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THE GODEN RATIO

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ABSTRACT:

Generally, in the art and architecture, the concept of the Golden Ratio has been used for 2,500 years to establish harmonious proportions [2]. Many designers and typography books are recommending the use of golden ratio in most of the engineering projects. Further, it is the ratio of two parts with different lengths of a line segment. In other words, the longer part divided by the smaller part in the segment is equivalent to the total of both lengths. The number has become most astonishing while they used it in architectural works, such as the Athens Parthenon splendid domes, which has been architected by Athens in 447 BC. During the European Renaissance, when Leonardo Da Vinci studied the physical proportions of man and depicted golden ratio in his unfinished canvas of St Jerome, along with other works such as the Mona Lisa and the Vitruvian Man, the ratio became even more pronounced and eternal. In this thesis, the review of basic properties, history and importance of the golden ratio are explained with the inclusion of examples.

INTRODUCTION:

The idea of Golden Ratio is a particularly savvy method of describing a number related equation that is as a proportion, 1:1.6. This number is utilized often in assorted zones, for example, nature, biology, computer science, and even art. This recipe can be utilized to plan various shapes including square shapes, twisting, and triangles, etc. The brilliant proportion has enormously captivated numerous mathematicians, savants, physicists, engineers, specialists, and even artists as it is exceptionally fundamental in each angle throughout everyday life. The foundation, history, use, causes, hypotheses, and models will show the noteworthiness of Phi.

WHAT IS GOLDEN RATIO?

Mathematics is a very important, everyday tool in life and the golden ratio brings out the best of it. The ratio is an irrational number that is found by dividing a line into two parts. The Golden Ratio is an extraordinary number found by isolating a line into two segments with the objective that the more drawn out and longer part separated by the tinier part is also equal to the whole length partitioned by the more expanded part.

$$\frac{a}{b} = \frac{a+b}{a} = 1.618 = \varphi$$

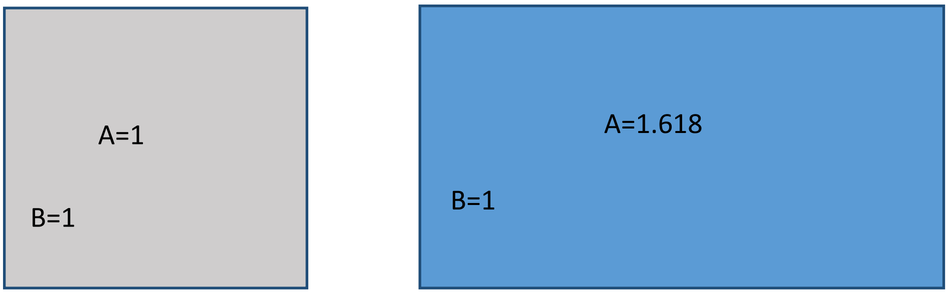
In an equation form, it looks like this:

$$\frac{a}{b} = \frac{a+b}{a} = 1.6180339887498948420 \ \ldots$$

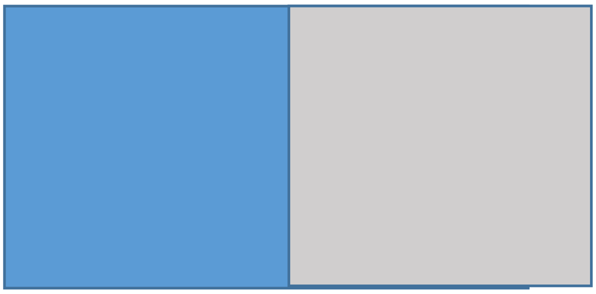
The digits go on and on into infinity however, it is usually rounded off to 1.618 for simpler use. It is frequently symbolized utilizing phi, after the 21st letter of the Greek alphabet.

The golden ratio has multiple names which corresponds to the number of times people have studied it, the names include divine section, divine proportion, golden proportion, golden section, golden mean, extreme and mean ratio, medial section, golden cut and golden number.

