs to V^{+}.

Theorem 2.4: Let $v \in V$ is an isolated vertex in a BFG $G(V,E)$. Then $v \in V^{-}$.

Proof: Let $v \in V$ is an isolated vertex in a BFG $G(V,E)$. Therefore $v \in V$ is a vertex dominating itself such that $v \in D$, D is a γ_{bf} set of $G(V,E)$. This implies $D-\{v\}$ is dominating set of a sub graph $< G-\{v\} >$. Clearly we get $\gamma_{bf}(G-u) < \gamma_{bf}(G)$. Therefore the vertex $v \in V^{-}$.

Example.2.2

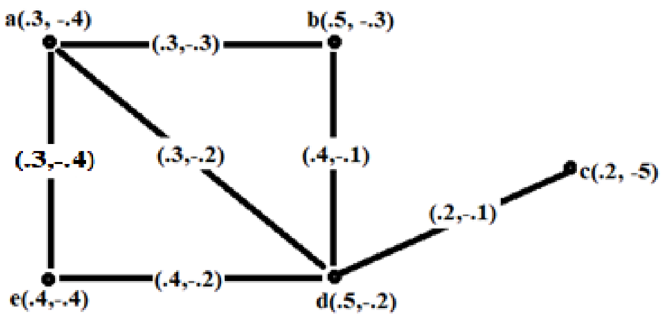


Figure 2.2: BFG $G(V,E)$

In the BFG $G(V,E)$, neighbourhood degree of the vertices are $d_N(a) = 1.75, d_N(b) = 0.45, d_N(c) = 0., d_N(d) = 0.45, d_N(e) = 45$. The minimal dominating set $D = \{a,c\}$. The null dominating critical $V^{0} = \{b,d,e\}$, positive dominating critical $V^{+} = \{a,c\}$ negative dominating critical $V^{-} = \{c\}$.

Definition 2.2: Let $A_1 = (\mu_{A_1}{}^{+}, \mu_{A_1}{}^{-})$ and $A_2 = (\mu_{A_2}{}^{+}, \mu_{A_2}{}^{-})$ be bipolar fuzzy subsets of V_1 and V_2. Let $B_1 = (\mu_{B_1}{}^{+}, \mu_{B_1}{}^{-})$ and $B_2 = (\mu_{B_2}{}^{+}, \mu_{\mathbf{B}_2}{}^{-})$ be bipolar fuzzy subsets of E_1 and E_2 respectively. Then we denote the union of two bipolar fuzzy graphs G_1 and G_2 of the graphs G_1^* and G_2^* by $G_1 \cup G_2 = (A_1 \cup A_2, B_1 \cup B_2)$ and defined as follows

$$\begin{cases} (\mu_{A_1}{}^{+} \cup \mu_{A_2}{}^{+})(x) = \mu_{A_1}{}^{+}(x) \ \ if \ \ x \in V_1 - V_2 \\ (\mu_{A_1}{}^{+} \cup \mu_{A_2}{}^{+})(x) = \mu_{A_2}{}^{+}(x) \ \ if \ \ x \in V_2 - V_1 \end{cases}$$

$$\begin{cases} (\mu_{A_1}{}^{-} \cup \mu_{A_2}{}^{-})(x) = \mu_{A_1}{}^{-}(x) \ \ if \ \ x \in V_1 - V_2 \\ (\mu_{A_1}{}^{-} \cup \mu_{A_2}{}^{-})(x) = \mu_{A_2}{}^{-}(x) \ \ if \ \ x \in V_2 - V_1 \end{cases}$$

$$\begin{cases} (\mu_{B_1}{}^{+} \cup \mu_{B_2}{}^{+})(xy) = \mu_{B_1}{}^{+}(xy) \ \ if \ \ xy \in E_1 - E_2 \\ (\mu_{B_1}{}^{+} \cup \mu_{B_2}{}^{+})(xy) = \mu_{B_2}{}^{+}(xy) \ \ if \ \ xy \in E_2 - E_1 \end{cases}$$

$$\begin{cases} (\mu_{B1}{}^{-} \cup \mu_{B_2}{}^{-})(xy) = \mu_{B_1}{}^{-}(xy) \ \ if \ \ xy \in E_1 - E_2 \\ (\mu_{B_1}{}^{-} \cup \mu_{B_2}{}^{-})(xy) = \mu_{B_2}{}^{-}(xy) \ \ if \ \ xy \in E_2 - E_1 \end{cases}$$

Theorem 2.5: Let $(G_1 \cup G_2)$ is a union of two BFG $G_1(V_1, E_1)$ *and* $G_2(V_2, E_2)$. The sets D_1 *and* D_2 are γ_{bf} dominating of sets of $G_1(A_1, B_1)$ *and* $G_2(A_2, B_2)$. Then $D_1 \cup D_2 \in V^+$ *and* $(D_1 \cup D_2) \in V^0$.