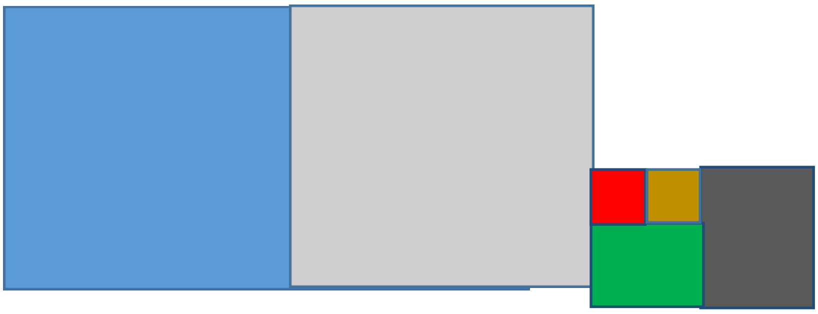
Another way to describe the golden ratio is by numbers and shapes but more specifically, square, rectangles, and spirals.
Take a square and multiply one side of by the golden ratio, 1.618, and a rectangle shape forms:



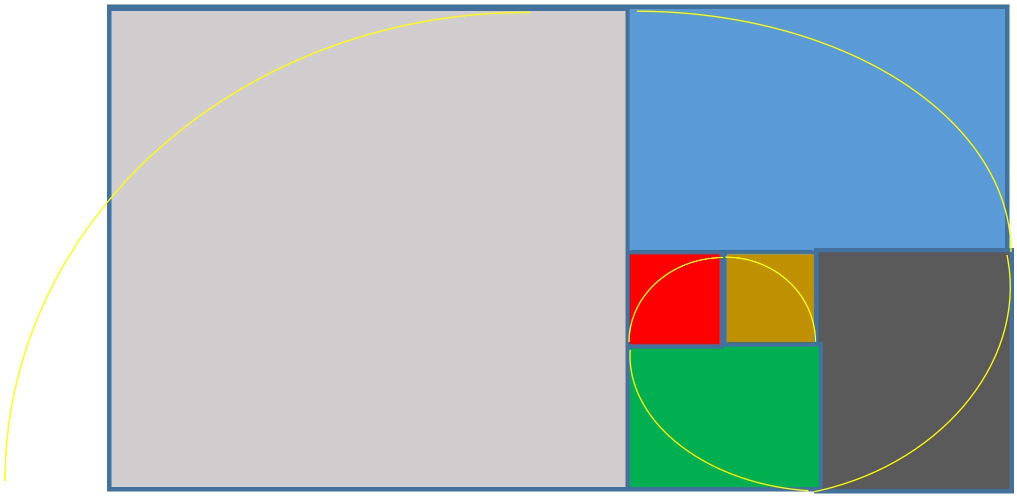
In this figure, A is multiplied by the golden ratio which results the shape in a rectangle shape. B has stayed the same.
If the square lays on top of the rectangle, the result will be a golden ratio.



On the off chance that you continue applying the Golden Ratio equation to the new rectangle above, you will in the long run get this outline with dynamically reduced squares:



With the figure above, draw an arc on each square, from one corner to the contrary corner which will result in the first curve of the golden spiral.



The figure above shows the Golden spiral, created using the golden ratio. This spiral can be found in numerous places such as the universe, nature and even the human body.

If circles are added to each square, the instead of arches, then the circles follow the 1:1.618 ratio and are proportion to each other.

HISTORY OF THE GOLDEN RATIO:

The golden ratio originated in ancient Greece by Greeks and from that day on, people of all branches of work would study this ratio and its characteristics up until current day. Phidias, a Greek sculptor and mathematician, studied the golden ratio, which was named Phi during 500 BC, to build upon his collection of sculptures.An example of his work using the golden ratio is the sculpture called Parthenon in Athens, Greece. Historians noticed that the concept of the golden ratio was found throughout the sculpture.

People started to witness the beauty and the brilliance of the formula so they adopted it into their own fields of work. Everyone agreed that it was “pleasing to the eye” and curiosity sparked which caused the ratio to be revised by many people. So later it was rediscovered by Plato, a Greek philosopher, who mentioned in his discourse called 'Timaeus' that the golden ratio was "the key to the physics of the cosmos andthe most combining of every mathematical relationship. In his 'Timaeus', he characterized five potential standard solids, which are known as cube, the tetrahedron, the Platonic solids, octahedron, icosahedron, and dodecahedron which had anrelation with the golden ratio.

Following Plato came the man who gave the first recorded definition of the golden ratio, which he named "extreme and mean ratio" but in Greek language. His name was Euclid. A greek mathematician who mentioned to separating a line at the 0.6180399… point as “dividing a line in the extreme and mean ratio.”This later offered arise to the usage of the term mean in the golden mean. This later offered ascend to the utilization of the term mean in the golden mean. He additionally connected this number to the building of a pentagram.

Later, the Leonardo of Pisa, better known as Fibonacci, he was recognized as the best mathematician of the medieval times. Fibonacci composed a book named“Liber Abaci” on how to do arithmetic in the decimal system. Even though it was Fibonacci himself that discovered the sequence of numbers, the French mathematician, Edouard Lucas who named the "Fibonacci numbers" to the series of numbers that was first mentioned by Fibonacci in his book.Since this revelation, it has been demonstrated that Fibonacci numbers can be found in an assortment of things today.

This Fibonacci sequence was very close to the special properties of the golden ratio.

if you choose any two successive Fibonacci numbers, the two numbers ratio is very close to the Golden ratio. As the numbers get higher, the proportion turns out to be considerably more like 1.618. For instance, the proportion of 3 to 5 is 1.666. Be that as it may, the proportion of 13 to 21 is 1.615. Getting much higher, the proportion of 144 to 233 is 1.618. These numbers are generally reformist numbers in the Fibonacci progressive.

The golden ration can happen anyplace. Many other mathematicians and philosophers carried on studying the properties and the use of the golden ratio and reached to the point where everyone, including biologists, artists, architects and many more unbelievable people would adopt it into their work.

WHRETO USE THE GOLDEN RATIO?

The golden ratio is used in many aspects of life by many talented people. Architecture is one of many uses of the golden ratio.Many masterpieces are professed to have been composed utilizing the golden ratio. For example, the Parthenon has numerous extents that are very close to the golden ratio.

Another use of the number is logos and trademarks. Utilizing the excellence of the golden ratio to settle on choices rapidly and create something very clever that can'tbe produced by any other ratios can be done in orderto create a logo of any sort,

Nature also uses this ratio, appears with astonishing frequency, to comprehend the physical extents and plan of living life forms.

The solar system and the universe have surprising connections to the golden ratio. The diameters of the earth and moon form a triangle which diameters are influenced by the scientific features of phi. Saturn's rings have a very similar dimension to the planet's diameter ratio. Also, the distance of the planets from the sun correspond nearly to phi. Scientists use this information for their hypothesis and observations to come up with theorems.

Last but not least, Phi is found in the human body in an interesting way. It is found in your face, your body, your teeth, your fingers, and our DNA. Phi is very similar to the proportions of our facial and body dimensions. As a textbook for producing excellent results, people use the golden ratio and apply it in plastic surgery and cosmetic dentistry.

WHY IS THE GOLDEN RATIO IMPORTANT?

Essentially, the golden ratio is used everywhere because it's a very basic recursive geometrical algorithm that portrays the straightforward case of how the placement and size of each subsequent item is the conclusion of the sum of the past two placements or sizes.

Without this number, artists, architectures, mathematicians, scientists, philosophers and many more people would have a hard time trying to find an answer to how everything is connected and use it as a base to create designs and masterpiece. The importance of the number is that one can create beautiful natural looking compositions in one's design work.

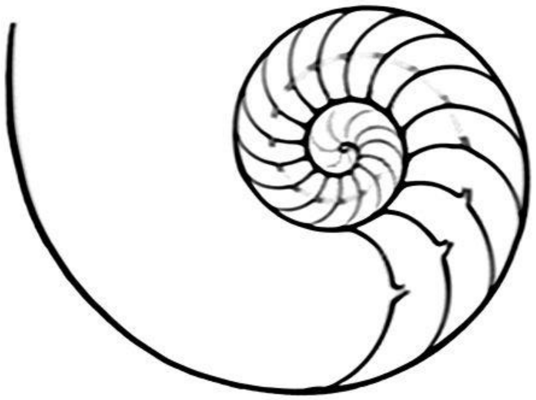
EXAMPLES OF THE GOLDEN RATIO?