Proof: Let $(G_1 \cup G_2)$ is a union of two BFG $G_1(V_1, E_1)$ *and* $G_2(V_2, E_2)$. The sets D_1 *and* D_2 are γ_{bf} dominating of sets of $G_1(A_1, B_1)$ *and* $G_2(A_2, B_2)$. Clearly by the definition of union $D_1 \cup D_2$ is the γ_{bf} dominating of sets of $(G_1 \cup G_2)$. Using theorem 3.3, $D_1 \cup D_2 \in V^+$ and $\{(V_1 - V_2) - (D_1 \cup D_2)\} \in V^0$. Hence proved

Example 2.3.

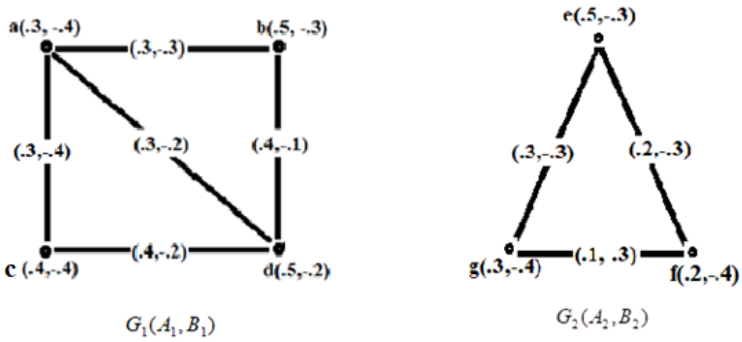


Figure 2.3

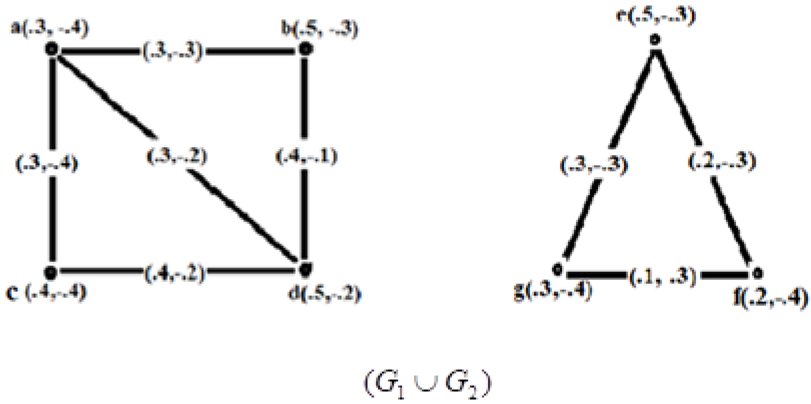


Figure 2.4

In Figure 3.3, the dominating sets of the BFG's $G_1(A_1, B_1)$ *and* $G_2(A_2, B_2)$ are $D_1 = \{a, c\}$ *and* $D_2 = \{e\}$.The domination number of $G_1(A_1, B_1)$ *and* $G_2(A_2, B_2)$ are $\gamma(G_1) = 0.66$, . In $G_1 + G_2$, $V^+ = \{a, c, e\}$ and $\{b, d, f, g\} \in V^0$.

Definition 2.3: Let $A_1 = (\mu_{A_1}{}^+, \mu_{A_1}{}^-)$ and $A_2 = (\mu_{A_2}{}^+, \mu_{A_2}{}^-)$ be bipolar fuzzy subsets of V_1 *and* V_2 in which $V_1 \cap V_2 = \phi$ and let $B_1 = (\mu_{B_1}{}^+, \mu_{B_1}{}^-)$ and $B_2 = (\mu_{B_2}{}^+, \mu_{B_2}{}^-)$ be bipolar fuzzy subsets of $V_1 \times V_2$ *and* $V_2 \times V_1$ respectively. Then we denote the join of two BFG G_1 *and* G_2 by $G_1 + G_2 = (A_1 + A_2,\ B_1 + B_2)$ and defined as follows