Examples:

the famous golden ratio has grabbed the attention from many people who adopted the golden ratio in their work. The golden ratio is sometimes found as a coincidence in some places such as the solar system or sometimes used deliberately to create masterpieces. Multiple examples are mentioned below:

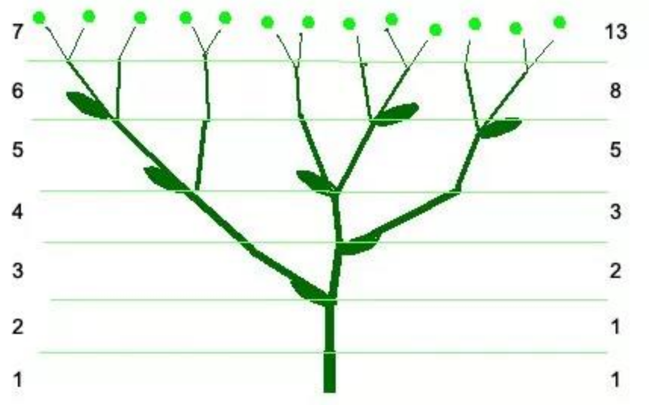
1. **Flower petals.**

   As seen in the image above, the petals curl around a spiral, which is created by the golden ratio. Also, the number of petals in a flower consistently follow the golden ratio.



2. **Tree branches**

The figure above shows another example that includes the golden ratio. It can be shown when the tree grows overtime and splits its branches. Each branch split into two, while the other line is dormant. It creates a pattern that satisfies the golden ratio.

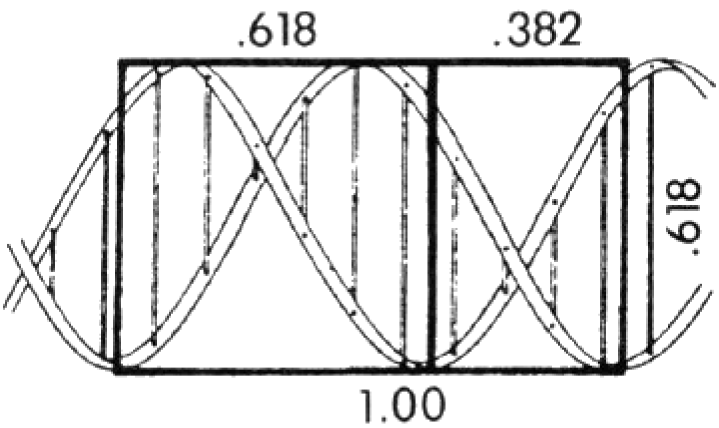


3. **Spiral galaxies**

The spiral of the galaxy demonstrates the golden ratio in action

4. **DNA molecules**

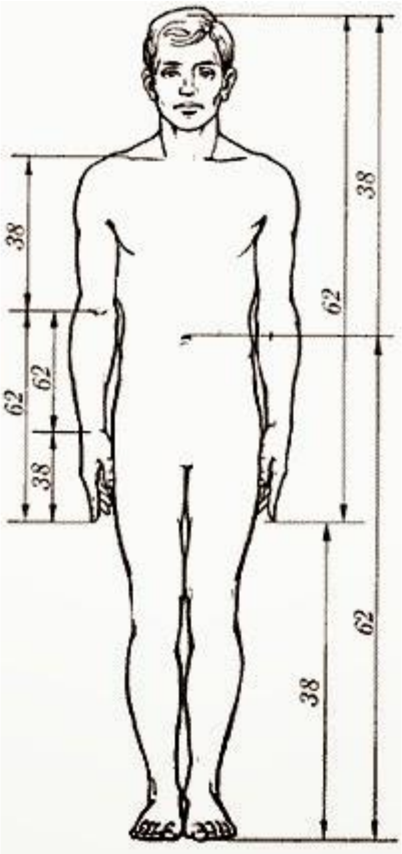
The DNA moecules shows the golden ratio pattern. For each complete cycle of its double helix spiral, the DNA molecule measures 34 angstroms in length and 21 angstroms wide. These numbers, 34 and 21, are values in the Fibonacci series, and their ratio 1.6190476 closely approximates the golden ratio, 1.6180339.



5. **Body parts**

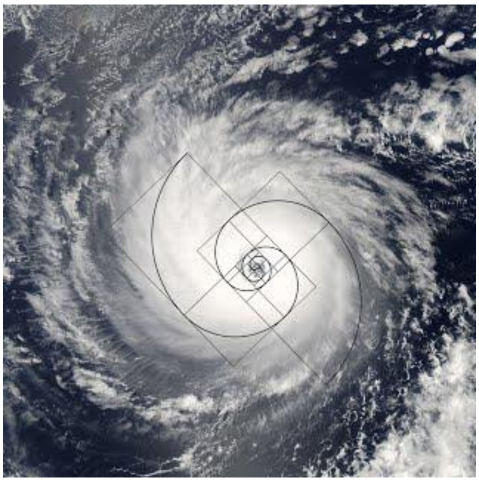
In the shape of the human body, the Golden Segment is depicted. The human body based on the value 5 and Phi.

- The number 5 appurtenance to the torso, in the head, leg and arms.
- On each of these, in the fingers and toes, 5 appendages
- On the face, 5 openings
- 5 sensory organs for vision, touch, smell, sound and taste.



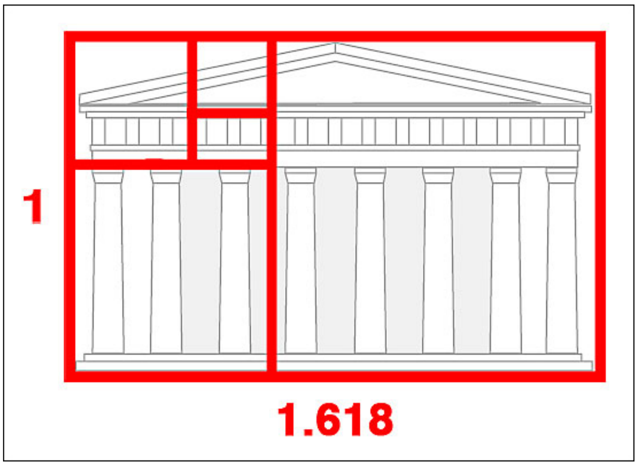
6. Hurricanes

Hurricanes mimic spirals just as the same as the golden spiral.



7. Architecture

Example of the golden ratio used in architecture is this Parthenon built during Ancient Greek times. It follows the ratio of phi, 1:1.618.



BASIC PROPERTIES OF THE GOLDEN RATIO:

The golden ratio appears in some exceptionally basic relationship including numbers from where a considerable lot, it’s possible to derive its properties.The property sequences provide one of the most important events of the golden ratio.A numerical sequence is a number set arrangement that is generated by a calculation characterized by all-around.A straightforward technique for creating a numerical sequence is utilizing at least one seed value and proper recursion connection. One of the known numerical sequences is the additive sequence which is created by the recursion connection.

$$A_{n+2} = A_{n+1} + A_n - - - (1)$$

This happens when each term refers to the inclusion of two previous terms. Two seed values require this sequence,A_0 and A_1. The straightforward case $A_0 = 0$ and $A_1 = 1$perhapsviewedas a model. This allows the sequence as follow:

$$0, 1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, 144, \ldots \quad (2)$$

By adding the recursion relation, this series can be extended indefinitely.It could also be applied to negative index numbers by applying a recursion relationship based on (Eq. 1) to the number shown in (Eq. 2), giving a sequence that continues indefinitely on both routes.;

$$\ldots 34, -21, 13, -8, 5, -3, 2, 1, 0, 1, 1, 2, 3, 5, 8, 13 \ldots \quad (3)$$

Even though they change in sign, In this case, the values of the negative index terms are numerically equivalent to the corresponding positive index terms.
This is an interesting property of this particular additive sequence, although it is not a property of additive sequences in general.

A simple numerical sequence, known as the geometric sequence, generated by the recursion relation, is also a geometric sequence.

$$A_{n+1} = aA_n \ldots 3 \ldots (4)$$

Each term is a constant factor multiplied by the last term.This sequence can be generated on the basis of one seed number and the constant factor number.A basic instance uses$A_0 = 1$ and $a = 2$. Which gives the well-known sequence of powers of 2.

$$1, 2, 4, 8, 16, 32, 64, 128, \ldots \quad (5)$$

It is also possible to expand this series to negative index numbers here.

$$\frac{1}{32}, \frac{1}{16}, \frac{1}{8}, \frac{1}{4}, \frac{1}{2}, 1, 2, 4, \ldots \quad (6)$$

The major contrasts amongst additive and geometric sequences is shown in the correlation of (Eq. 2) and (Eq. 5). Be that as it may, these distinctions are the outcome of the specific decision of the multiplicative consistent in (Eq. 3). Extreme merent results are given from the amount of distinctive decisions. In some case, for instance, The probability of a sequence being both additive and geometric might be; which might fulfil both (Eq. 1) and (Eq. 3). To yield a compelling relationship, these two equations can be consolidated. Out of (Eq. 3) We're able to write

$$A_{n+2} = aA_{n+1} = a^2 A_n \ldots (7)$$

(Eq. 1) and (Eq. 6) gives the below equation

$$a^2 A_n = aA_n + A_n \ \ldots (8)$$

Or straightforward:

$$a^2 - a - 1 = 0 \ \ldots (9)$$

The name of this equation is the Fibonacci quadratic equation and to give the two roots, it's clearly solved.