$$\begin{cases} (\mu_{A_1}{}^{+} + \mu_{A_2}{}^{+})(x) = \max(\mu_{A_1}{}^{+}(x), \mu_{A_2}{}^{+}(x)) \\ (\mu_{A_1}{}^{-} + \mu_{A_2}{}^{-})(x) = \min(\mu_{A_1}{}^{-}(x), \mu_{A_2}{}^{-}(x)) \ \ if \ \ x \in V_1 \cup V_2 \end{cases}$$

$$\begin{cases} (\mu_{B_1}{}^{+} + \mu_{B_2}{}^{+})(xy) = \max(\mu_{B_1}{}^{+}(xy), \mu_{B_2}{}^{+}(xy)) \\ (\mu_{B_1}{}^{-} + \mu_{B_2}{}^{-})(xy) = \min(\mu_{B_1}{}^{-}(xy), \mu_{B_2}{}^{-}(xy)) \ \ if \ \ xy \in E_1 \cap E_2 \end{cases}$$

$$\begin{cases} (\mu_{B_1}{}^{+} + \mu_{B_2}{}^{+})(xy) = \min(\mu_{B_1}{}^{+}(x), \mu_{B_1}{}^{+}(y)) \\ (\mu_{B_1}{}^{-} + \mu_{B_2}{}^{-})(xy) = \max(\mu_{B_1}{}^{-}(x), \mu_{B_2}{}^{-}(y)) \ \ if \ \ xy \in E_1{}^{/} \end{cases}$$

Where $E_1{}'$ is the set of all edges joining the nodes of V_1 and *and* V_2.

Theorem 2.5: Let $(G_1 + G_2)$ is a join of two BFG $G_1(V_1, E_1)$ *and* $G_2(V_2, E_2)$. The sets D_1 *and* D_2 are dominating of sets of $G_1(A_1, B_1)$ *and* $G_2(A_2, B_2)$.

i) If $|D_1| < |D_2|$, $D_1 \in V^{+}, D_2 \in V^{0}$

ii) If $|D_2| < |D_1|$, $D_2 \in V^{+}$, $D_1 \in V^{0}$

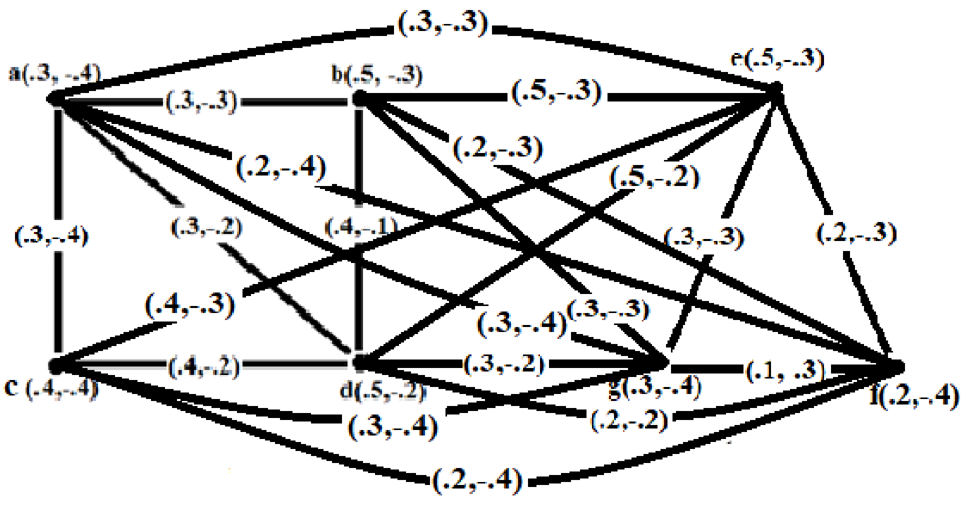
Proof: Let $(G_1 + G_2)$ is a join of two BFG $G_1(V_1, E_1)$ *and* $G_2(V_2, E_2)$. The sets D_1 *and* D_2 are dominating of sets of $G_1(V_1, E_1)$ *and* $G_2(V_2, E_2)$.