$$a_1 = \frac{1 + \sqrt{5}}{2} = \tau \ldots (10)$$

And

$$a_2 = \frac{1+\sqrt{5}}{2} = \frac{1}{\tau} \quad \dots (11)$$

A geometric sequence that fits with the figure of $\boldsymbol{a_1}$ as a constant factor and a seed value of (for example) $A_0 = 1$. is very easy to construct.

$$1, \tau, \tau^2, \tau^3, \tau^4, \tau^5 \dots (12)$$

Expanding this to negative indices provides

$$\dots, \tau^{-3}, \tau^{-2}, \tau, 1, \tau, \tau^2, \tau^3, \dots (13)$$

From (Eq. 11), utilizing the seed numbers of $A_0 = 1$ and $A_0 = \tau$, Using the recursion relation of (Eq. 1), a corresponding additive sequence could be generated. For positive and negative indices this sequence is given as

$$\dots, -3\tau + 5, 2\tau - 3, -\tau + 2, \tau - 1, 1, \tau, \tau + 1, 2\tau + 1, 3\tau + 2, \dots (14)$$

In terms of numbers, the values in this series are the same as in the geometric sequence in the (Eq. 12). When these terms are equated betweenpowers and linear expressions in τsome interesting relationships are given.A couple are shown below:

$$2\tau - 3 = \tau^{-3}$$

$$-\tau + 2 = \tau^{-2}$$

$$\tau - 1 = \tau^{-1}$$

$$1 = 1$$

$$\tau = \tau$$

$$\tau + 1 = \tau^2$$

$$2\tau + 1 = \tau^3 \dots (15)$$

Usually, powers of the golden ratio could be saw as

Table 1. In the relationship given by (Eq.16), some coefficients and exponents.

n	a_n	a_{n+1}
-8	-21	34
-7	13	-21
-6	-8	13
-5	5	-8
-4	-3	5
-3	2	-3
-2	-1	2
-1	1	-1
0	0	1
1	1	0
2	1	1
3	2	1
4	3	2
5	5	3
6	8	5
7	13	8
8	21	13

$$a_n\tau + a_{n-1} = \tau^n \qquad (16)$$

If the $a_n$coefficients as shown in Table 1 are the A_n of the additive series in(Eq.3).

The other root of the quadratic equation can be derived from another sequence that is both geometric and additive, as given by (Eqs.11 and 15),$a_2 = -\tau^{-1} = 1 - \tau$.The sequence is given by this.

$$\dots, -\tau^3, \tau^2, -\tau, 1, -\tau^{-1}, \tau^{-2}, -\tau^{-3}, \dots \qquad (17)$$

and In order to be defined, the equivalent sequence based on the additive recursion relationship is

$$\dots, -3 - \frac{2}{\tau}, 2 + \frac{1}{\tau}, -1 - \frac{1}{\tau}, 1, -\frac{1}{\tau}, 1 - \frac{1}{\tau}, 1 - \frac{2}{\tau}, 2 - \frac{3}{\tau} \dots \quad (18)$$

equating terms from (Eqs. 17 and 18) permits for the derivation of relations of the equation

$$a_{n+1} + \frac{a_n}{\tau} = \tau^n \quad (19)$$

This demonstrates that the coefficients are as well the terms in the additive sequence of the (Eq. 3). These expressionscan be shown to be algebraically the same to those of (Eq. 16) by multiplying both sides of (Eq.19) by τ.

As mentioned before, the relation of the golden ratio to certain important characteristics of numbers with regard to numerical sequences shows. If a geometric strategy is adopted, it is possible to obtain additional knowledge of the properties of the golden ratio. This is the case of the golden ratio that is responsible for the philosophers' curiosity and the root of where it got its name, the golden ratio. (Fig. 1) outlined shows a considered line from point A to C that is portioned by point B so The portion of the length of the two parts corresponds to the proportion of the length of the longer segment of the entire line. If the AB length is set arbitrarily to equal to 1 and theentire line length is called, x then the section$BC = x - 1$ and the length proportions can be seen as

$$\frac{x}{1} = \frac{1}{x - 1} \quad (20)$$

Or

$$x^2 - x - 1 = 0 \quad (21)$$

The Fibonacci equation is shown below, the roots given by (Eqs. 10 and 2.11); τ and $-1/_{\tau}$in terms of the golden ratio. Clearly, In the sense of this problem, it is the positive root that has possible physical significance. On the other hand, the entireline length could be set to 1 and segment ABcould bearbitrarily called$x$. Then the ratios are shown as$\frac{1}{x} = \frac{x}{1-x}$ (22)

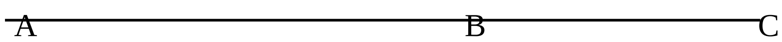


Fig 1.Dividing of the line

Or

$$x^2 + x - 1 = 0 \quad (23)$$

There are roots in this quadratic equation that could be seen in the name of the golden ratio as

$$x_1 = \frac{\sqrt{5}-1}{2} = \frac{1}{\tau} \qquad (24)$$

and

$$x_2 = \frac{\sqrt{5}+1}{2} = -\tau \qquad (25)$$

Also,individually the positive root has physical importance and provides the ratio of lengths to be connected to the golden ratio.

Combining powers of τcan derive the golden ratio, which shows a few fascinating mathematical connections. For instance, a straightforwardexamination of connectionslike those provided in (Eq. 15) and Table 1,It will allow expressions containing both negative and positive golden ratio powers to be derived. The most straightforward of these is

$$\tau^n + (-1)^n \tau^{-n} = L_n \qquad (26)$$

L is shown as an integer that takes on numbers$L_n = 1, 3, 4, 7, 11, 18, ...$ for $n = 1, 2, 3, 4, 5, ...$These are the numbers of the so-called Lucas.This expression is exceptional since it provides that a rational value can be equal to the addition of two irrational values.

Another interesting relationcan begivendirectly from the Fibonacci quadratic formula (Eq. 9) including golden ratio. This might be expressed for τas

$$\tau = \sqrt{1+\tau} \qquad (27)$$

On the right-hand line, substituting the left-hand side for r in the square root yields

$$\tau = \sqrt{1+\sqrt{1+\tau}} \qquad (28)$$

This methodmight be continued indefinitely to give

$$\tau = \sqrt{1+\sqrt{1+\sqrt{1+\sqrt{1+\sqrt{1+\cdots}}}}} \qquad (29)$$

It is alsofamiliar that (Eq. 23)'s positive root is $1/_{\tau}$. This expression could be passed around and thesubstitution in the square root performed indefinitely to achieve the outcome below

$$\frac{1}{\tau} = \sqrt{1 - \sqrt{1 - \sqrt{1 - \sqrt{1 - \sqrt{1 - \cdots}}}}} \qquad (30)$$

The expression in (Eq. 30) provides one way of measuring the golden ratio using a computer to a high degree of accuracy. In any case, it takes less time to calculate X, based explicitly on (Eq. 10) by first finding out the square base of 5. Using a simple iterative scheme, an irrational square root can be measured to an arbitrary precision. It takes different critical math tasks to figure a square root to an accuracy of N figures, which is in proportion to N^2.An prior study of the use of a computer to measure the golden ratio to elevated accuracy gave τ to 4599 decimal points which can be found in Berg, 1966. On an IBM 1401 frame computer, it took about twenty minutes. Nowadays, in about 2 seconds, this measurement can be performed on an IBM Pentium personal computer. The validity of the measured values can easuly be determined. A technique is to replace the determined number of τ in the formula of Fibonacci (Eq. 9) and perform the procedures with the necessary value of the decimal positions and to ensure that the specification maintains. A similar method is to measure the accompanying magnitude and to ensure that $1/_{\tau} = \tau - 1$ keeps to the necessaryaccuracy.

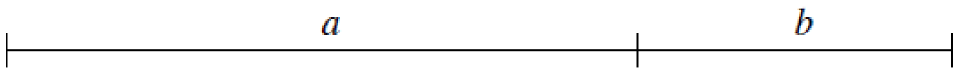
THEOREMS:

Theorem:every number is significant
Evidence:
For the nonnegative numbers that are non-significant, let the organization be U. Presuming that the value U is nonempty, it is limited underneath and, in this manner, has a biggest lower bound a. On the off chance that a is in Uat that point there is a littlest non-interesting value; yet this in interesting. Another interesting is that if a is not in U, then there is no littlest non-interesting number. Along these lines, every single positive valueare interesting. Comparative contentions might be provided for negative values. Consequently, all values are intriguing. While every sense is significant, a few are more significant than others. A good example is that the number is

$$\frac{1 + \sqrt{5}}{2}$$

Our subject will be the relation between mathematics and the golden mean.