i). Assume $|D_1| < |D_2|$. Every vertex $v \in V_1$ such that there exist a vertex $u \in D_1$ such that u dominates v. since D_1 is a minimal dominating set of $G_1(V_1, E_1)$. By the definition of $(G_1 + G_2)$, there is a strong edge between vertices in D_1 and V_2. clearly D_1 is the minimal dominating set of $(G_1 + G_2)$ since $|D_1| < |D_2|$. In the bipolar fuzzy sub graph, $< G - u >$, here $u \in D_1$. Therefore there is some vertex $v \in V_1$ is not dominated by D_1. This implies $(D_1 - u) \cup \{v\}$ is a dominating set of $(G_1 + G_2)$. Hence we get $\gamma(G - u) < \gamma(G)$ here $G = (G_1 + G_2)$, $D_1 \in V^{+}$. In the bipolar fuzzy sub graph, $< G - u >$, here $u \in D_2$. Note that every vertex is dominated by D_1. This implies $\gamma(G-u)=\gamma(G)$, hence we get $D_2 \in V^0$.

ii). Assume $|D_2|<|D_1|$. Every vertex $v \in V_2$ such that there exist a vertex $u \in D_2$ such that u dominates v. since D_2 is a γ_{bf} set of $G_1(V_1, E_1)$. By the definition of (G_1+G_2), there is a strong edge between vertices in D_2 and V_1. clearly D_2 is the γ_{bf} set of (G_1+G_2) since $|D_2|<|D_1|$. In the bipolar fuzzy sub graph, $<G-u>$, here $u \in D_2$. Therefore there is some vertex $v \in V_2$ is not dominated by D_2. This implies $(D_2-u)\cup\{v\}$ is a dominating set of (G_1+G_2). In $G=(G_1+G_2)$ we get $\gamma(G-u)<\gamma(G)$, therefore $D_2 \in V^+$. In the bipolar fuzzy sub graph, $<G-u>$, here $u \in D_1$. Note that every vertex in $<G-u>$ is dominated by D_2. This implies $\gamma(G-u)=\gamma(G)$, Hence we get $D_1 \in V^0$.

Example 2.3.

The join of graphs $G_1(A_1, B_1)$ *and* $G_2(A_2, B_2)$ in the figure 3.3 is represented in the figure 3.5



G_1+G_2

Figure 2.5

In Figure 3.3, the dominating sets of the BFG's $G_1(A_1, B_1)$ *and* $G_2(A_2, B_2)$ are $D_1 = \{a, c\}$ *and* $D_2 = \{e\}$.The domination number of $G_1(A_1, B_1)$ *and* $G_2(A_2, B_2)$ are $\gamma(G_1) = 0.66$, . In $G_1 + G_2$, $V^+ = \{e\}$ and $\{b, d, f, g\} \in V^-, \{a, c\} \in V^0$.

Conclusion

Further we promote the dominating critical idea to split dominating set, and connected dominating set in bipolar fuzzy graphs and investigate theproperties of dominating critical in BFG.

References:

[1]. Rosenfeld, Fuzzy Graphs, In Fuzzy Sets and their Applications to Cognitive and Decision Processes, L.A. Zadeh, K.S Fu, M. Shimura, Eds; Academic Press, New York, (1975) 77-95.

[2]. J.N.Mordeson, P.S Nair , Fuzzy graphs and Fuzzy Hyper graphs, Physica-Verlag, Heidelberg, 1998, second edition, (2001).

[3]. W.R. Zhang, Bipolar fuzzy sets and relations, a computational framework for cognitive modeling and multi agent decision analysis, Proc. of IEEE Conf., 1994.

[4]. W.R. Zhang, Bipolar fuzzy sets, Proc. of FUZZ-IEEE, 1998.

[5]. M. Akram, Bipolar fuzzy graphs, Information Sciences, 181 (2011) 5548–5564.

[6]. M. Akram, M.G. Karunambigai, Metric in Bipolar Fuzzy Graphs, World Applied Sciences M. Akram, W.A. Dudek, Regular bipolar fuzzy graphs, Neural Computing Appl., 21 (2012) 197–205. 10.

[7]. M.G. Karunambigai, M. Akram, K. Palanivel, S. Sivasankar, Domination in Bipolar Fuzzy Graphs, FUZZ-IEEE International Conference on Fuzzy Systems, Hyderabad , India ,(2013) 1-6 11.

[8]. N. Vinothkumar, G. Geetharamani, Edge domination in fuzzy graphs, Pensee Multi-Disciplinary Journal, 76 (2014) 61-70. 12.

[9]. N. Vinothkumar, G. Geetharamani, Vertex edge domination in operation of fuzzy graphs, International Journal of Advanced Engineering Technology, 7 (2016) 401-405. 13.