Starting with an interesting name in Euclid: “To divide a linesegment in extreme and mean ratio.” Which implies that a line section is split such that the whole is to the greater section as the larger section is to the smaller section.



Designate the largersectionwitha and the smallersectionwith b, this demand is

$$\frac{a+b}{a}=\frac{a}{b}, \quad or \quad a^2=a(a+b).$$

The next formula on the top shows that the geometric mean of the smaller section and the whole is the larger section. Like $x={}^{a}/_{b}$, it demonstrates:

$$x^2=x+1 \qquad (1)$$

The positive root of this formula is the golden mean, commonly seen as $\emptyset$:

$$\emptyset=\frac{1+\sqrt{5}}{2}=1.6180\ldots \qquad (2)$$

It is readily apparent from (1) that the golden mean has the property that adds one generates his square and one creates his inverse by subtracting one:

$$\emptyset^2=\emptyset+1, \;\; \emptyset^{-1}=\emptyset-1.$$

A negative number is another root of the equation (1)

$$\alpha=1-\emptyset=-\frac{1}{\emptyset}=\frac{1-\sqrt{5}}{2}=-0.6180\ldots$$

ConstantGolden Mean Fraction:

The positive root of the equation can be defined as $\emptyset$

$$x=1+\frac{1}{x}$$

Using iteration to solve this:

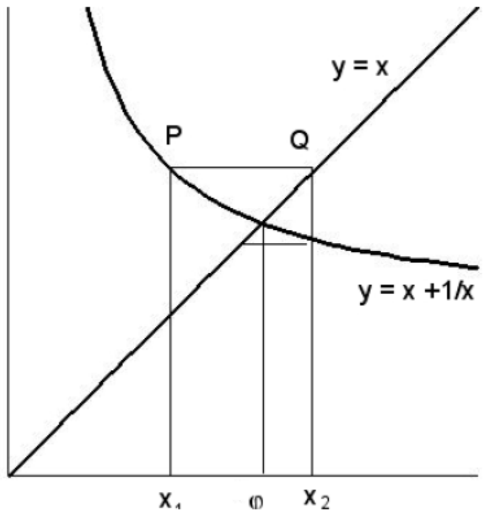


Figure 2: The sequence $x_{n+1} = 1 + {}^{1}\!/_{x}$

$$x_1 = 1 \;\; x_{n+1} = 1 + {}^{1}\!/_{x}\,, n = 1, 2, 3, \ldots$$

Whichgives the sequence

$$x_2 = 1 + \frac{1}{1}$$

$$x_3 = 1 + \frac{1}{1 + \frac{1}{1}}$$

Figure (2) shows this. For the golden mean, this providesan term as an endless continuous fraction including ones:

$$\emptyset = 1 + \frac{1}{1 + \frac{1}{1 + \frac{1}{1 + \cdots}}}$$

An analytic verification that x_n converges to f might be provided like below. Note that

$$|\emptyset - x_{n+1}| = \left|\emptyset - \left(1 + \frac{1}{x_n}\right)\right| = \left|\frac{x_n(\emptyset - 1) - 1}{x_n}\right| = \left|\frac{x_n - \emptyset}{\emptyset x_n}\right| \leq \emptyset^{-1}|x_n - \emptyset|$$

This gives from above

$$|\emptyset - x_{n+1}| \leq \emptyset^{-1}|x_n - \emptyset| \leq \emptyset^{-1}|\emptyset - x_{n-1}| \leq \cdots \leq \emptyset^{-1}|\emptyset - x_1|.$$

Since $\emptyset > 1$it is see that x_n converges to $\emptyset$.

CONCLUSION:

In conclusion, the golden ratio has many uses, significance, and fascination in the world. It occurs in everyday activity and things. The ratio is cherished by numerous people of numerologists, mystics, philosophers, mathematicians, scientists, and even everyday human beings. The golden ratio is very useful in life, it is the foundation of most things, from huge to microscopic.

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